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[*Continued on page* [redacted] *Cover.*

THE HIGH-PRESSURE MERCURY-VAPOUR LAMP IN PUBLIC LIGHTING

By G. H. WILSON, B.Sc.(Eng.), Associate Member, E. L. DAMANT, Commander R.N.(Retd.), Associate Member, and J. M. WALDRAM, B.Sc.(Eng.).

(Communication from the Staff of the Research Laboratories of the General Electric Co., Ltd., Wembley.)

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SUMMARY

This paper records the experience which has now been gained over three years in the use of high-pressure mercury-vapour lamps in public lighting.

The engineering and photometric characteristics of the lamps and the design of the auxiliary apparatus are discussed. The overall characteristics of the lamp and its auxiliaries are given, together with certain peculiarities of street-lighting circuits which affect its performance. The effect of the illuminating engineering aspects upon the realization of the efficiency of the lamp is discussed, and the developments of the theory of street lighting which its use has brought about are indicated. Novel designs of lanterns, which are rendered necessary by the peculiar shape of the source, are described; and special optical, thermal, and constructional problems which arise from the use of the lamps, and also other points which have occurred in practical experience, are recorded.

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INTRODUCTION

The fundamental problems in highway lighting concern not electrical but illuminating engineering. Nevertheless, they bear heavily on the electrical part of any installation and are therefore as important as purely electrical matters to the electrical engineer.

The high-pressure mercury-vapour lamp has not only provided a light source of three times the efficiency previously available with incandescent lamps, but its use has brought about unexpected developments in the knowledge of the mechanism of street lighting and a radical change in the basis on which installations are designed.

The lighting of streets has long been treated as a special case of interior lighting. Lamps have been suspended over the street in lanterns designed as though a street were an elongated living-room. But at the long spacings dictated by economics, and with the peculiar problems of vision concerned, the whole mechanism by which the installation works is quite different. Recent studies have shown that the success which has been achieved hitherto has not always been by design, and the empiricism born of experience is now beginning to be understood and explained. A lamp and its use are equally important; for, unless its light is intelligently controlled, the advantage of its efficiency can be lost.

There have always been popular opinions about lighting matters. To the extent to which it is an art, every individual has his own ideas as to the treatment of a lighting problem. But in public lighting, if not in the more decorative branches of the work, there are fundamental truths connected with visibility into which questions of individual taste and preference do not enter, and every effort should be made to avoid confusion between the science and the art. Beyond a certain point the problem is only aesthetic, and here popular opinion can take its place. Providing the fundamental requirements of a lighting installation are fulfilled, the aesthetic side can be treated as liberally as financial considerations will permit. These considerations and the difficulty of obtaining unanimity of opinion on artistic points are the only limitations to this side of the problem.

The new discharge lamps as applied to public lighting are technical achievements. Their physics has been fully dealt with previously* and mention has been made before the Institution of the developments taking place.† The practical experience gained in the last four years now makes it possible to review the engineering aspects.

Two types of discharge lamp have been used for public lighting. In one the discharge takes place in sodium vapour, and in the other in mercury vapour at high pressure. Although the former was the first type to be used, the installations were abroad. In this country the high-pressure mercury-vapour lamp was first employed and has had the wider application. Over 10 000 lamps are now in use in Great Britain for public lighting alone, and a large number are now being used in addition for industrial lighting. It is the important part which the mercury lamp has played in public lighting and the results of investigatory work on this subject, stimulated by the development of the lamp, which this paper seeks to describe.

(1) THE LAMP AND ITS AUXILIARIES

(a) The Lamp

The high-pressure mercury-vapour lamp‡ consists essentially of a tubular glass vessel containing rare gases at low pressure and a small quantity of mercury. At either end of the tube is sealed an electrode. On the application of a sufficiently large difference of potential between the electrodes, a discharge is made to pass. This discharge raises the temperature of the electrodes and the bulb. This, in turn, increases the vapour pressure of the mercury and, if the pressure is sufficient, the discharge takes the form of a narrow, intensely bright column between the two electrodes.

It was due to J. W. Ryde and his colleagues that this sequence of events was produced in a practical form of lamp capable of operating at high efficiency for a long life on the ordinary supply mains. The first successful lamps were used experimentally for public lighting in June, 1932; and in March, 1933, the first full-scale installation, erected in Wembley with the co-operation of Captain J. M. Donaldson, the North Metropolitan Electric Supply Co., and the Wembley Urban District Council, was put into operation. Since that time improvements in details of construction and performance have continually been taking place, but the lamps have not changed in their essentials.

Fig. 1 shows details of a standard construction. At each end of an inner tubular bulb is sealed an electrode consisting of a stick of rare-earth oxides, held in a tungsten spiral. These oxides are activated during the construction of the lamp by heating the tungsten wire. The bulb contains rare gases, such as argon, at low pressure, and a carefully measured quantity of mercury. In order to ensure that all the mercury is easily vaporized and to obtain an adequate and reasonably constant mercury-vapour pressure, local cooling of the bulb has to be avoided, and it is therefore sealed into a second tubular bulb. This outer bulb contains oxygen at a low pressure, as this has been found to reduce, in some in-

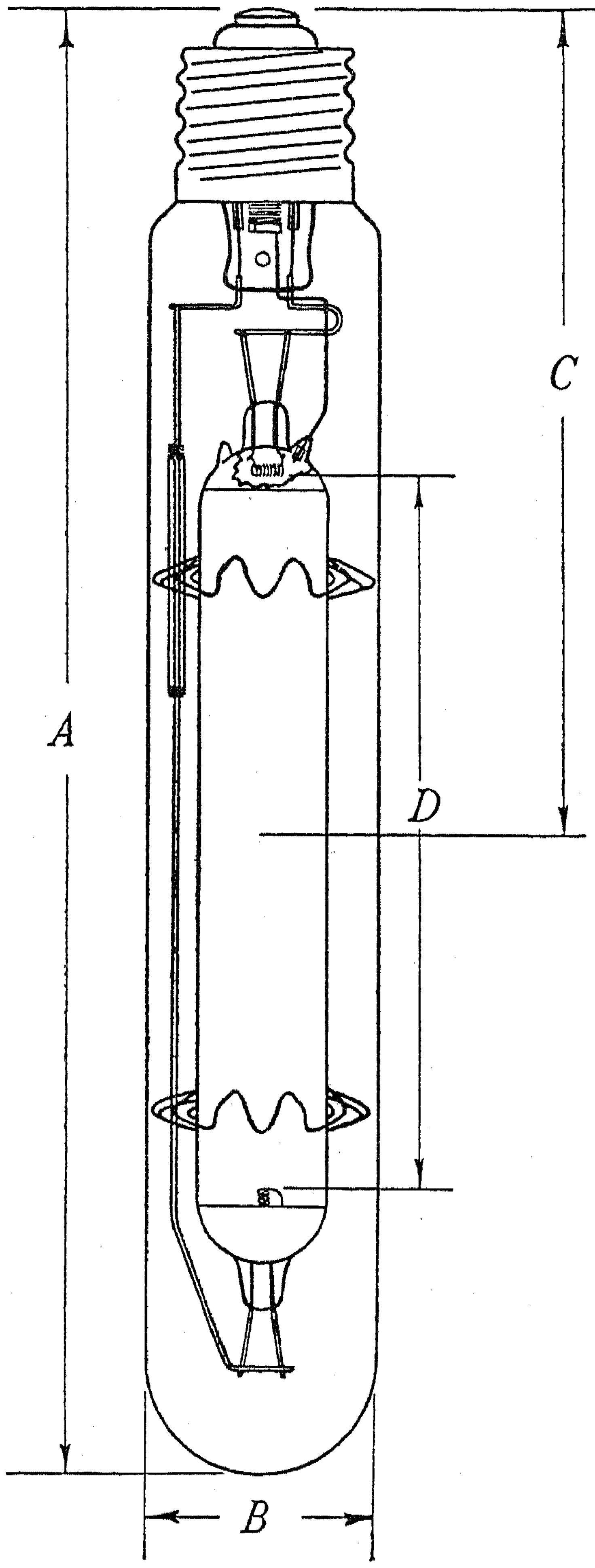


Fig. 1.—Typical construction and dimensions of h.p.m.v. lamps.*

	250-watt	400-watt
A	290 ± 10 mm	325 ± 15 mm
B	50 mm	50 mm
C	177 ± 13 mm	190 ± 8 mm
D	120 ± 5 mm†	160 ± 5 mm†

* For data on 150-watt lamp see page 275.
† Dimension of one manufacturer's lamps.

* J. W. Ryde: *G.E.C. Journal*, 1933, vol. 4, p. 199; also J. W. Ryde: *Journal of the Royal Society of Arts*, 1934, vol. 82, p. 623.
† C. C. Paterson: "Inaugural Address," *Journal I.E.E.*, 1931, vol. 69, p. 1.
‡ Referred to throughout the paper as the h.p.m.v. lamp.

direct way, the formation of an absorbing film on the inside of the inner bulb. It is usual also to apply some metallic coating to the ends of the inner bulb in order to prevent low temperatures at these parts. The two electrodes are connected to the screw cap at the end of the outer bulb.

An auxiliary electrode is provided to facilitate starting. This is usually a wire sealed into the inner bulb near one main electrode. The auxiliary electrode is connected, through a high resistance placed in the foot-tube or upper part of the lamp, to the opposite pole of the cap to that to which the nearby main electrode is connected.

When connected to the supply, the full voltage immediately occurs between the auxiliary and main electrodes. This causes a discharge between these electrodes which initiates the discharge between the two main electrodes, filling the whole tube with light. The electrodes are heated by the bombardment, and as they and the bulb warm up, the mercury is vaporized. As the temperature and vapour pressure rise, the luminous column becomes narrower and brighter.

As the effective resistance of any discharge lamp falls as the current through the lamp increases, it is necessary to employ some form of current control to prevent the lamp current from rising to catastrophic values. A choke is normally used for this purpose on account of the small loss of energy which occurs, and the standardized lamps are constructed for burning on a.c. supplies. Other possibilities for current limitation are discussed on page 247.

The two standardized sizes of lamp which have been developed up to the present have ratings of 400 and 250 watts.* These are the nominal wattages of the lamps themselves. For reasons which will be apparent later, the lamps are constructed for various voltage-ranges.

The dimensions of these lamps have been standardized, and details are given in Fig. 1. In both cases the goliath Edison-screw cap is used.

Photometric and Optical Characteristics.

It is the high luminous efficiency of the new lamps rather than any other one characteristic which has resulted in their wide application. When first introduced, the initial efficiency of the 400-watt lamp was 40 lumens per watt.† As more knowledge of the factors controlling efficiency was gained and manufacturing technique improved, the efficiency was raised. The rate of progress has been rapid, and in the autumn of 1935 the efficiency had risen to 45 lumens per watt. This corresponds to a total light output of 18 000 lumens for the 400-watt lamp. The corresponding figures for the 250-watt lamp are 36 lumens per watt and 9 000 lumens respectively.*

A successful lamp must have a reasonable life. This, on the average, is now not less than 1 500 hours, and many public lighting contracts are based on this figure. During life, blackening gradually occurs at the ends of the inner bulb and the efficiency falls. Conservative figures for the average efficiency throughout life are 80 per cent of the initial value for the 400-watt lamp and 85 per cent for the 250-watt lamp.*

* On the 1st April, 1936, the 150-watt lamp was introduced. For data on this lamp see page 275.

† The figures of efficiency quoted are based on the wattage in the lamp. Losses in the auxiliaries will give a figure for the overall efficiency of the complete unit, about 2 lumens per watt lower than for the lamp alone.

The other characteristics of particular importance to the illuminating engineer are the size and brightness of the source. The dimensions are controlled in the main by the luminous efficiency, the maintenance of light output during life, and the stability of the discharge. Variations in the spacing of the electrodes, for example, will improve some of these factors at the expense of the others, and the final design is a matter of compromise.

The light source in these lamps is the narrow, bright column between the electrodes. The average brightness of this column, which is closely the same for both ratings of lamp, is approximately 1 candle per mm^2.* As would be expected, the column is brighter at the middle than at the edges, the values across the arc stream midway between the electrodes of the 400-watt lamp varying as shown in Fig. 2. Towards the electrodes the column becomes narrower than at the middle, and the brightness distribution is also somewhat altered.

The lengths of the light columns are 160 mm and 120 mm for the 400-watt and 250-watt lamps respectively. An accurate description of the width of the column can-

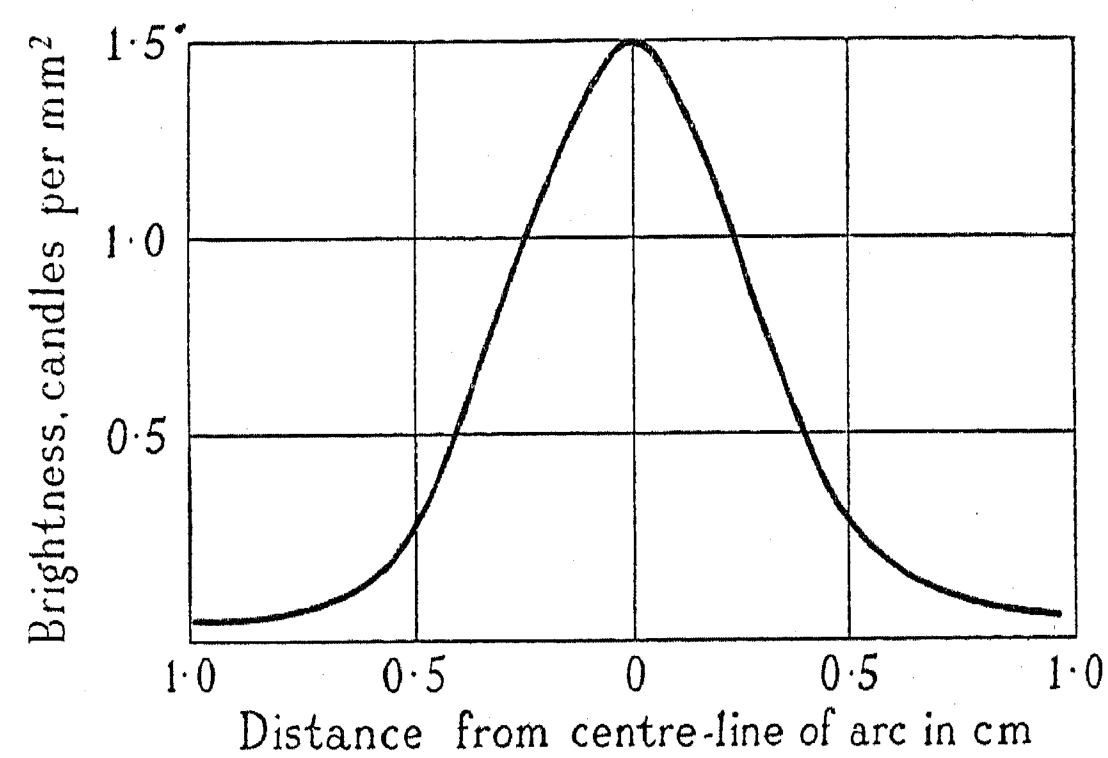


Fig. 2.—Distribution of brightness across arc stream of 400-watt lamp.

not be given, as its boundary is not clearly defined. The application of a common definition employed for allied problems in illuminating engineering suggests that the edge of the column could be considered to be the place at which the brightness is one-tenth of the maximum. By this definition, as will be seen from Fig. 2, the width of the column in the 400-watt lamp is approximately 12 mm and the corresponding width in the 250-watt lamp is 9 mm. It is interesting to note that when the lamps are viewed casually at close quarters, the effectively bright part of the discharge appears to be considerably narrower than the above figures suggest.

It will be seen that in length and width of light column the 250-watt lamp has dimensions three-quarters of those of the 400-watt lamp. As the brightness is substantially the same for the two, the intensity in any direction and the light output will be a function of the projected area of light source, and the intensity of the smaller lamp will be $(0 \cdot 75)^2$ or approximately half that of the larger.

The light distribution from the lamps, shown in Fig. 3 as a polar curve in a plane containing the lamp axis, has a form which would be expected from the shape

* Compare with brightness of:—
Tungsten filament of 500-watt lamp = 5 candles per mm^2.
Pure carbon arc = 150 candles per mm^2.

of the source. The mercury discharge radiates as though it were an incandescent solid, and the luminous intensity is therefore proportional to the projected area in any direction. If the lamp is considered to be in its usual position with the axis vertical (actually the only position in which it can burn without risk of failure unless a magnetic deflector is used), the curve shown refers to a vertical plane. It will be seen that the intensity is a maximum in a horizontal direction and falls off as the angle from the horizontal increases, having a low value in the direction of the downward vertical (if the lamp is considered to be burning cap-up) and zero in the opposite upward direction. As drawn, the curve relates to a 400-watt lamp giving 18 000 lumens. To convert the values to those applying to the 250-watt lamp, the ordinates should be halved.*

Twice in every cycle the arc intensity falls to a low value on account of the reduction in voltage. The period of low intensity is short, however, and on a 50-cycle supply at street-lighting levels of illumination no flicker is perceptible. Even on a 25-cycle supply no flicker can be seen under street-lighting conditions, although it is perceptible at higher brightness levels.

The greater part of the light emitted by the lamp consists of line spectrum radiation. Although, as will be seen from Fig. 4, the main lines occur in similar positions throughout the spectrum to those from the low-pressure mercury-pool lamp, their relative intensities are different. In the low-pressure lamps the green line at about 546 mμ is considerably stronger than the yellow-green lines at about 577–9 mμ. The higher mercury-vapour pressure in the modern lamp causes the yellow-green lines at about 577–9 mμ to be the stronger, and it also introduces some continuous radiation.

The colour of the light from the high-pressure lamp may appear white, blue, blue-green, or green, depending on the previous history of the observer's eyes, his distance from the lamp, the proximity of other light sources, and the colour of the surroundings. It is a common experience that after a few minutes' acquaintance with an installation the colour appears white, but blue-green is probably the best general description which can be given for comparison with other sources. The colour rendering of blues, greens, and yellows, under the light from the lamps is good, but red colours look brownish on account of the small red component in the radiation.

Early in the application of the lamps, some potential users were apprehensive of the colour effects produced, especially in shopping and residential areas. Colour modification was achieved by the use of auxiliary tungsten-filament lamps,* by the introduction of cadmium and zinc into the mercury lamp as described below, and recently by the incorporation of a filament in the mercury lamp itself as described on page 247. All these methods involve a reduction in the overall efficiency of the unit, and experience has shown that in practice the colour has proved no drawback. The wide application of the unmodified lamps to all classes of thoroughfare bears witness to this fact.

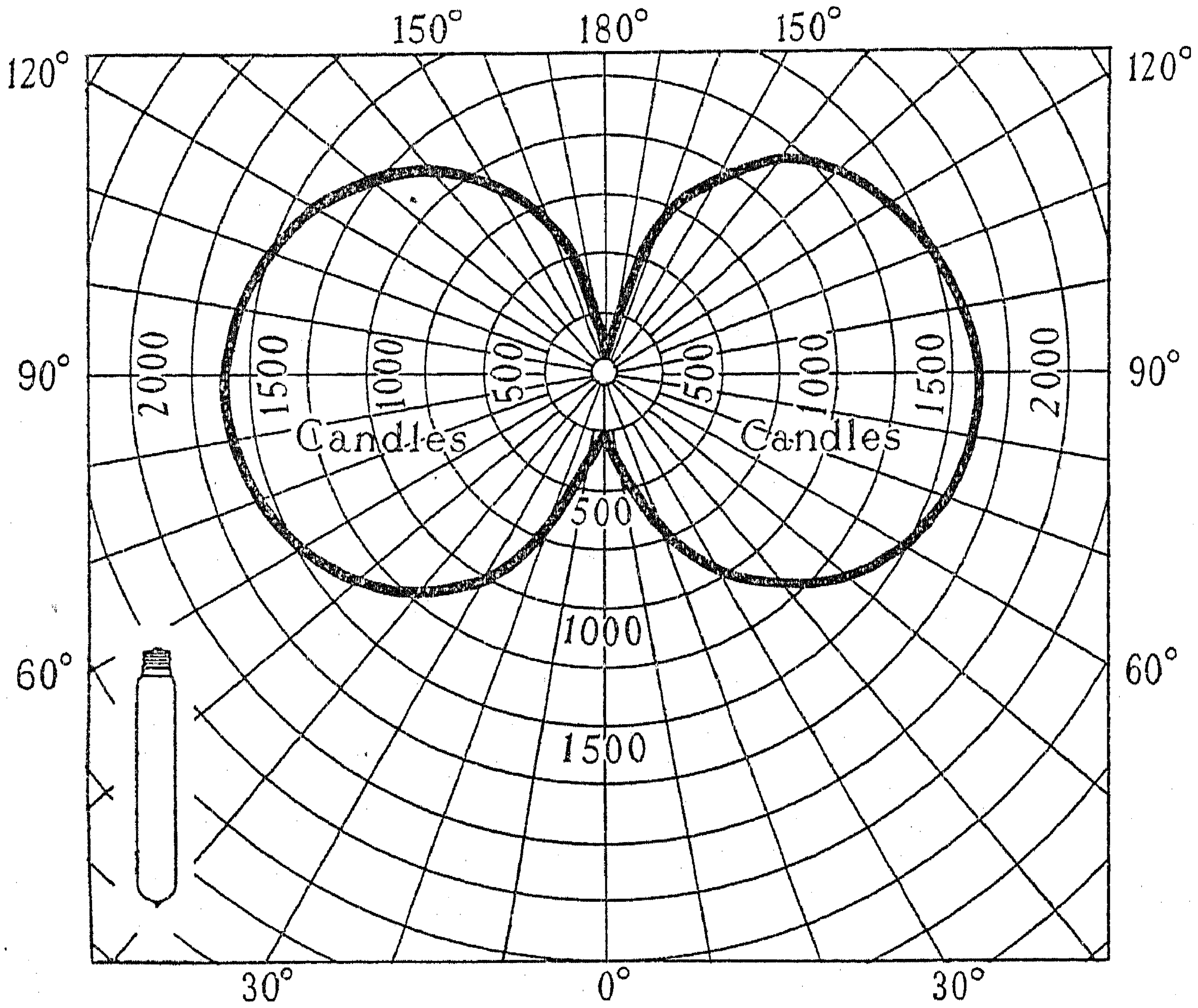


Fig. 3.—Polar curve showing light distribution in a vertical plane from 400-watt h.p.m.v. lamp: light output = 18 000 lumens.

Further, the experimental data available show that in public lighting the colour quality of the light from these lamps does not appreciably affect the ability to see.† This work has confirmed the impression of many observers who have compared installations of incandescent, mercury-vapour, and sodium-vapour lamps.

The introduction of cadmium and zinc into the lamp adds a small amount of red and a good deal more blue to the light emitted.‡ Such lamps are available com-

* For data on 150-watt lamp see page 275.

* G. H. Wilson: Association of Public Lighting Engineers, Margate, 1933.
† K. M. Reid and H. J. Chanon: *General Electric Review*, 1935, vol. 38, p. 580.
‡ J. W. Ryde: *Journal of the Royal Society of Arts* 1933, vol. 82, p. 624.

mercially in the 400-watt size, with an initial efficiency of 37 lumens per watt.

On account of the colour of the light emitted, the question of fog penetration has frequently been raised. An experiment was conducted in a dense fog to compare the transmission of the light from an h.p.m.v. lamp with that from an incandescent-filament lamp having the same luminous intensity. The distances at which both light sources disappeared in the fog were practically identical. Such a result in dense fog would have been anticipated from the known light-scattering characteristics of fog particles. Various workers have found that in a dense fog the transmission of all wavelengths for visible light is approximately the same,* and that it is only in light mists that the blue end of the spectrum is transmitted less than the red. It is on this account that in a light haze distant filament-lamp sources appear yellower or redder and the mercury-vapour lamps greener than they do at close range. In public lighting, the problem of fog penetration does not occur in a light haze, however, and for practical purposes there is therefore nothing to choose between light sources of different colours.

The arc discharge emits a considerable quantity of short-wave ultra-violet radiation, but this is absorbed by the glass envelopes of the lamp. The shortest wavelength emitted is about 312 mμ, which is well outside the biologically harmful range.

Physical Characteristics.

The choice of rating which caused the 400-watt lamp to be selected as the first type for production was a matter of considerable importance. It was known that, within limits, the efficiency increased regularly with the wattage of the lamp. At a wattage of 400 or 500 there was a prospect of an efficiency of about $2\frac{1}{2}$ times that of the equivalent tungsten-filament lamp. With this gain in efficiency, a lamp was available, at a consumption of only 400 watts, having a light output about equal to that of a 1 000-watt filament lamp and therefore capable of producing first-class public lighting, as judged by modern standards, when used at common spacings of 100–200 ft. This rating of lamp was therefore first chosen, and the 250-watt rating has followed as a second type.* Consideration is now being given to other lamp ratings, but in the important applications of public lighting the sizes of lamp which are for some considerable time bound to play the most important part are those giving a light output within the above two ratings.

The electrical characteristics of the lamps have already been given by Ryde,† and it is only factors which affect the design of the auxiliaries and the application of the lamps which need be dealt with here.

The forms of the steady characteristics connecting the voltage V_t across the lamp with its wattage W_t are shown in Fig. 5. For a given external temperature, as the lamp wattage is increased so the mercury vapour pressure rises and the lamp voltage increases, as shown by the curve AB. At the wattage corresponding to the point B, all the mercury is vaporized and, for further increases in W_t, V_t falls slightly. It will be appreciated from this curve that the effective resistance of the lamp decreases with increasing wattage and that the current would therefore increase, with catastrophic effect, unless some current-limiting device were employed. If more mercury were present, the curve AB would continue to extend as shown by the broken line. For other external

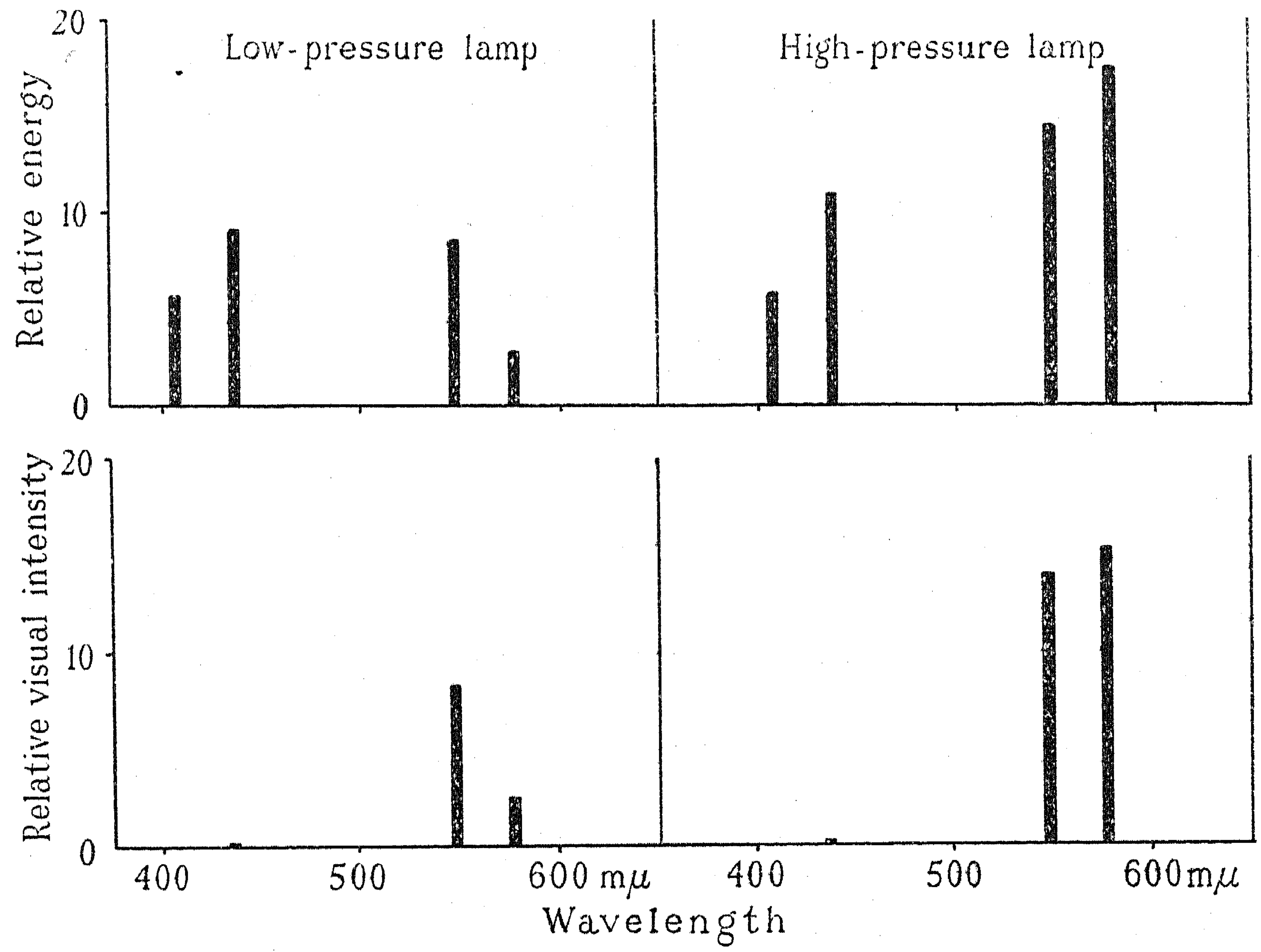


Fig. 4.—Diagrammatic representation of spectral energy and visual intensity distribution for low-pressure and high-pressure mercury-vapour lamps of approximately equal wattage.

* M. G. Bennett: *Illuminating Engineer* (London), 1933, vol. 26, p. 75; also W. S. Stiles: *ibid.*, 1935, vol. 28, p. 125

* The 150-watt lamp was introduced on the 1st April, 1936. For data of this lamp see page 275.

† J. W. Ryde: *G.E.C. Journal*, 1933, vol. 4, p. 199.

temperatures, typical characteristics are shown in the curves CDG and EFG.

In practice the lamp is so designed that when operating normally all the mercury is vaporized. The reason for this will be clear from Fig. 5. When some unvaporized mercury is present in the lamp, the lamp voltage for a given wattage, such as that indicated by the line OH, will change with every variation in the external temperature. If this ambient temperature rose for any reason, V_t would rise and might eventually reach such a value that the supply voltage would be unable to maintain the discharge.

In order to operate a lamp with mercury present and with a reasonable factor of safety under these conditions, it would be necessary to arrange for the value of V_t normally to be low and the current for a given wattage would therefore be high. High current is clearly undesirable and V_t is therefore usually maintained at the highest satisfactory value. By arranging to operate the lamp with all the mercury vaporized, the lamp voltage becomes independent of changes in temperature over wide limits, and the difficulty of determining the normal value of V_t at which the lamp will operate in practice is simplified.

A peculiarity, which has to be considered in the design of the lamps and their auxiliaries, is the extinction of the discharge on a sudden drop in mains voltage below a certain value. If the mains voltage is reduced slowly, the lamp voltage remains practically constant until the point B on the curve in Fig. 5 is reached. After this, a further reduction in the mains voltage causes some mercury vapour to condense and the lamp voltage falls, as shown by the part of the curve AB. If, however, the mains voltage is reduced suddenly, Ryde has explained that the instantaneous characteristic of the lamp and current-limiting device is involved, and the lamp voltage will rise. When the reduction in mains voltage is such that the potential available is below the lamp voltage, the lamp will go out. The maximum permissible voltage-drop for the lamp not to be extinguished is affected by the design of the current-limiting device. In practice the drop permissible is usually of the order of 20 to 40 volts, but the question is more fully discussed under the heading "The Choke."

The wave-form of the current through these lamps does not follow strictly the form of the applied voltage. Although the latter may be sinusoidal, the current wave will be slightly distorted and, whilst the current cannot be said either to lag behind or to lead the voltage, the distortion causes the lamp to have an inherent power factor of about 0·92. This is independent of the effect of any choke coil which may be used as a stabilizer. The question of overall power factor is dealt with on pages 251 and 252.

It will be shown later that when the high-pressure lamps with their relatively long light sources are burned vertically, it is comparatively easy to obtain a light distribution well suited to street-lighting requirements. There are, however, some advantages to be gained from burning the lamp horizontally, as described on page 258. As the discharge in the high-pressure lamp takes the form of an arc, there are limitations to the positions in which the lamp can be operated in safety without auxiliary apparatus. When the electrodes are one above the other, the arc stream is vertical, but if the axis of the

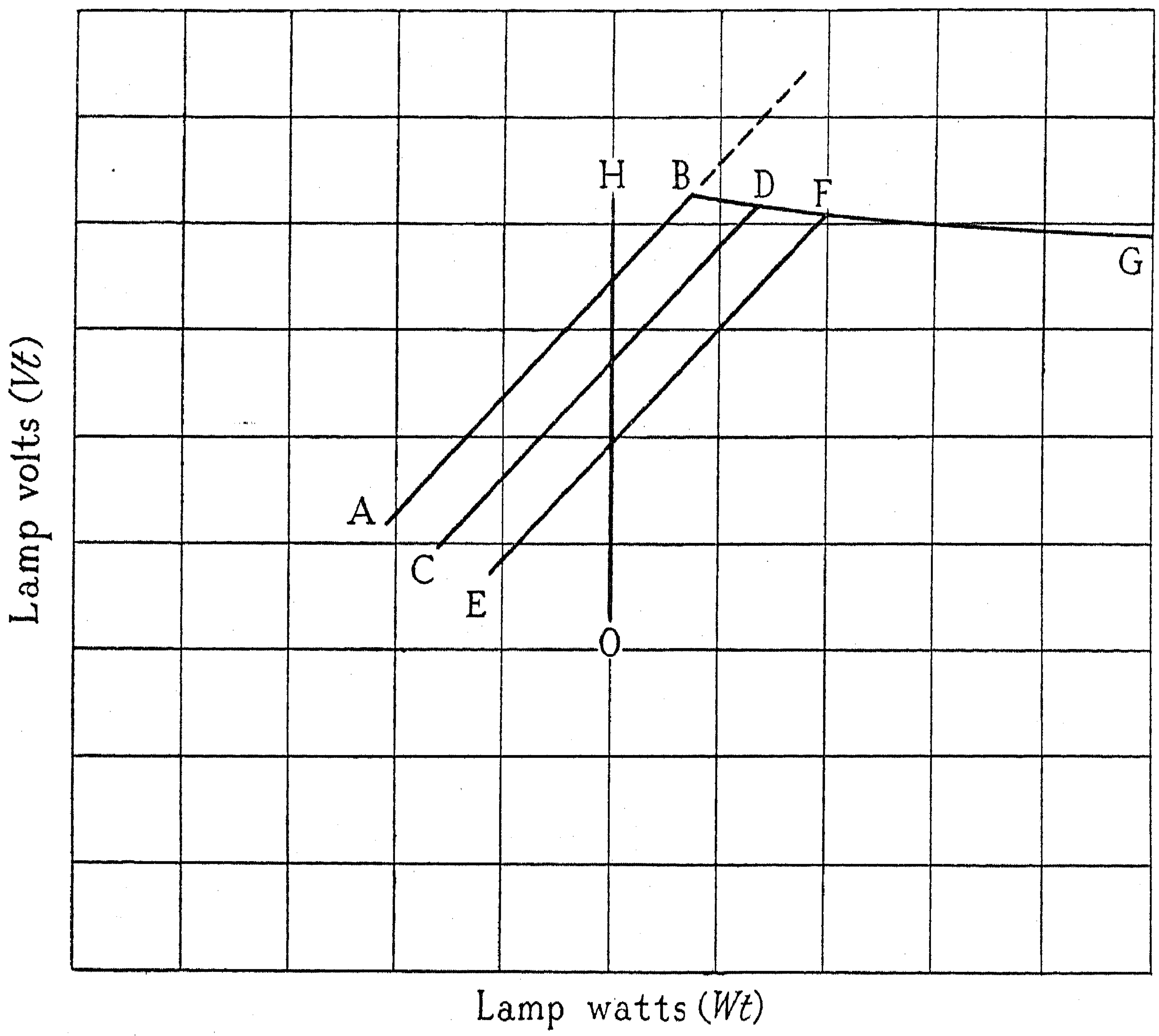


Fig. 5.—Relation between lamp watts and lamp volts, showing effect of external temperature.

lamp is inclined, the arc tends to bow upwards on account of the rising convection currents in the hot mercury vapour. If the lamp is tilted too far, the upward bow of the arc will soften the inner glass tube and a bulge will result from the difference in pressure between the inner and outer bulbs. In practice a 5° tilt is the maximum which should be permitted.

The temperature attained by various parts of the lamp, and the heating effects on the unit surrounding it, are matters of importance. The 400-watt h.p.m.v. lamp radiates approximately 20 per cent of the total energy input as visible light, and a further 10 per cent as ultra-violet and near infra-red. This radiation, together with the visible, will pass through the glass bulbs and the glassware of any fittings, with practically no absorption and, consequently, no heating. The remaining energy is longer-wavelength infra-red radiation from the inner bulb of the lamp, which has a maximum temperature, in the case of the 400-watt lamp, of the order of 500–600° C. Much of this energy is absorbed by the outer bulb, which has the property, common to all glasses, of strongly absorbing long-wavelength radiation. As a result, the final radiator of heat becomes the outer glass, which runs at a maximum temperature of about 350° C. in air. Any glassware used round the lamp will absorb a high proportion of the long-wavelength radiation from the outer bulb, and will actually attain a higher temperature than if the source were a filament lamp of the same wattage as the discharge lamp. The reason is that, although the percentage of total energy radiated as heat with the two lamps is not very different, that from the incandescent lamp is emitted mainly by the filament, which in a 500-watt lamp operates at about 2 850° C., whilst that from the discharge lamp comes from the bulb operating at a temperature of about 350° C. The longer-wavelength heat radiation from the bulb is much more absorbed by any surrounding glassware than the shorter-wavelength radiation from the filament. This is a point which needs attention in the design and construction of any fittings for the lamps.

One other factor is affected by the lamp operating temperature—the time to restrike in the event of an interruption in the supply. Before a lamp will restrike, sufficient mercury vapour must be condensed for the vapour pressure to fall to the required value. The hotter the surroundings of the lamp, the longer the cooling period. A well-glass fitting, in which the glass takes the form of a closed cylinder 6 in. in diameter and 13½ in. long with a hemispherical bottom 2½ in. below the bottom of the lamp, results, on the average, in a restriking time of 5 minutes, as compared with 3 minutes for the lamp in still air. In these tests a 400-watt lamp was used and the ambient air temperature was 15° C.

(b) The Choke

Under practical conditions the most usual application of discharge lamps is on an a.c. distribution system of nominally constant potential, in which case it is necessary to use some form of impedance in series with each discharge lamp. The impedance may be an ohmic resistance or, on a.c. supplies, a choke coil.

It will be shown later that the voltage-drop across the impedance should preferably be of the same order as that across the discharge lamp. The wattage loss in a resistance will therefore be approximately the same as the lamp wattage, and the overall efficiency of the unit as a source of light will be reduced to about half value if a resistance is employed. If, as is possible, the resistance takes the form of an independent glowing filament, the overall efficiency will again be lowered, but only to about 60 per cent instead of to 50 per cent of that of the discharge lamp.

In a lamp recently developed, the auxiliary filament has been incorporated below the discharge tube, in the same outer bulb. An automatic switch, operated by the heat from the discharge tube, short-circuits half the filament when the lamp is partially run up, so that the current is limited to appropriate values both for starting and for running. The use of the filament as the current-limiting device avoids the use of a choke and condenser, and at the same time produces some colour modification by the admixture of white light. The lamp is made in the 500-watt rating and has an overall initial efficiency of 25 lumens per watt.

It has been found in practice that, on the usual a.c. supply systems which are being standardized in this country, a choke coil forms the most economic and satisfactory impedance for use with h.p.m.v. lamps. Its design can be arranged to give desirable starting and running conditions and the wattage loss can be kept low.

In the consideration of the combined characteristics of lamp and impedance, it is therefore necessary to have in view the characteristics of the modern h.p.m.v. lamp using a properly designed series choke coil as a stabilizer.

Combined Characteristics of a Lamp-choke Unit.

A discharge tube alone has, for the engineer, no independent existence, and consideration must always be given to the complete lamp-choke unit.

For example, the permissible over-voltage of the unit may be limited by the temperature-rise in the series impedance as much as by the effect produced in the discharge tube itself. Even the latter effect will be dependent upon the voltage/current characteristic of the impedance.

The consideration of the combined characteristics is complicated by the fact that the current in the lamp, and therefore in the choke, is not sinusoidal. As a result the ordinary vector calculations do not apply, and the current, voltage, and wattage in the various parts of the circuit, cannot be accurately related by the usual methods of calculation. Crude approximations can be obtained by assuming voltages and currents of sine form, or even by considering the case of an ohmic resistance as the series impedance, but more detailed calculations are required to obtain adequate accuracy.

Current/Voltage Characteristics of the Choke.

The first step in the design of the lamp-choke unit must be to arrange for the choke to have a current/voltage characteristic satisfactory both for starting and for running conditions. The characteristic of the h.p.m.v. lamp at starting is different from that under steady running conditions. The voltage across the

lamp is much lower when starting, and the choke characteristic must suit both sets of conditions.

If the current in, and the voltage across, the choke are related somewhat as shown in Fig. 6, the choke will be operating at some point such as S under starting conditions, and at another point such as R when the lamp has settled down to steady burning.

The objective in the design of the choke will be to fix the points R and S, and, further, to regulate as far as possible the slope of the curve at R and S so as to give the best operating characteristics under starting and running conditions.

Current and Voltage under Steady Running Conditions.

Under steady running conditions, the voltage across the lamp for different values of lamp wattage is of the

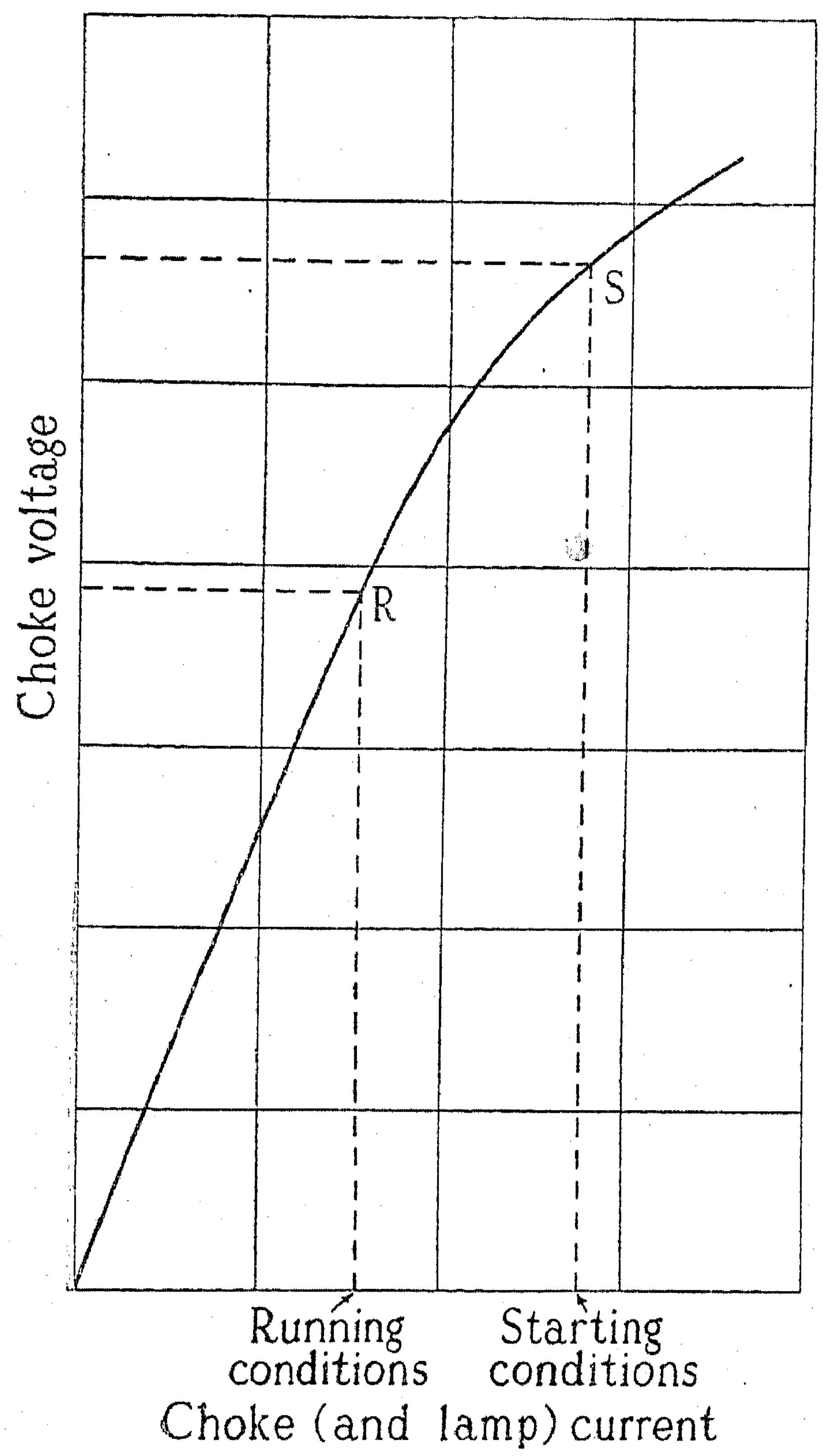


Fig. 6.—Current/voltage characteristic of choke.

form shown by the solid line ABCD in Fig. 7. The running condition must be at some point such as C, for the reasons given on page 246.

The quantity of mercury in the lamp will fix the value of lamp voltage V_t at which the lamp operates, lamps with more or less mercury having steady voltage characteristics such as AEF and AGH respectively. It is possible, with a given mains supply voltage, V_m, to choose alternative values of V_t and of corresponding impedance to give the designed lamp wattage. The designed value of V_t for a given V_m will be chosen according to its effect upon the combined operating characteristics of the lamp and choke. The characteristics affected include the following:—

(1) The variation of lamp wattage caused by small manufacturing variations of V_t on either side of the designed value of V_t.

(2) The variation of lamp wattage caused by variations of mains voltage V_m on either side of the nominal voltage of supply for which the lamp is designed and the choke adjusted.

(3) The maximum permissible sudden drop in mains voltage V_m, which will not cause the lamp to be extinguished.

(4) The power factor of the lamp-choke unit.

The effect of the designed ratio V_t/V_m upon these four relationships is shown in Table 1, with the values usually obtained when a well-designed choke is used in series with the present h.p.m.v. lamp.

The summary indicates that a good compromise and satisfactory operation of the lamp-choke unit will be obtained if a ratio of V_t/V_m is chosen in the neighbourhood of 0·6. This is, in fact, the value usually adopted with the present h.p.m.v. lamps. From the foregoing it is clear that for ideal results a different lamp and choke would be required for each value of mains voltage V_m, those for the higher supply voltages having a high lamp voltage and low current, and vice versa. In this country, the general practice is to cover the usual supply voltages by four different lamp ratings:—

One lamp for 200- and 210-volt supplies with a lamp voltage of about 125 volts.

One lamp for 220-volt supplies with a lamp voltage of about 130 volts.

One lamp for 230-volt supplies with a lamp voltage of about 145 volts.

One lamp for 240- and 250-volt supplies with a lamp voltage of about 155 volts.

For a given lamp, the value of V_t will determine, for a certain supply voltage, the values of choke voltage V_c under running conditions, and the rated lamp wattage will determine the lamp current and therefore the choke current under running conditions. The point R on the curve in Fig. 6 is thus fixed.

Current and Voltage under Starting Conditions.

The next consideration must be the starting period, at the beginning of which the lamp voltage is only about 20 volts and therefore the voltage across the choke practically equal to the supply voltage. The time taken by the h.p.m.v. lamp to reach full brilliance depends upon the lamp wattage during the running-up period. The value of current on the choke characteristic when V_c is practically equal to V_m will consequently determine the starting current and wattage, and therefore the time taken by the lamp to reach full brilliance. The quickest starting is desirable within limits which will be set by the carrying capacity of the supply wiring. This wiring will normally be rated to carry the running current, and a practical compromise is an initial starting current approximately double the running current. This gives reliable starting and an interval of about 4 minutes for the lamp to reach 80 per cent of its full light output. Few wiring systems are rated so closely that they will not stand an overload of 100 per cent falling to 50 per cent within 3 or 4 minutes and to normal

value in a further similar interval. These considerations will fix the point S on the curve in Fig. 6.

Slope of Characteristic Curve.

Alternative characteristics can be designed to pass through the points R and S in Fig. 6.

an undue starting period or too severe an overload on the supply cables.

Construction.

The desirable choke characteristic described is obtained with a choke having a small air-gap in the magnetic

Table 1

Designed ratio V_t/V_m	Percentage change of W_t for a 1 % change of V_t	Percentage change of W_t for a 1 % change of V_m	Permissible sudden drop in V_m for lamp not to be extinguished	Power factor of lamp-choke unit	Designed ratio V_t/V_m
0·1	Very small (1 % or less)	Small (less than 2 %)	Large (Over 20 % of V_m)	Low (0·4 or less)	0·1
0·2					0·2
0·3					0·3
0·4			Satisfactory (about 10 to 20 % of V_m)		0·4
0·5		Satisfactory (about 2 %)		Satisfactory (0·5 to 0·6)	0·5
0·6	Satisfactory (1 or 2 %)				0·6
0·7		Large (3 to 15 %)	Small (less than 10 % of V_m)	High (0·7 or over)	0·7
0·8	Large (3 to 10 %)				0·8
0·9					0·9

The minimum effect of lamp wattage due to small variations in V_t will be given by a choke with a moderately steep characteristic at the running point. The minimum effect due to variations in mains voltage

circuit. The latter may be of either shell or core construction.

The choke as considered up to this point is suitable only for one particular lamp and supply voltage, and one

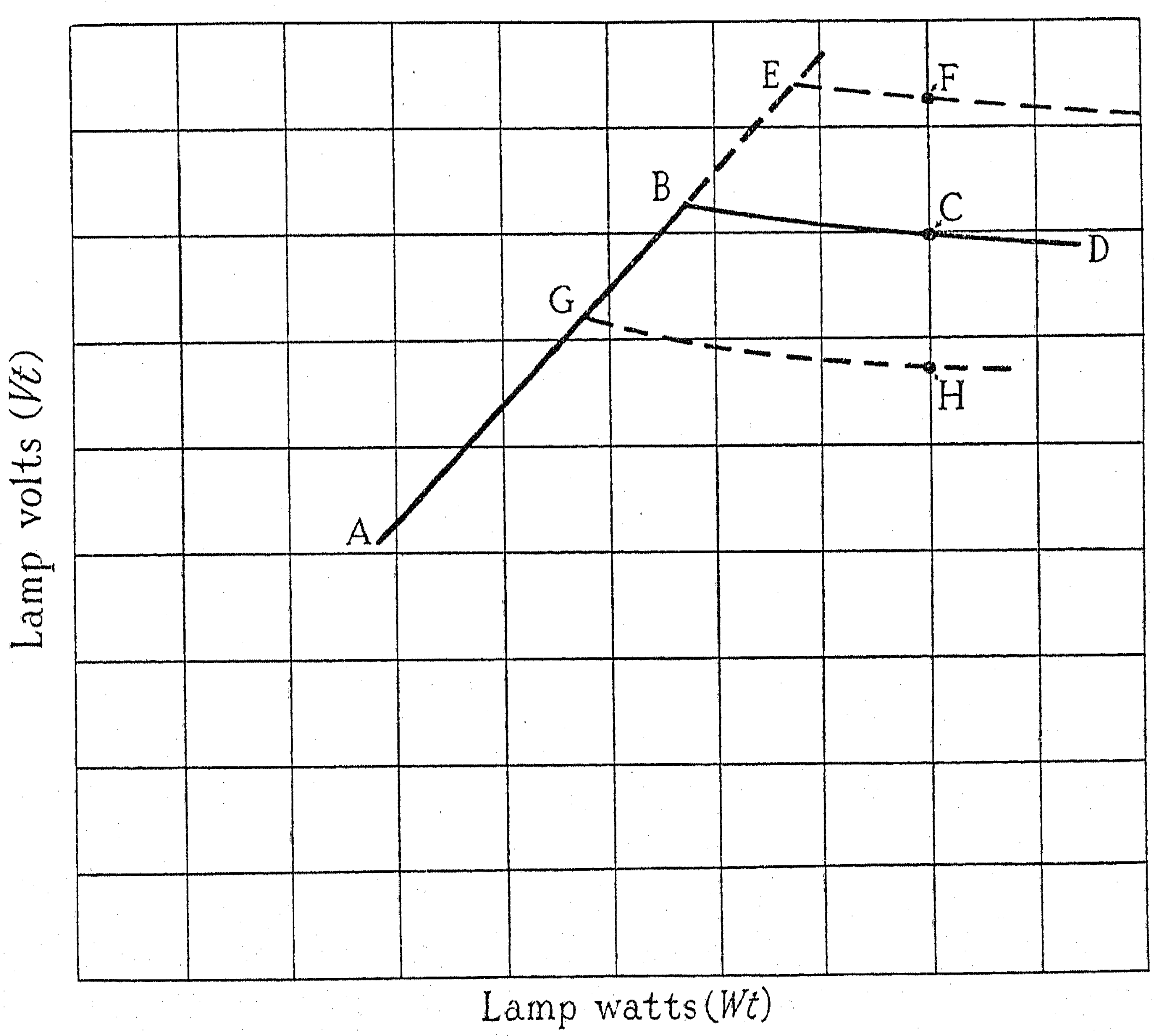


Fig. 7.—Relation between lamp watts and lamp volts, showing effect of the amount of mercury.

V_m will be given by a choke with the steepest possible slope at both the running and the starting points of the characteristic. With a choke running near saturation at the starting current, variations in V_m may cause either

frequency. In practice it is desirable to provide chokes to cover the different 50-cycle supplies operating at voltages between 200 and 250, and making allowance for conditions on different parts of a street-lighting network

where the voltages may differ by an appreciable percentage from the declared value. In these circumstances it is advantageous to provide chokes which can be adjusted with ease and certainty by the user to suit the actual average voltage available at the post during burning hours. Available adjustments to cover $\pm$ 6 per cent on the declared voltage will normally be sufficient.

Adjustment with ease and certainty to the nearest 5 volts is found to meet all requirements outside those of a photometric laboratory. This can be arranged by specified changes of either the air-gap or the number of turns. The latter method, for which there is a sound precedent in standard transformer practice, is more generally adopted. Although it is slightly more expensive in construction it has the advantages of leaving the choke construction rigid and of providing a series of settings of higher repetitional accuracy than can consistently be obtained by air-gap adjustments, because each setting, being checked by the maker, is not subject to errors in adjustment on site.

Changes in the weight of iron, the number of turns, and the air-gap, would be necessary in order to preserve the ideal choke characteristic for each voltage, but in practice a compromise is accepted, and in the case of the tapped choke two or three types are usually designed to cover the range required.

The general size and shape of the choke will be a matter of compromise between choke efficiency and cost. It will also be affected by the space available for mounting such auxiliaries. In street-lighting work it is usual to house the choke and other apparatus in the base of the lamp column, and excessive choke dimensions may necessitate a larger and therefore more expensive pole.

Adjustment and Tolerances.

In chokes of equal dimensions, with the same weight of iron and the same number of turns, there is a spread in inductance owing to inevitable variations in the quality of the iron and the usual manufacturing tolerances.

In the case of tapped chokes, the necessary final adjustment during manufacture may consist of an accurate setting of the air-gap, which must be rigidly fixed to prevent any subsequent alteration. Adjustment of the choke voltage by this method is desirable and possible to within $\pm$ 1 per cent.

As the performance of a discharge lamp is controlled not only by its own characteristics, but also by those of the choke, the spread in the performance of lamp-choke units of nominally the same wattage and light output is governed by the manufacturing tolerances of both the lamp and the choke. It is, therefore, evident that both the design and the manufacture of chokes for discharge lamps need to be carried out in the closest collaboration with the design and the manufacture of the discharge lamps with which they are to be used. Routine tests on chokes, similar to those employed in the manufacture of lamps, are necessary in order to maintain the desirable minimum spread in performance.

Efficiency.

In a well-designed choke the losses will only total about 5 per cent of the lamp wattage. For a given design and temperature-rise this percentage will tend to be larger in chokes for lamps of lower wattage, owing to the greater number of turns on the choke and the consequent extra copper loss. Clearly, the wattage loss can be reduced by a more lavish design, and the designer is faced with the problem of making an economical compromise. The choke loss can be considered as a part of the annual cost of the choke, in which case an optimum design will be that in which the sum of the annual cost of the losses and the annual capital charge and depreciation on the choke itself is a minimum.

Under average street-lighting conditions and existing costs, a figure of 5 per cent for choke loss relative to lamp wattage gives results at about this point of optimum economy. For equal annual cost the price and wattage loss of chokes of varying efficiency can be related.

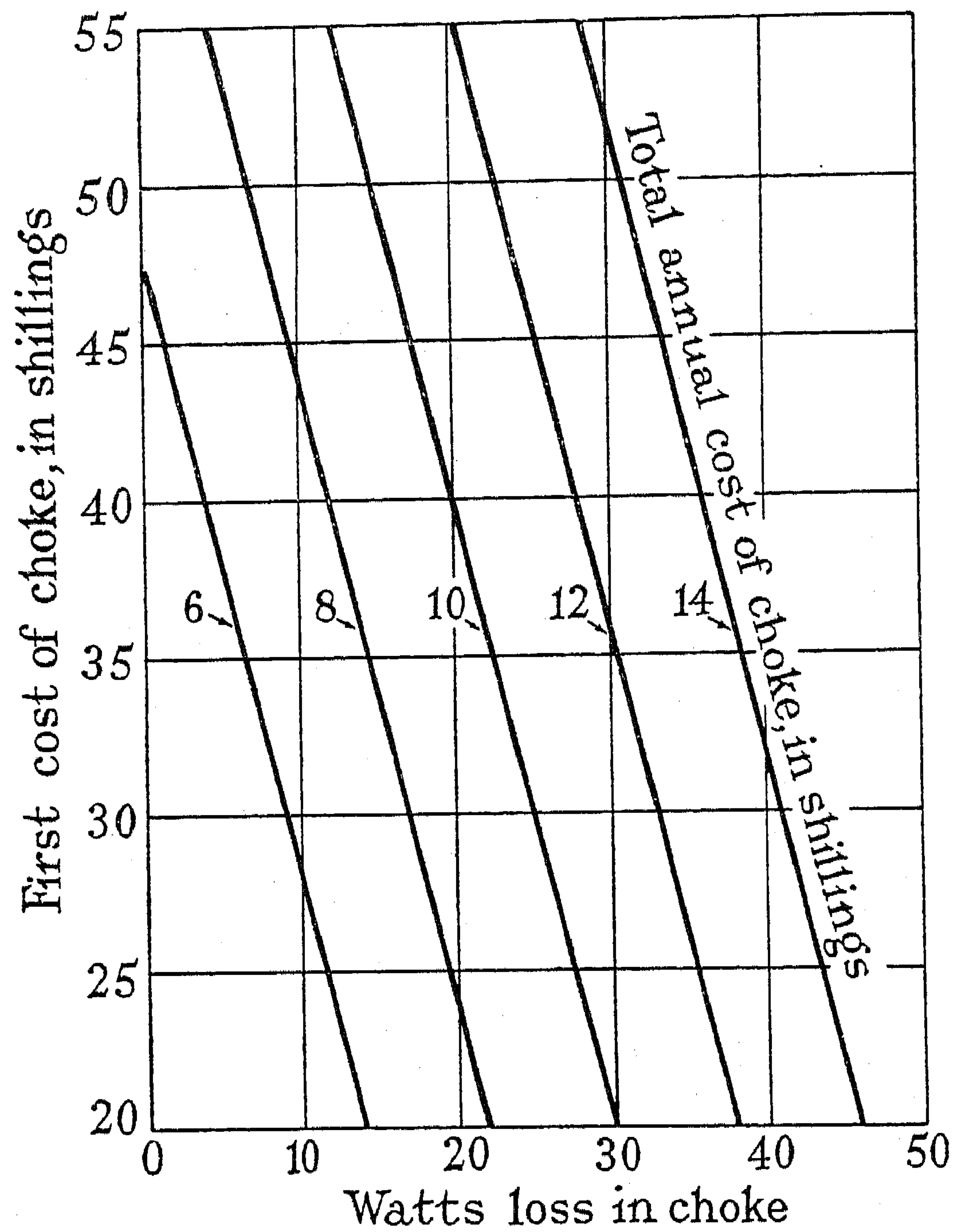


Fig. 8.—Diagram for calculating the effective cost of the choke.

Fig. 8 shows the total annual cost of chokes, which varies according to the capital charges and the cost of the wattage wasted. The diagram assumes 3 000 burning hours per annum, power at 1d. per unit, and the annual capital charges as one-eighth of the first cost. A similar diagram can be constructed for other charges. The sloping lines represent chokes of equal total annual cost; for example, a choke costing 40 shillings and wasting 20 watts and a choke costing 20 shillings and wasting 30 watts will each cost 10 shillings per annum.

General Design.

Experience has shown that a choke, like other street-lighting gear, is often handled more roughly than most electrical apparatus of equal accuracy. It is therefore necessary to construct it to meet these conditions. The design requires to be robust and rigid, and the general arrangement should be such as to protect the windings and tappings from accidental damage.

Insulation.

The insulation of the choke should be of the usual standard for 250-volt apparatus. Oscillograph records have failed to show any abnormal voltage-surges across the windings under operating conditions.

The housing provided for chokes in street-lighting installations may be the base of the lamp column. This is by no means an ideal situation for electrical apparatus. Condensation, dirt, deposits of salt, and, in industrial areas, acid- or sulphur-laden atmospheres, tend to produce corrosion and to break down the insulation. It is found in practice that air-cooled chokes which are vacuum-impregnated with a bituminous compound will withstand such conditions, providing that they are not subjected to water splash or drip which may leave standing water on the windings. The life of the choke will normally be limited by that of the insulation, and the windings and tappings should be insulated by equally durable methods. On account of the conditions of use, there are strong arguments in favour of tough-rubber-sheathed flexibles for connection to the choke, but terminals are also used on some designs.

(c) The Condenser

A condenser is generally used to improve the power factor of the lamp and choke. The size necessary to produce the maximum correction is given roughly by the expression $C = 3\ 185 I/V_m$, where C is the capacitance in microfarads, I the lamp current in amperes, and V_m the mains voltage.

The power factor of the complete lamp-choke-condenser unit will vary under running conditions according to the capacitance of the condenser in use, as shown in Fig. 9. The capacitance shown as 100 is that given by the approximate formula above. It will be noted that lamps designed for lower-voltage supplies will need larger condensers in order to give equal correction of power factor—the value of I being greater and that of V_m less in the formula quoted. In practice it is usual to compromise and to provide one size of condenser for a given wattage of lamp, accepting a slightly reduced power factor with the lower-voltage lamps.

The British Electrical Development Association in 1933 recommended a minimum power factor of 0·8 for discharge-tube signs. Correction to this value can be obtained with h.p.m.v. lamps by the use of a condenser having a capacitance about half that calculated by the formula above (20 μF is commonly used with the 400-watt lamps) and giving a power factor of 0·8 with the lower-voltage lamps and about 0·9 with the highest-voltage units.

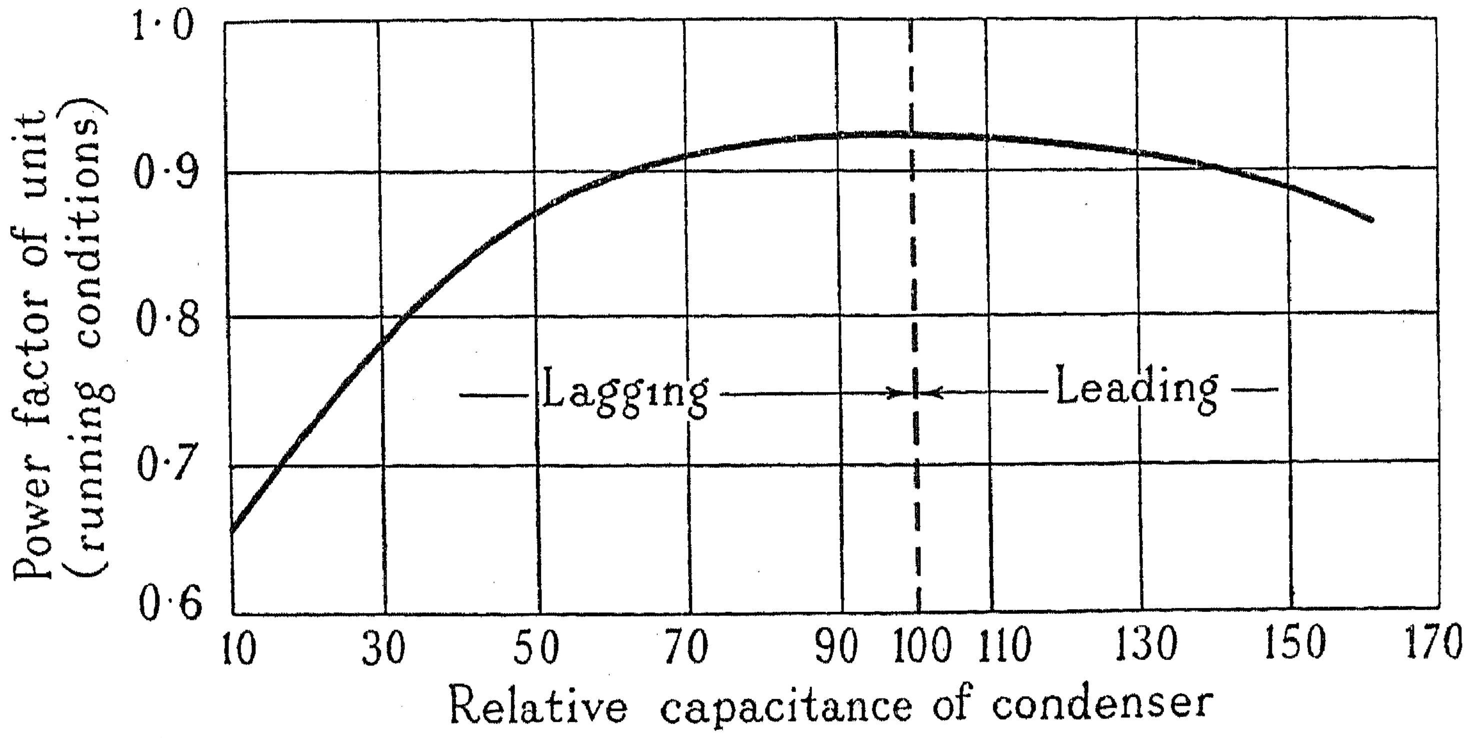


Fig. 9.—Effect of condenser capacitance on power factor of unit.

Condensers to stand up to street-lighting conditions should be totally enclosed and efficiently sealed against damp. The finish needs to be proof against corrosion by salt- or acid-laden atmospheres and humid conditions. They should be capable of standing up to tropical weather, and any wax filling should therefore have a high melting point.

To guard against danger from shock, an internal leak of about $\frac{1}{4}$ megohm is often fitted, which rapidly discharges the condenser when the supply is cut off.

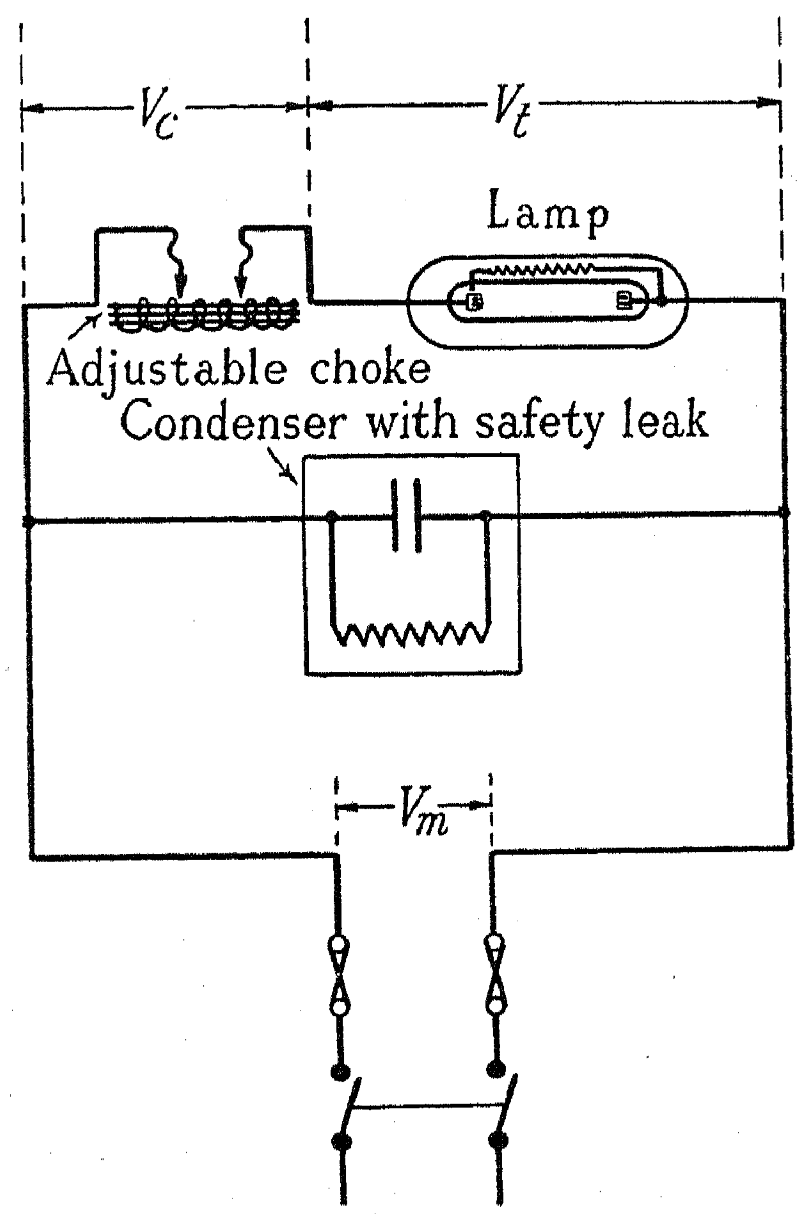


Fig. 10.—Connections of the complete discharge-lamp unit.

The wattage loss in the condenser is negligible, compared with the wattage of the complete unit.

(d) Operation

The connections of the complete discharge-lamp unit with choke and condenser are shown diagrammatically in Fig. 10.

The operating characteristics of the complete unit are different during the periods of striking, starting, and steady running. Immediately the switch is closed the full voltage of the supply is impressed on the main and auxiliary electrodes, initiating the discharge. There is at this instant no current flowing and consequently no voltage-drop in the choke: the voltage available for striking is thus the mains voltage V_m, and is independent of the choke in use or of its setting.

The initial emission from the electrodes will constitute a switching surge of similar magnitude to that experienced for a filament lamp of approximately the same light output as the h.p.m.v. lamp will be satisfactory for the discharge-lamp unit.

The changes in lamp voltage, current, wattage, power factor, and lumen output, during the starting period are shown in Fig. 11. The values, although those for a particular 230-volt lamp, choke, and condenser, are typical. Lamps of different rating have different values of starting current, and these are shown in Table 2, together with the values of current and power factor under steady running conditions.

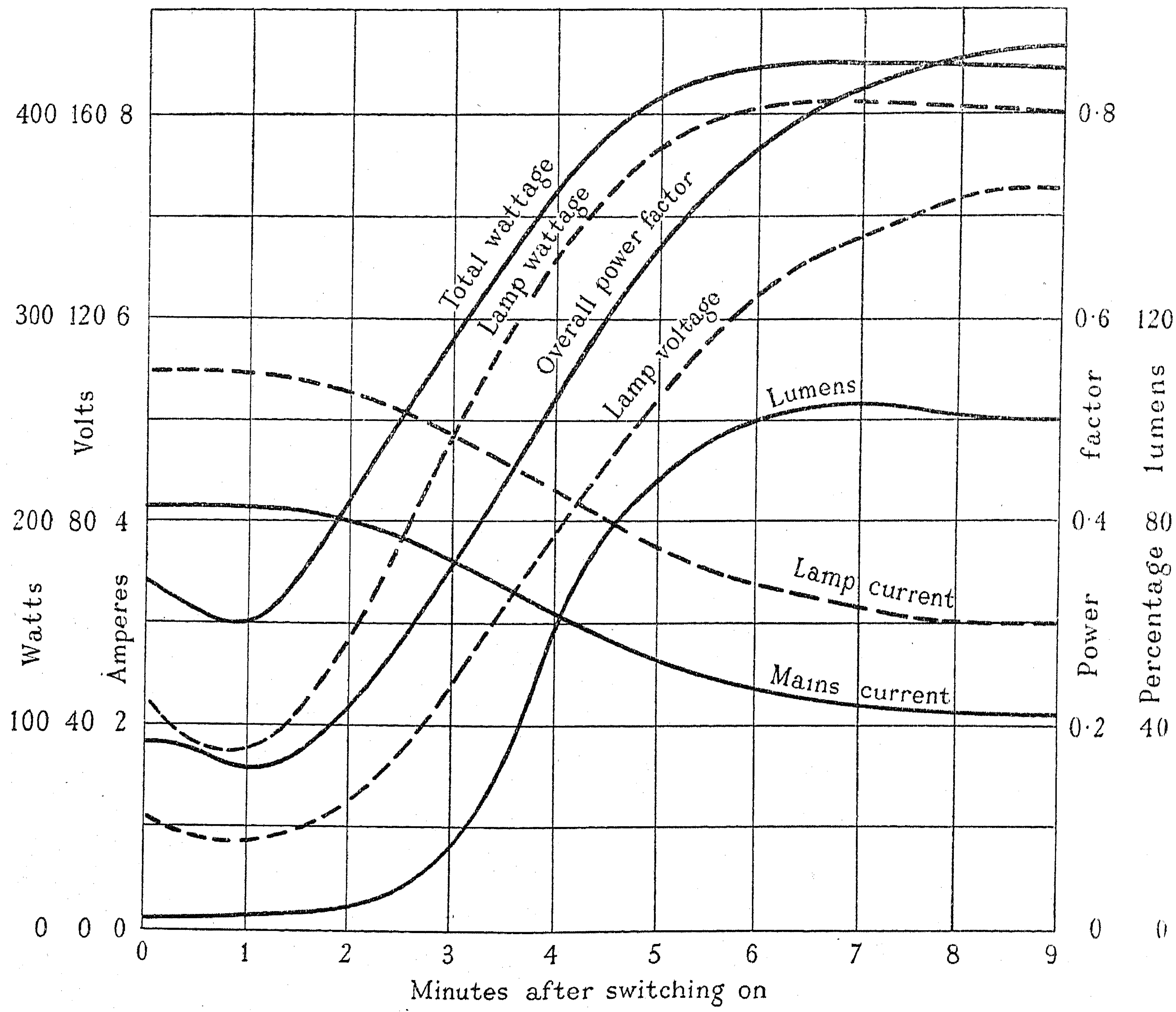


Fig. 11.—Starting characteristics of complete discharge-lamp unit.
230-volt 50-cycle supply.
230-volt 400-watt lamp with choke and 20-μF condenser.

with tungsten-filament lamps, though the causes are different. This surge dies away within a few cycles and can be kept down to a reasonable value by special care in the treatment of the electrodes during lamp manufacture.

Another initial surge of current which will occur is the charging current in the condenser. It will vary in amount according to the point of the supply-voltage cycle at which the switch is closed. It will last for half a cycle at the longest. Both these effects are far too momentary to show to scale on the diagram of starting current and voltage.

It has been found in practice that switchgear suitable

Variations in mains voltage will affect the current and wattage in the circuit, the permissible voltage-drop, the lumen output, the lamp efficiency, and the power factor. Curves showing the changes in these factors for an average lamp are given in Figs. 12 and 13.

(e) The Supply System

Accessory Apparatus and Wiring.

The essential accessories which have to be mounted in connection with each lamp are the choke and condenser, and a switch and fuse. These are often placed in the base of the column, where they are concealed and easily

accessible if necessary; but there is little available space, and adequate protection against water-splash is required. A typical arrangement inside a street-lighting column is shown in Fig. 14. An alternative position, which is in many ways preferable, is in a specially designed box which can be properly ventilated and designed with convenient clearances. Fig. 15 shows the arrangement of the choke, condenser, and fuses, in such a box. The two incoming leads would normally have to be supplied through a switch. It is sometimes suggested that the choke should be placed in the fitting, but, apart from the problems of high temperature, there is danger of the choke field affecting the lamp, and there is seldom room for the condenser as well. To avoid interaction, the choke, if in an iron or steel box, should be placed at least 18 in. from the lamp. If two chokes are arranged in the same box, they should be so oriented that their stray fields do not interact; with some designs this can be achieved by mounting their axes at right angles.

The connection of the components is, of course, of great importance, and cases of trouble from accidental wiring errors—such as condensers being connected on the lamp side of the choke—are not unknown. Adequate space for wiring and for checking connections has been found to be well worth while.

It is desirable that the choke (and the series magnetic deflector if used) should be wired in the phase side, since a fault to earth or a break in continuity only extinguishes the lamp without damage. Some engineers have used in addition to the above gear a relay, with its control coil either in series with the lamp or in parallel with the choke, arranged to switch in a tungsten pilot-lamp in the event of the h.p.m.v. lamp being extinguished by a surge.

Time switches, fog switches, and remote-control devices, follow standard practice.

A caution is necessary with regard to the switching of h.p.m.v. lamps for test purposes on an installation. The lamps should always be allowed to reach full brilliance before switching off again. If lamps are repeatedly switched on and off without this precaution, the active coating of the electrodes, which is instrumental in initiating the discharge, may be removed and reliable starting of the lamp may be prejudiced. If the lamp is allowed to attain full brilliance, the arc discharge causes this coating to re-form.

Table 2

Rating of lamp in use	Mains voltage (actual) at unit	Installation without condensers			Installation with condensers. 250-W lamp, 15 μF; 400-W lamp, 20 μF		
		Maximum starting current in amps. (excluding switching surges)	Average running conditions		Maximum starting current in amps. (excluding switching surges)	Average running conditions	
			Current in amps.	Power factor		Current in amps.	Power factor
200/210-volt 250-watt	200	$3\frac{1}{2}$	2·3	0·55 to 0·6	$2\frac{3}{4}$	1·7	0·8 to 0·85
	210	$3\frac{1}{2}$	2·3		$2\frac{1}{2}$	1·6	
220-volt 250-watt	220	$3\frac{1}{4}$	2·1		$2\frac{1}{4}$	1·4	
230-volt 250-watt	230	$3\frac{1}{2}$	2·0	0·55 to 0·6	$2\frac{1}{2}$	1·3	About 0·9
240/250-volt 250-watt	240	$3\frac{1}{4}$	1·9		$2\frac{1}{4}$	1·2	
	250	3	1·9		2	1·2	
200/210-volt 400-watt	200	$5\frac{1}{2}$	3·6	0·55 to 0·6	$4\frac{1}{4}$	2·7	About 0·8
	210	$5\frac{1}{4}$	3·6		4	2·6	
220-volt 400-watt	220	5	3·4		$3\frac{3}{4}$	2·4	
230-volt 400-watt	230	$5\frac{1}{2}$	3·0	0·55 to 0·6	4	2·2	0·85 to 0·9
240/250-volt 400-watt	240	5	2·8		$3\frac{3}{4}$	2·0	
	250	$4\frac{3}{4}$	2·8		$3\frac{1}{4}$	1·9	

Supply Cables.

In order finally to decide on the appropriate choke tapping, it is necessary to check the voltage actually available at the unit under running conditions. In the design of a new installation the probable voltage-drop in the cables can be calculated by the usual methods,* so

* L. B. W. Jolley, J. M. Waldram, and G. H. Wilson: "Theory and Design of Illuminating Engineering Equipment" (Chapman and Hall), p. 553.

that an undue voltage-drop is avoided. It is worth noting, however, that the h.p.m.v. lamp and tapped choke have an advantage over the tungsten-filament lamp in this respect. It is unusual to provide filament lamps of different voltage ratings in the same installation, and a considerable difference in light output may result

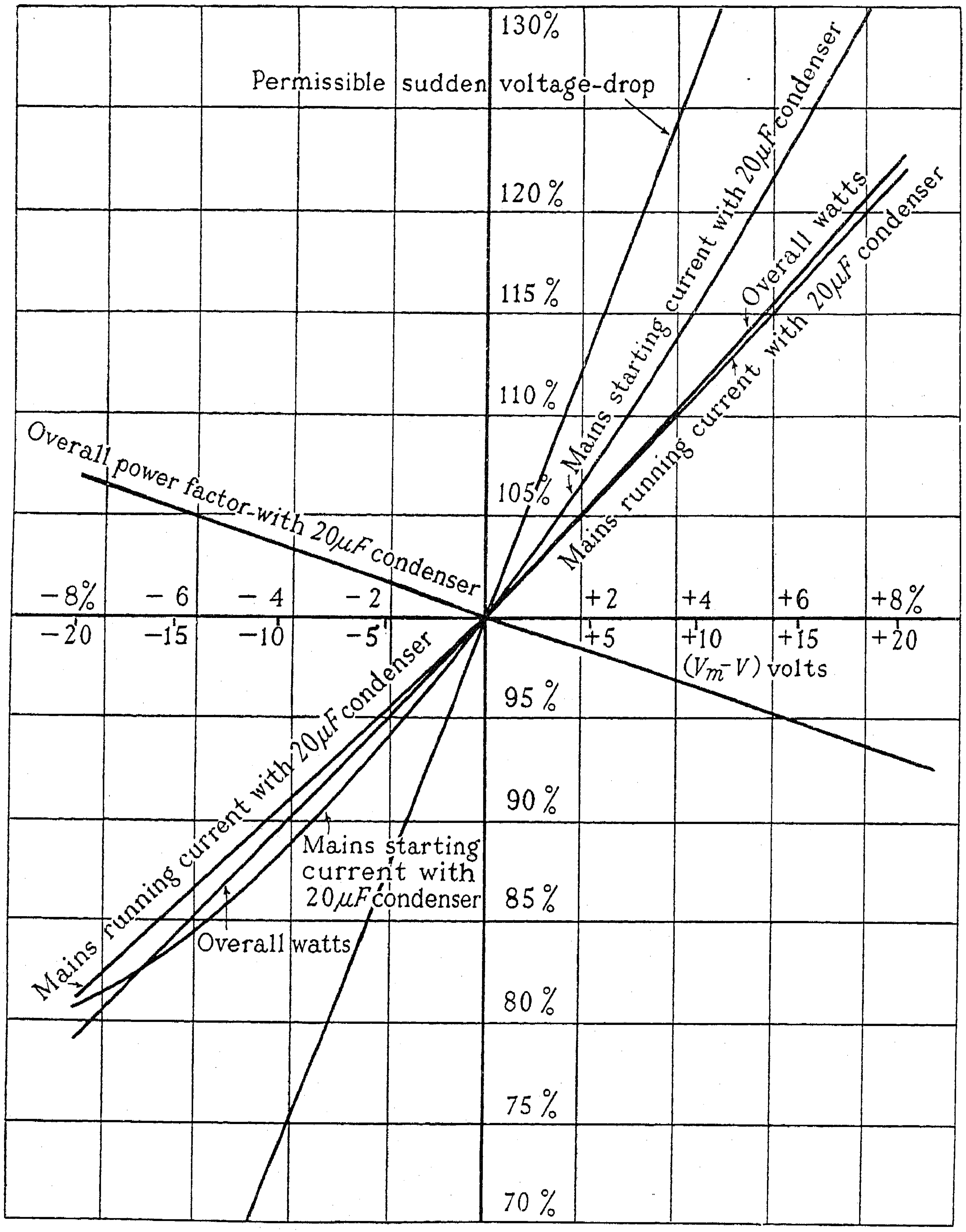


Fig. 12.—Variation in performance of lamp, choke, and condenser unit, with changes in mains voltage. V_m = mains voltage. V = voltage setting of choke (230 V).

Variable	Value at $V_m = V = 230$ volts
Overall watts	420
Mains starting current	4·10 amperes
Mains running current	2·12 amperes
Overall power factor	0·86
Permissible sudden voltage-drop ..	29 volts

as between the beginning and end of a long cable-run. With the h.p.m.v. lamp this effect can be counteracted by the use of different tappings on the chokes, and a more uniform light output can be provided.

Sufficient voltage must be available for striking the lamps, the minimum required for one make of lamp being given as:—

200/210-volt lamp	..	185 volts
220-volt lamp ..	..	200 volts
230-volt lamp ..	..	210 volts
240/250-volt lamp	..	215 volts

In a single-phase system or a balanced multi-phase system the voltage-drop in the mains will be lower under starting conditions, owing to the lower wattage of the units; for, although the starting current is about twice the running current, the power factor is very low, and

consequently the voltage-drop due to the starting current is nearly in quadrature with the mains voltage. This effect may, however, introduce unusual voltage-drops in unbalanced 2- or 3-phase systems, but normally if the voltage under running conditions is up to the specified minimum, correct striking will be assured. In exceptionally cold weather the required minima may be about 5 volts higher than the figures given.

Wave-Form and Interference.

The majority of street-lighting cables in this country are run underground. With this system there will be no radio or telephone interference from h.p.m.v. lamp units. If overhead distribution is used in a residential area where sensitive radio receivers may be used in first-floor or second-floor rooms a few feet from the units, some interference may be detected during the first minute or two after the lamps have been switched on. This very infrequent effect is not considered to be of serious importance, but it may be mitigated, if necessary, by connecting a small condenser (about $0 \cdot 1\,\mu F$ for 400-watt lamps) across the lampholder connections.

It has been suggested that the slightly irregular wave-form of current taken by the lamp might affect the supply to near-by consumers on a street lighted by these lamps. Consideration will show that the street-

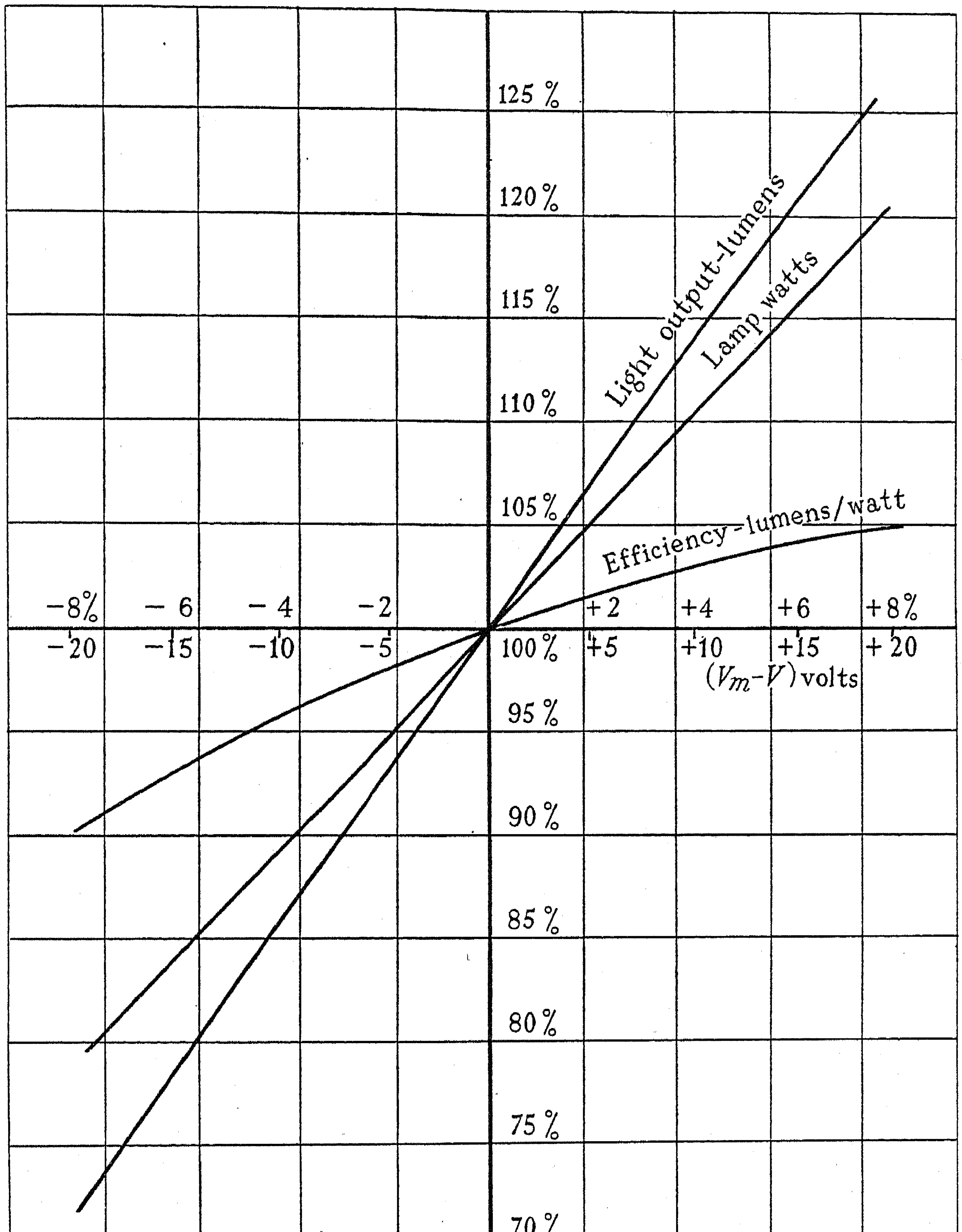


Fig. 13.—Variation in performance of lamp, choke, and condenser unit, with changes in mains voltage.
V_m = mains voltage. V = voltage setting of choke (230 V).

Variable	Value at $V_m = V = 230$ volts
Lamp watts	400
Lumen output	18 000
Efficiency	45 lumens per watt

lighting load would have to be much greater than the domestic load, and the cables common to both to be of high resistance, for any effect whatever to take place at the consumers' terminals. Oscillograph records show that the supply voltage is not, in fact, affected. The current taken by the condenser will be affected to some extent by the wave-form of the supply voltage, and the condenser will provide a path for any high-frequency ripple in the supply.

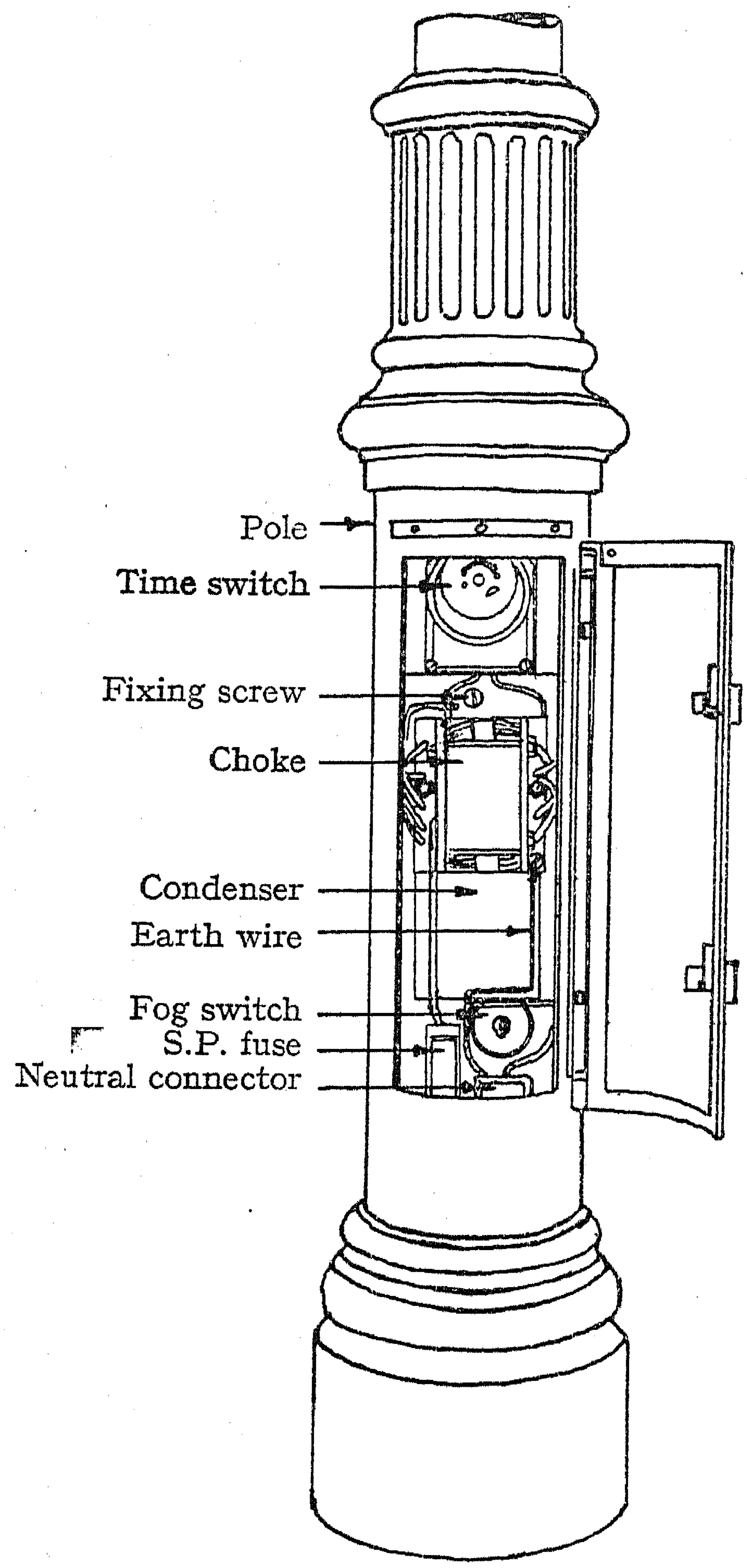


Fig. 14.—Typical arrangement of apparatus in base of column.

Frequency of Supply.

Supplies at frequencies other than 50 cycles per sec. will require special chokes and condensers of different rating. For 25-cycle supplies, two 50-cycle chokes in series and two standard condensers in parallel will meet the case, whilst the reverse arrangement will cover 100-cycle supplies.

For intermediate frequencies it may be possible to adjust the standard chokes so as to give the correct running wattage, but the starting current will depart slightly from normal in most cases. It is to be hoped that the standardization of frequency now taking place will soon eliminate these special requirements.

Voltage of Supply.

The 200/210-volt h.p.m.v. lamp will strike at about 185 volts and over. If the supply voltage is lower, a step-up transformer will be necessary; alternatively, special lamps with heating arrangements for one or both electrodes can be used. The use of a transformer to supply a group of posts will have the advantage of reducing the copper loss in the distribution system and will enable smaller cables to be used.

(2) THE LANTERNS

(a) Optical Design

In lighting a street it is clearly desirable to redirect into more useful directions light which the bare lamp emits towards the sky. It is further desirable to redirect up and down the street some of the light which would be emitted by the bare lamp beyond the road margins.

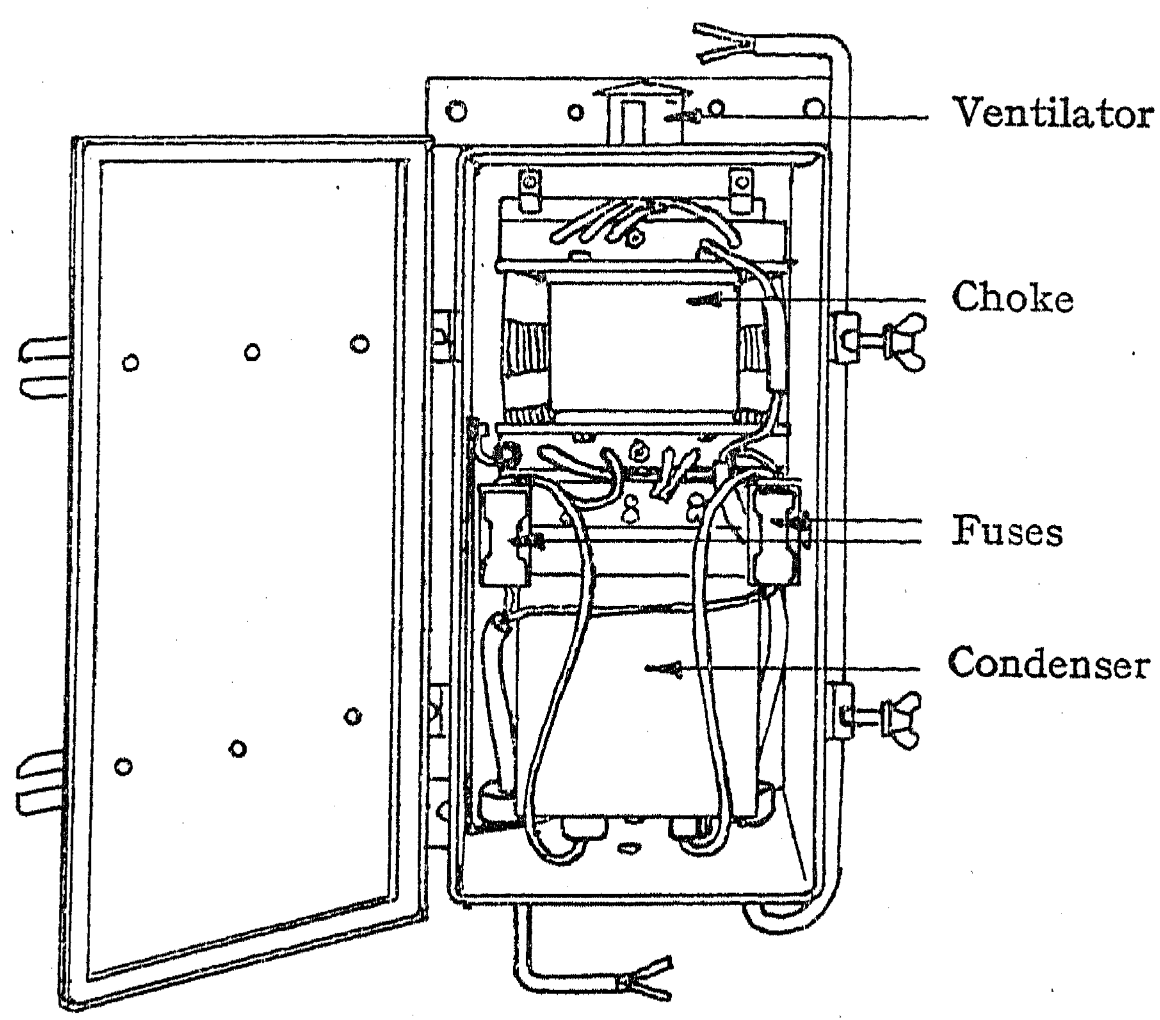


Fig. 15.—Ventilated box, housing choke and condenser.

An effective type of light distribution is now known and is given by most of the lanterns to be described.

The vertical-burning lamp has proved a most convenient source for the production of the desirable form of light distribution discussed on page 263. Various units generally similar in appearance but differing in details of construction or in the optical system have now been produced and are in common use.*

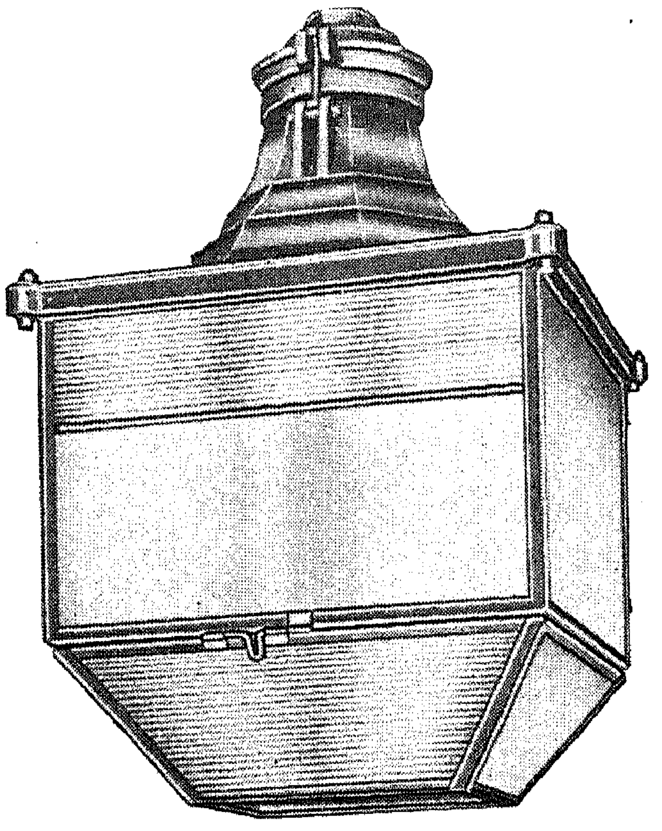
In the four typical forms of lantern shown in Fig. 16 (see Plate 1), the main redirection of the light from the lamp is obtained by means of pressed glass plates having vertical prisms moulded on the surface, as shown at R in Fig. 16(*d*). Such an optical device can be made to fine limits at a relatively low cost, and accurate control of the light is possible. The angles of the prisms are so designed that each prism or gang of prisms redirects the light incident upon it into a predetermined zone, and the desired form of distribution can thus be obtained from the complete refractor. Very little redirection by the refractor is attempted in a vertical plane. Any attempt to do this with a set of horizontal prisms

* S. ENGLISH: *Illuminating Engineer* (London), 1934, vol. 27, p. 352; also S. S. BEGGS and G. H. WILSON: *G.E.C. Journal*, 1935, vol. 6, p. 127.

Fig. 16.—Examples of refractor-type lanterns.

(*Facing page* 256.)

Plate 2



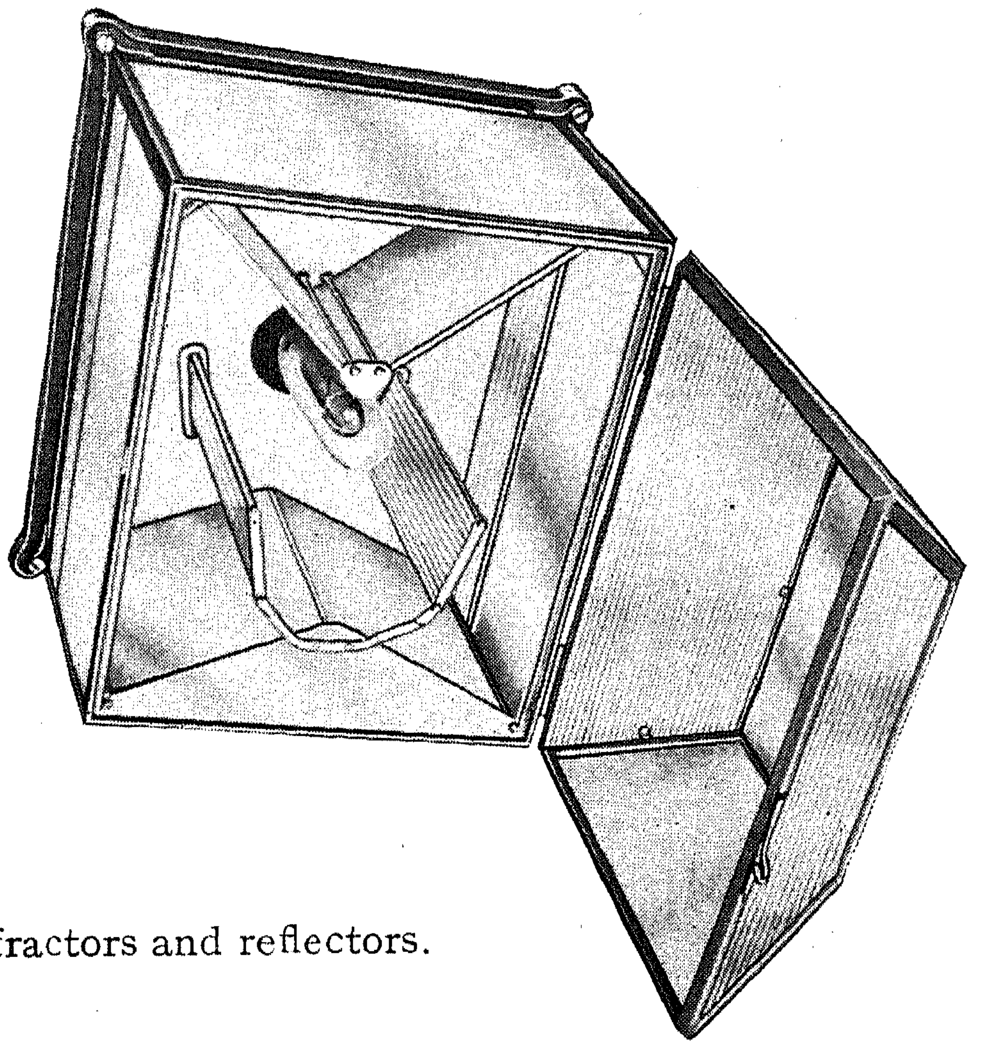


Fig. 19.—Lantern using refractors and reflectors.

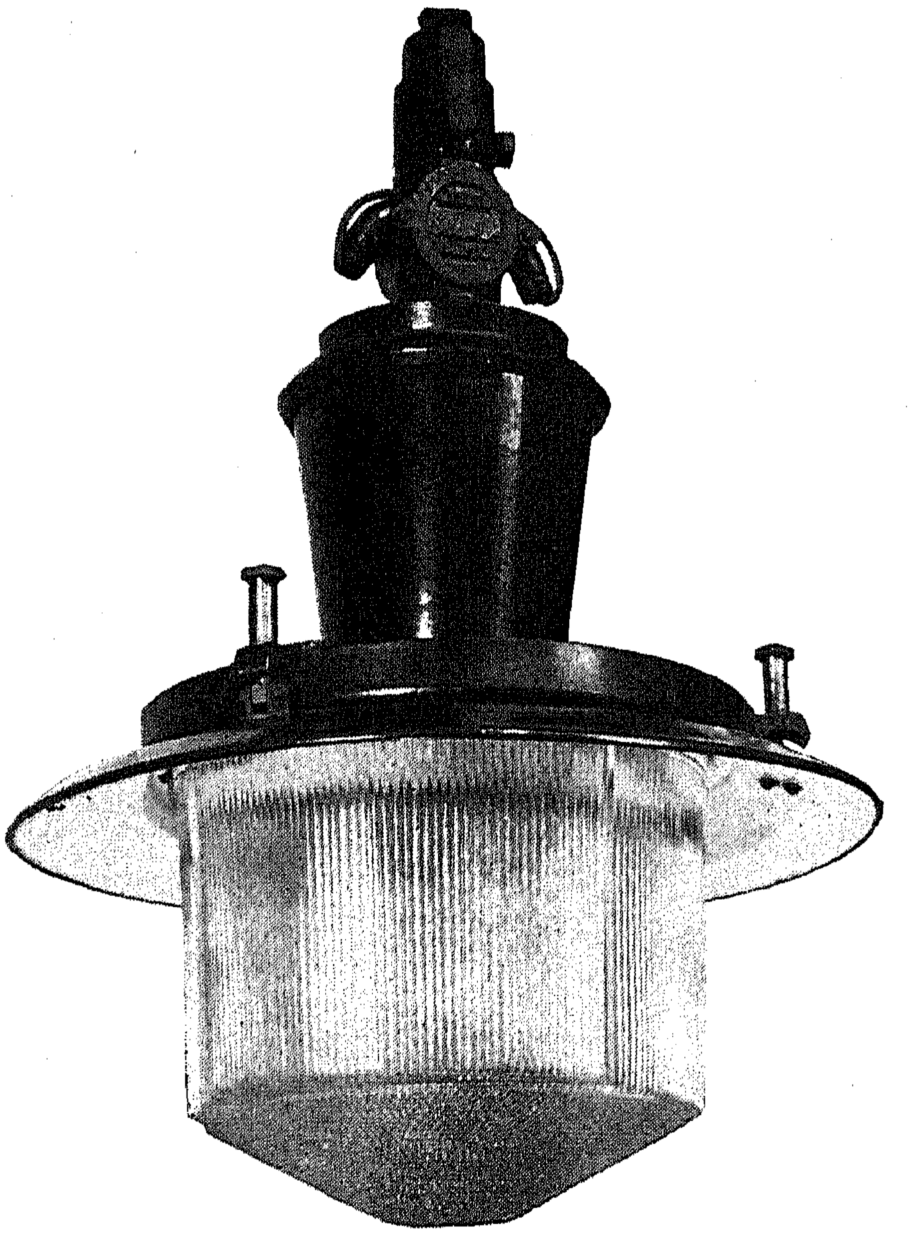


Fig. 20.—Lantern with spun body and single-piece optical system.

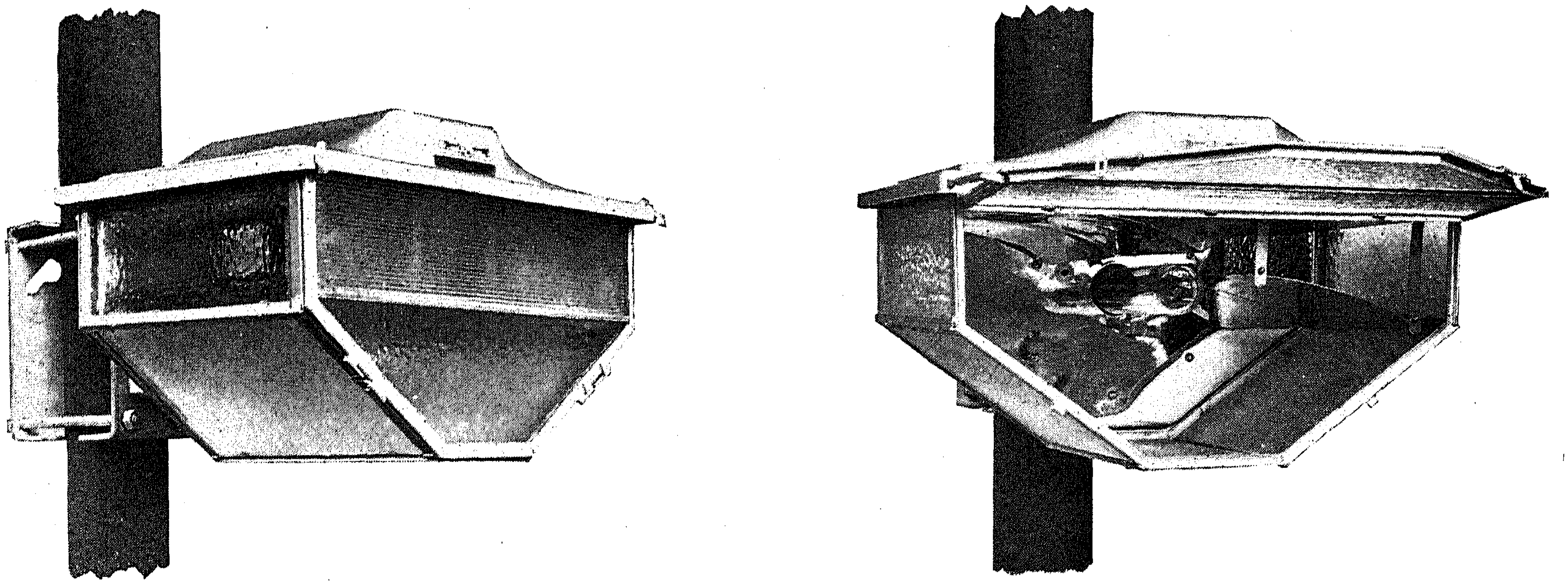


Fig. 21.—Reflector unit for horizontal-burning lamp.

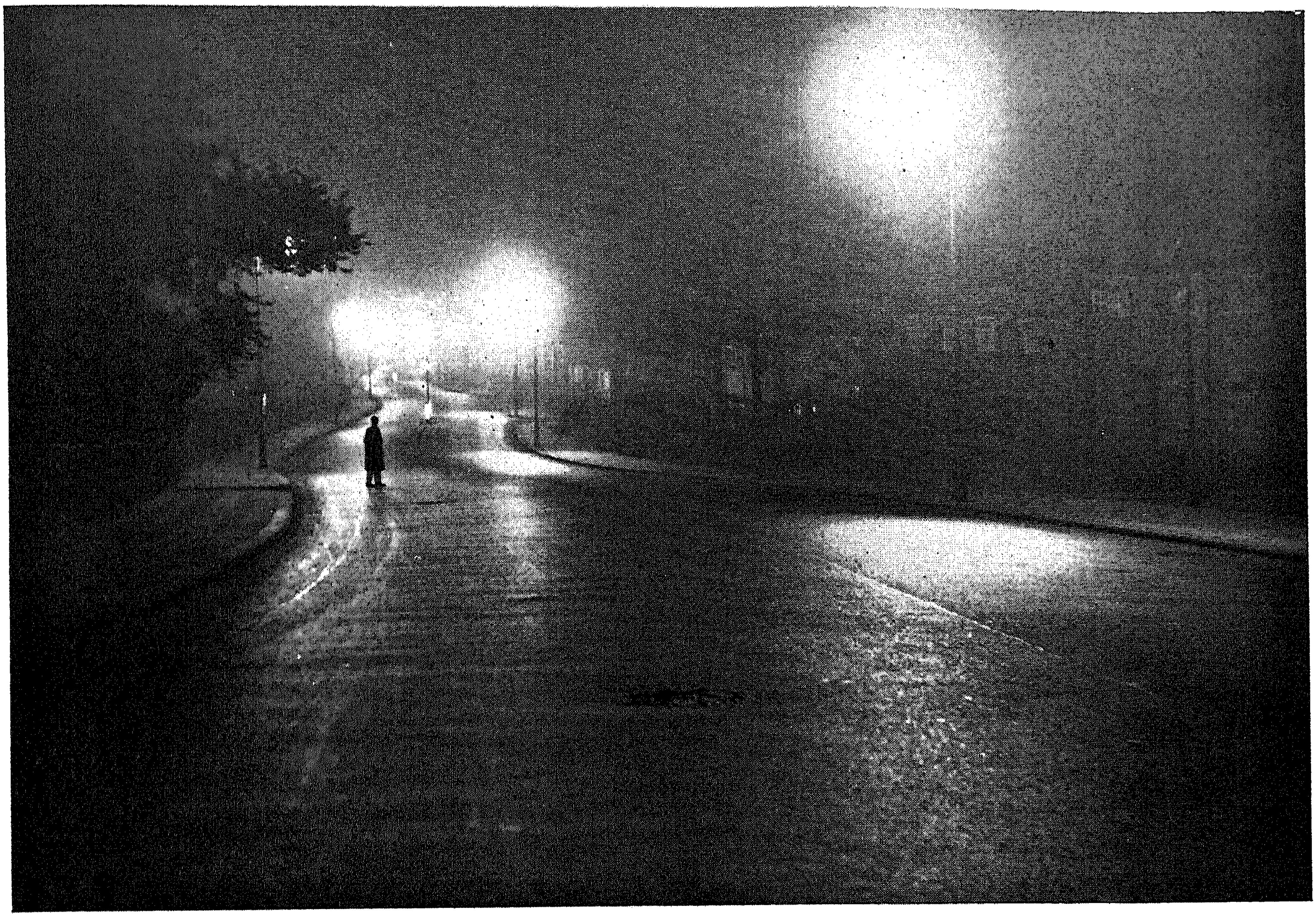


Fig. 23.—Street showing superimposition of bright patches.
(View at 10 inches for correct perspective.)

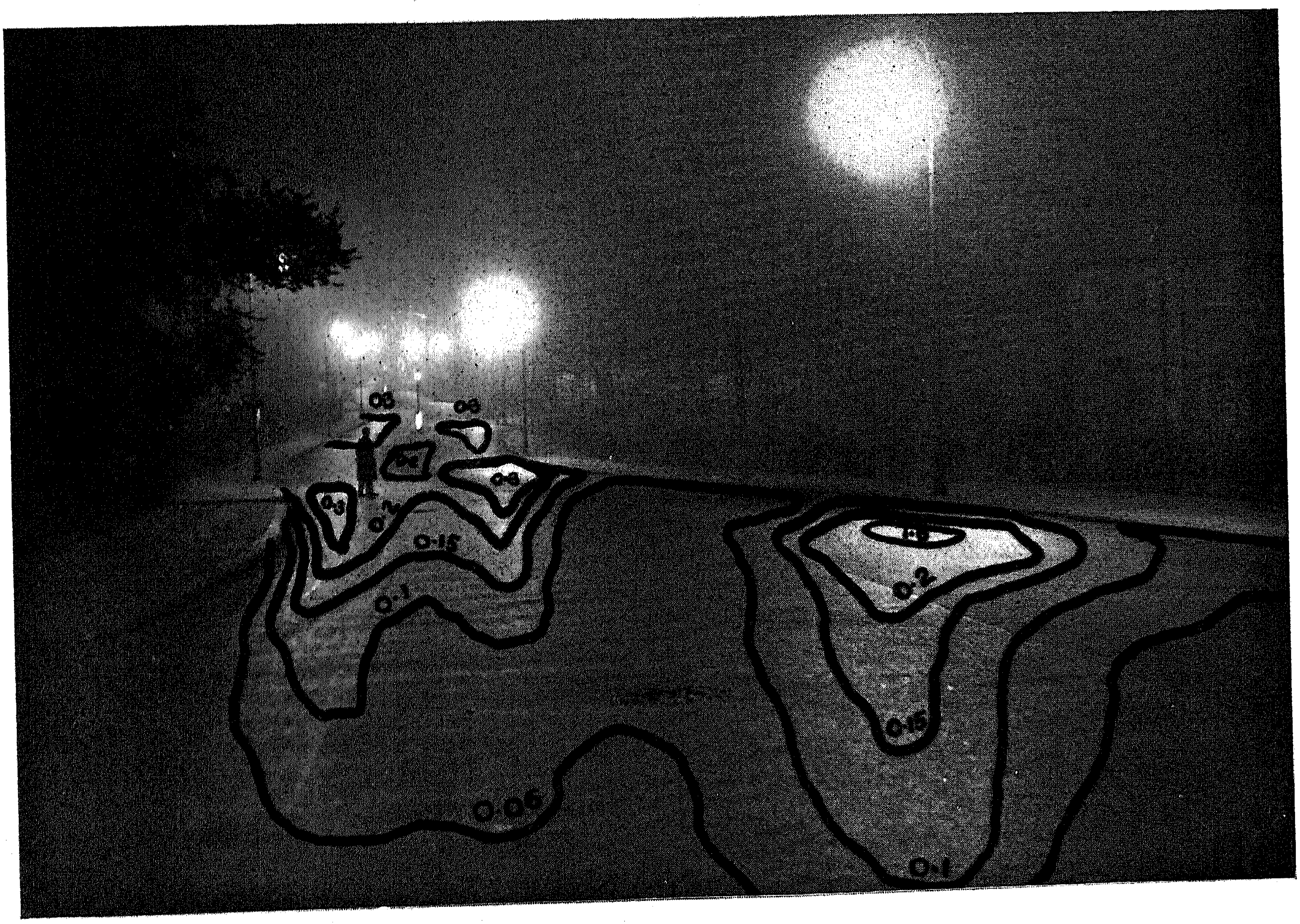
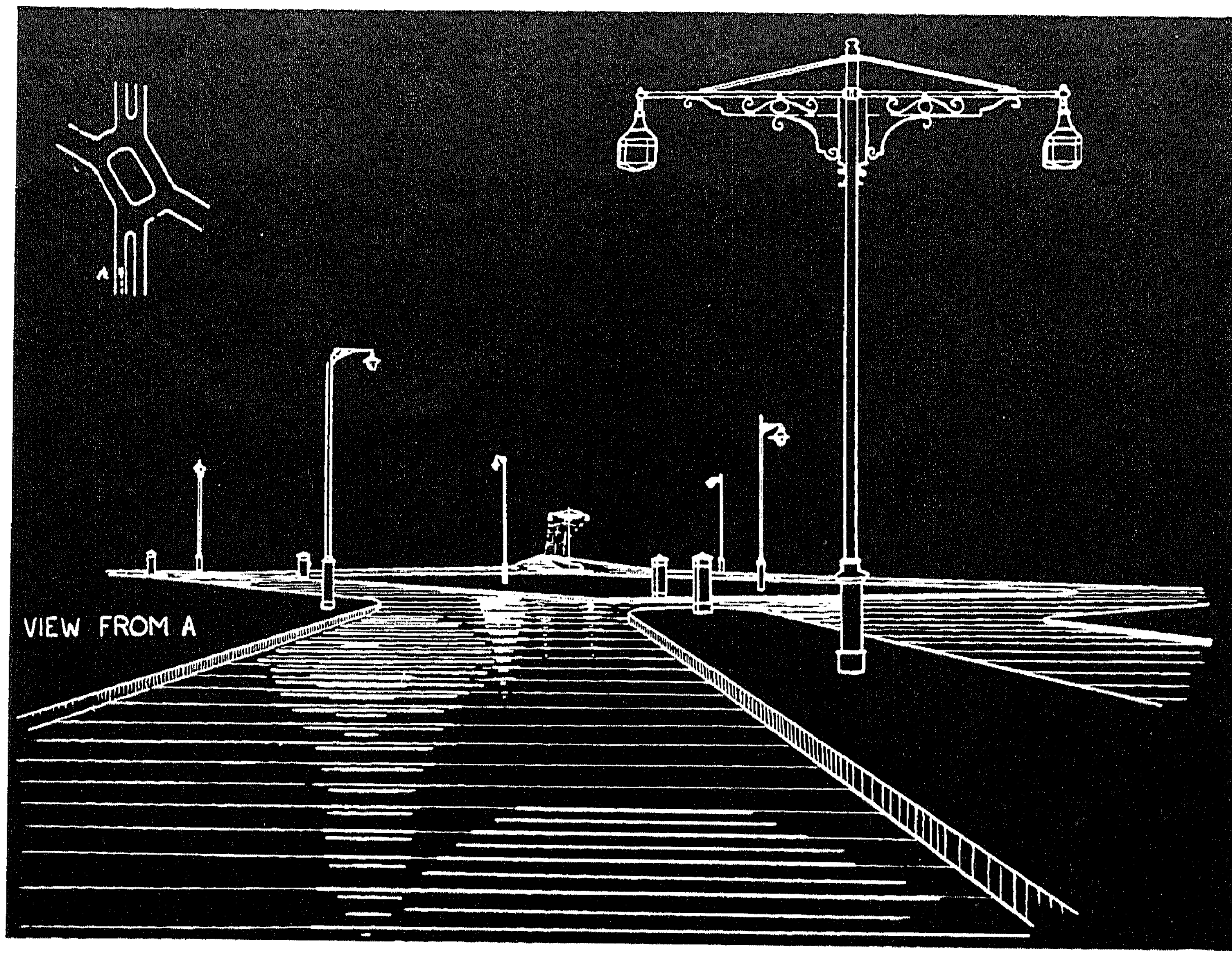


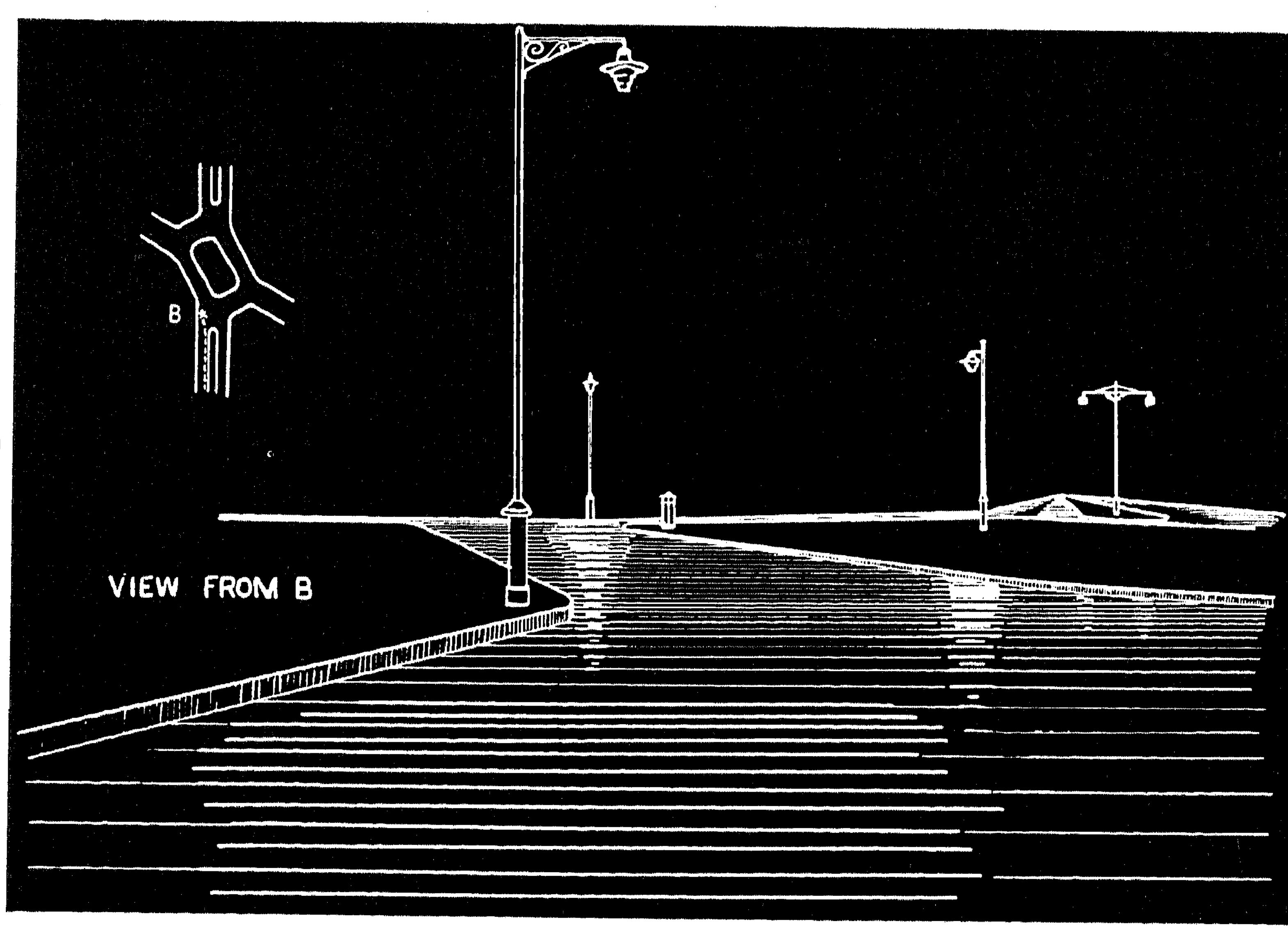
Fig. 24.—Brightness distribution in a well-planned installation of h.p.m.v. lamps. Values given are in equivalent foot-candles.
(View at 10 inches for correct perspective.)

Plate 4

(*a*)



(*b*)



(*c*)

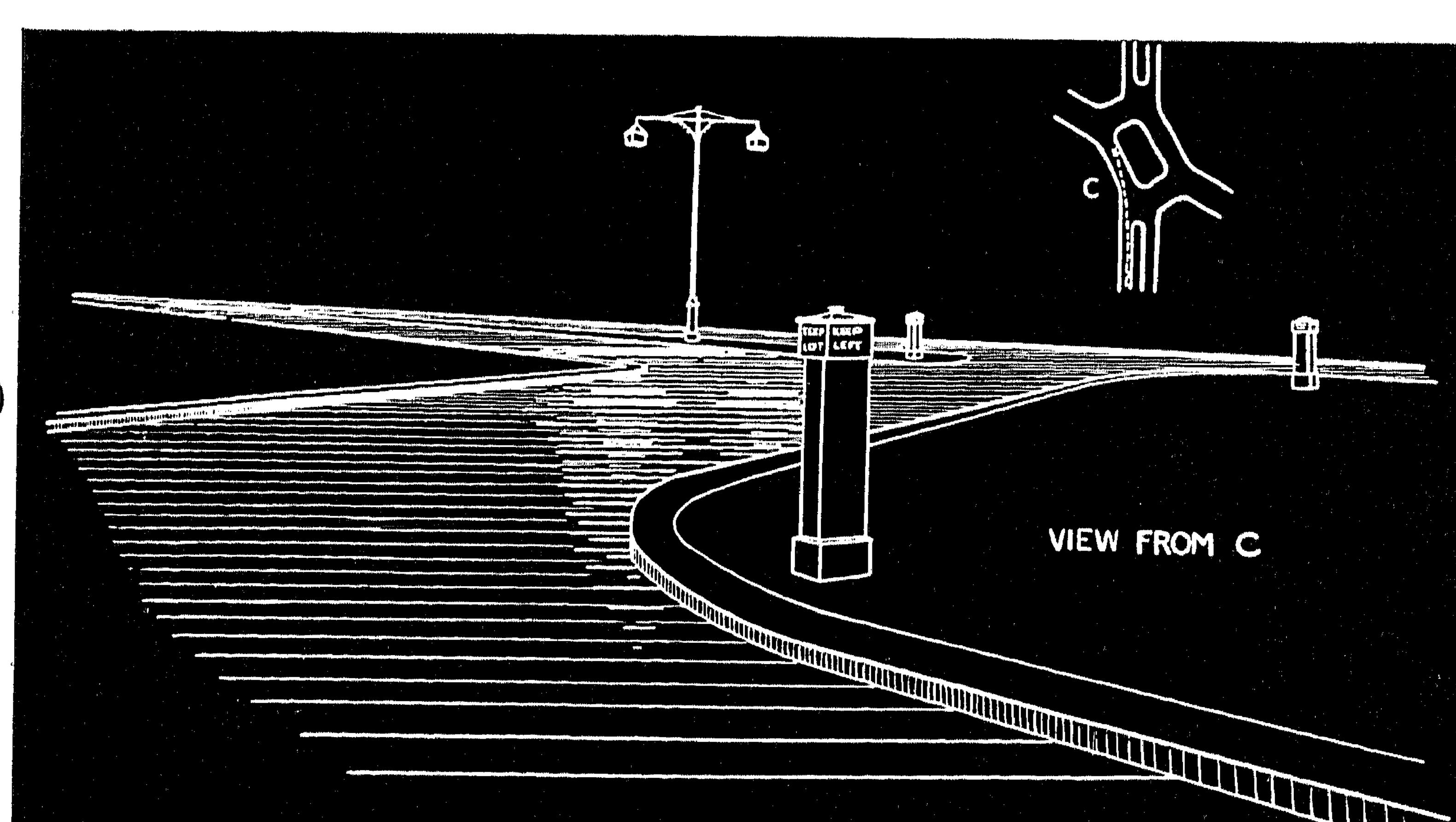


Fig. 26.—Perspective diagrams of installation for intersection with "roundabout."
(View at 4 inches for correct perspective. (If it is desired to focus for correct perspective, view through lens of 4 inches focal length.)

results in a displacement of the bare-lamp distribution in such a way that the intensity increases rapidly from about 20° to the downward vertical to 60° or 70°. This form of distribution inevitably results in a patchy distribution of brightness in any normal roadway, for the following reason. The road reflectivity increases rapidly with increasing angle, and if the intensity from the fitting increases at too rapid a rate the resulting increase in brightness will produce the often-condemned "pools of light."

Some further redirection is required into zones not covered by the refractor. It will be seen from Fig. 3 that the lamp itself has a relatively low intensity in a downward direction. This low intensity, if not augmented, would result in a dark patch under the unit. In the built-up type of lantern a vitreous-enamelled or specular top reflector and diffusing glass or enamelled side panels augment the illumination on a diffusing bottom panel. If this panel is of sufficient area and transmission, an adequate intensity can be produced in the downward direction and at angles near to that direction, so that the desirable flat-bottomed polar curve described on page 263 results.* Fig. 17 shows for lantern (*a*) in Fig. 16 the polar curve in a vertical plane through the direction of maximum intensity.

The region of the distribution seen in plan, which contributes to the effectively bright patch produced by a distant source, is less than 1° wide. Nevertheless, it is necessary for the lighting unit to give sufficient intensity in directions making greater angles with the direction of the street, so that, as a source on the far side of the road is approached, the road surface appears adequately bright and, at the same time, sufficient light is available for the general illumination of roadway, sidewalks, and building fronts. The combination of refractor plate, diffusing bottom, and diffusing end, can be made to produce the requisite form of distribution. The typical performance diagram for one side of a lantern, shown as a series of isocandle curves in Fig. 18, indicates the way in which the bare-lamp distribution is modified by the lighting unit.

In some other units, reflectors alone or combinations of reflector and refractor produce the redirection of light. The interior view of the lantern illustrated in Fig. 19 (Plate 2) shows the combined use of plate refractors and opal glass reflectors.

The particular optical arrangements of the units described above are robust and simple to incorporate in a lantern. In their manufactured form the lanterns are accessible and easy to maintain.

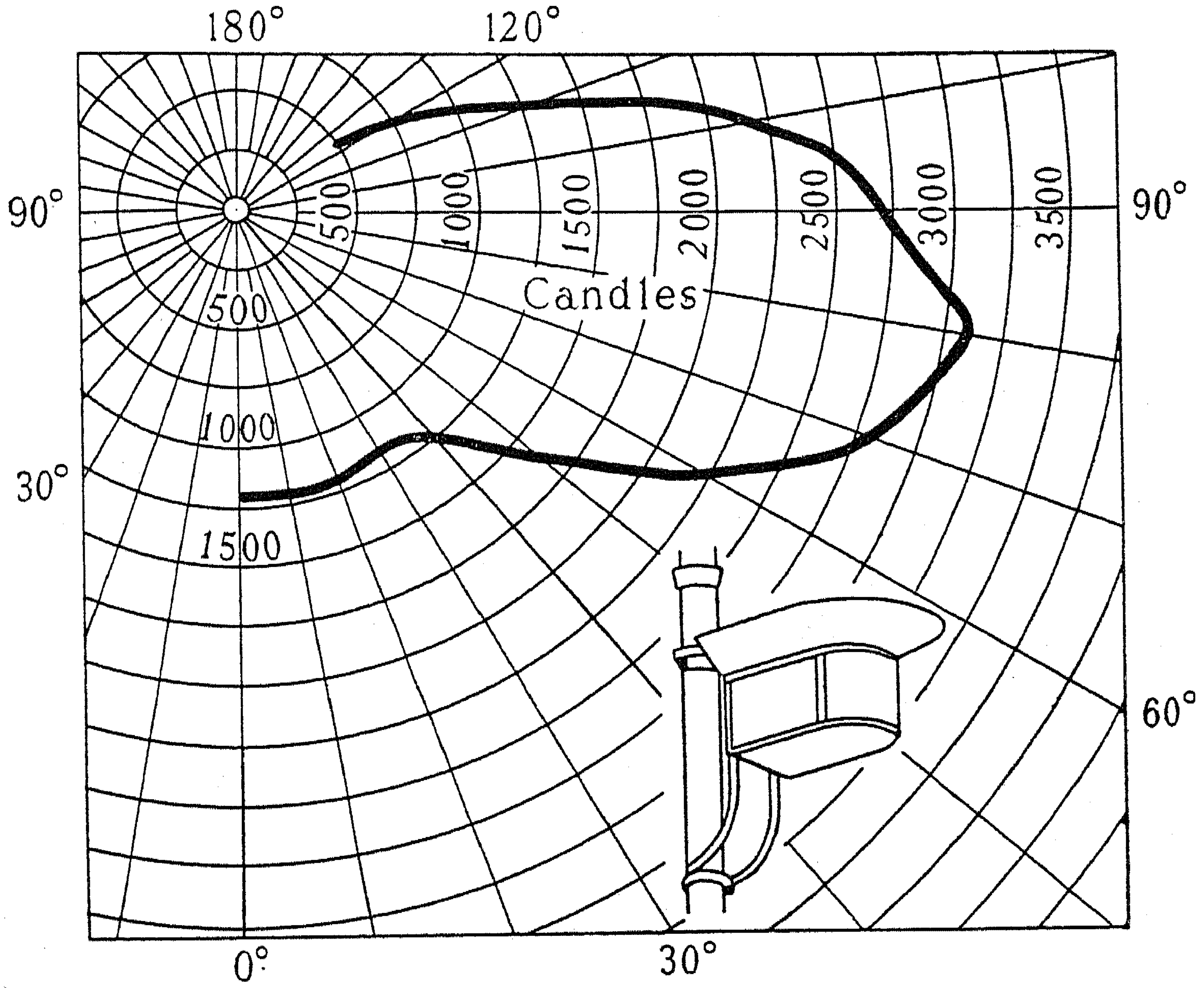


Fig. 17.—Polar curve showing light distribution in a vertical plane in direction of street, from lantern illustrated in Fig. 16(*a*) with 400-watt lamp; 18 000 lumens.

The built-up unit is considered by many to be pleasing in appearance. It is, however, expensive by comparison with a lantern comprising a spun body supporting a single-piece optical system. With this application in mind, units employing a single-piece refractor have been made which combine the features of the built-up lanterns in a unit which is both simple to construct and to maintain. In one design, illustrated in Fig. 20 (Plate 2), concentrating prisms are pressed on the inside of the glassware and shallow diffusing flutes on the outside. The effect of this combination is to spread the brightness of the refractor over the whole width of the prismatic section so as to minimize glare and give to the lanterns a pleasing appearance. The same principle is used in lanterns (*a*) and (*d*) in Fig. 16 (Plate 1). The diffusing ends of the built-up unit are replaced by diffusing prisms situated between the main gangs of diffusing flutes. The light is reflected by the sides of the prisms and emerges

* For the reasons for this see G. H. Wilson: *Illuminating Engineer*, (London), 1933, vol. 26, p. 151.

from the tips, so that dirt lodging in the interstices between them has little effect. The bottom of the glassware is pressed in a series of small cones of various angles, which produce an effect comparable with that of the diffusing glass in the rectangular units and increase the intensity in the downward direction.

Consideration of the performance diagrams for the lanterns will indicate that quite a considerable proportion of the light is radiated above the horizontal. It will be shown that, in spite of this, the distribution is effective carriage-ways of average width. But they also showed that a high gain could only be obtained by robbing the distribution of the characteristics which produce a high-brightness installation, viz. a flat-bottomed distribution with high intensities in the region of 80° to the downward vertical.

When a unit is designed to produce this optimum form of distribution in a vertical plane with a horizontal-burning lamp, much light is radiated at high angles on to the sides of the road, for the distribution is wide and it is

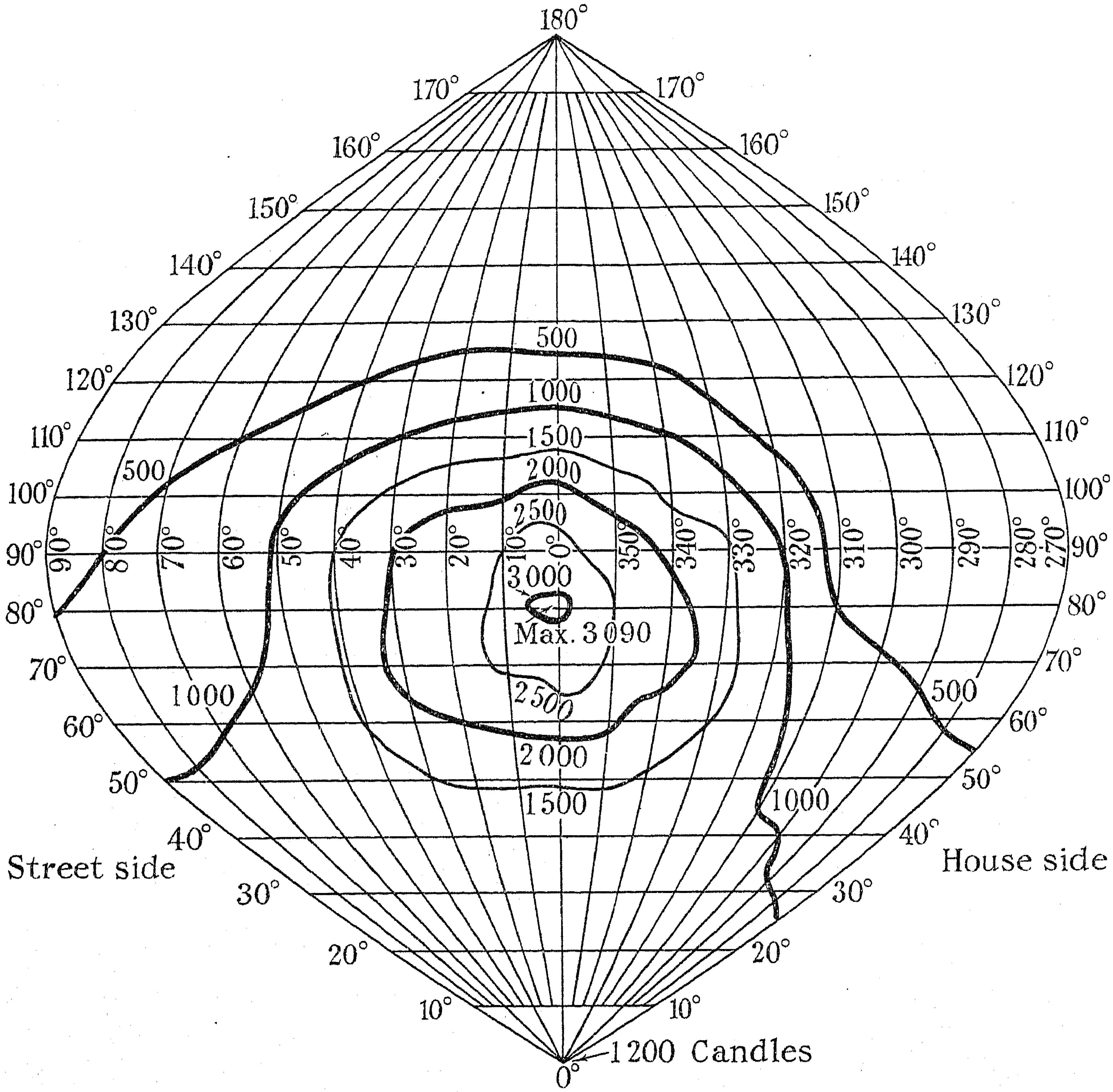


Fig. 18.—Isocandle diagram for lantern shown in Fig. 16(*a*): 400-watt lamp; 18 000 lumens.

in producing high and uniform road brightness, on account of the shape of the distribution and the maintenance of high intensities at high angles. Nevertheless, it was apparent at an early stage that if the distribution from the vertical-burning lamp and fitting could be turned through 90° (which would entail burning the lamp horizontally) less light would be radiated above the horizontal and there would be some possibility of producing better lighting of the roadway, although the distribution would be extremely wide.

Measurements on experimental fittings showed that in this way more light could, in fact, be concentrated on to desirable that the maximum intensity should occur at angles in the region of 80° or more to the downward vertical. Results from an experimental fitting of this kind have shown that for a normal road 30 ft. in width, with the units kerb-mounted about 25 ft. high, there was a gain of only approximately 10 per cent in the amount of light reaching the carriage-way. The gain will increase, however, as the road width increases, and, further, with this type of unit higher maximum intensities are possible, and therefore higher road brightness, without any reduction in the width of distribution across the street.

If the same maximum intensity were obtained with the vertical-burning lamp, the distribution would be too narrow for satisfactory lighting. One form of lantern, burning the lamp horizontally as described and using reflectors for the redirection of the light, but giving its maximum intensity at an angle of 75° to the vertical, is shown in Fig. 21 (Plate 2).

Running the lamp horizontally involves the use of some auxiliary apparatus to prevent the arc from bowing upwards, and this inevitably adds to the cost of the unit. A magnetic deflector, connected in series or in parallel with the lamp, can conveniently be used for this purpose. Various mechanical arrangements of the deflector are possible, the requirements being a reasonably uniform horizontal magnetic field of the necessary strength through the axis of the lamp and in the correct direction to maintain the arc substantially horizontal. In the lantern described, the deflector is situated above the reflector and over the lamp. The position and field strength of the magnet have to be so designed that the requisite deflection is obtained over the normal ranges of mains-voltage fluctuation and spread in rating between individual lamps.

(b) Mechanical Design

The mechanical construction of the fittings will be determined mainly by the available manufacturing facilities. Cheapness is important, but it is clearly unwise to reduce costs to the bare minimum if it leads to unserviceable fittings. A single drop of water may crack an h.p.m.v. lamp, and therefore great care has to be taken to ensure a watertight construction and to avoid the possibility of condensed water from the bracket arm entering the lantern. With non-enclosed fittings it is usual to employ a cylinder or well glass to protect the lamp. This protecting glass needs to be of such a size that it operates at a sufficiently low temperature to avoid cracking during rain.

For the construction of the lanterns, copper sheet and bronze or iron castings have been found most serviceable. If steel is adopted for lanterns in which magnetic deflectors are used, it may considerably affect the deflector field. The greater heating effect of the h.p.m.v. lamp as compared with a tungsten lamp of similar wattage has already been remarked. It has often been necessary to employ heat-resisting glass, specially developed for the purpose, for the manufacture of prismatic glassware for these lamps.

Ventilation of the lantern tends to result in severe soiling of the interior and in access of insects, and a construction as nearly airtight as possible is preferable. It has been found that totally-enclosed lanterns can be designed which adequately dissipate the heat from the lamp.

The design of brackets calls for little comment, except that it is desirable that the bracket should be rigid, and that the lantern should be so attached that it can be aligned accurately with the street and securely locked in position. When adjustment for alignment is permitted, it is not simple to ensure a watertight connection between lantern and bracket. A special form of union for suspended lanterns, shown in Fig. 22, has been developed to overcome this difficulty. This union permits a positive locking after alignment and prevents leakage of condensation into the lantern. An effective but unconventional bracket shown in Fig. 16(*a*) (Plate 1) was designed for the first h.p.m.v. lamp lanterns and overcomes the above difficulties.

(3) THE INSTALLATION

Experience has repeatedly shown that the success of an installation is determined—perhaps more than by any other factor—by the way in which the sources are located and the installation is designed as a whole. The correct planning of installations is essential to their success, just as are sound electrical engineering and suitable photometric design of the lanterns: some knowledge of the underlying principles of street lighting is therefore indispensable to the electrical engineer who has to deal

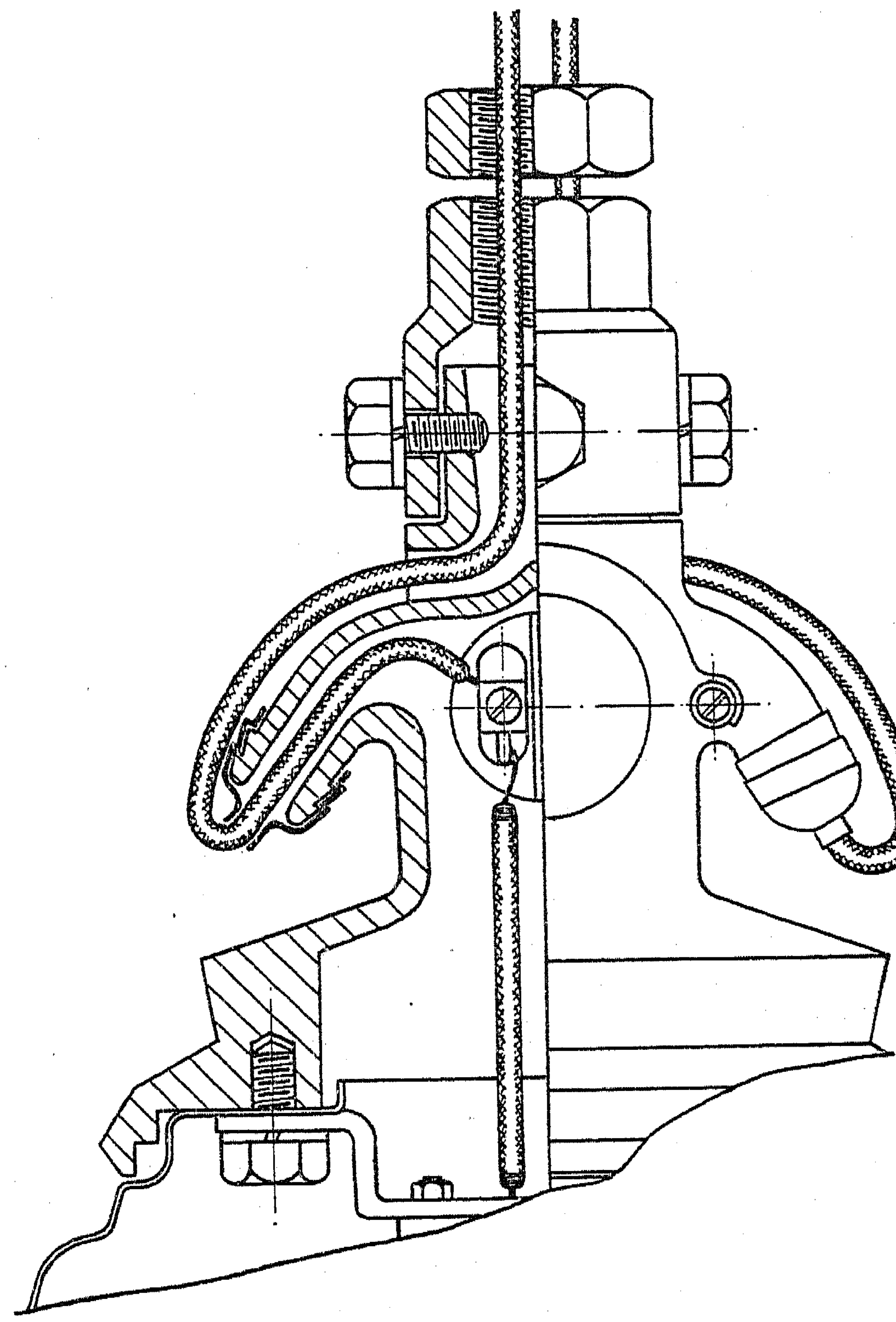


Fig. 22.—Watertight union for suspended lantern.

with public lighting. The relevant results of recent work will therefore be summarized and some new conclusions given which it has now been possible to draw from them.

It was realized in the early experiments that success would not depend alone upon having a lamp of high efficiency: it was necessary to know how to use its light most effectively. It so happened that certain features of the h.p.m.v. lamp led to a better understanding of the fundamentals of street lighting and showed the way to more effective installations. From the design standpoint the high efficiency of the lamp appeared at first to be offset by its length. The long, vertical light column was awkward, and accurate control of the light in vertical planes was difficult. As a result, in the first lanterns designed higher intensities than had been usual

were produced near the horizontal, and the first full-scale installation on the Watford Road, Wembley, was erected with some fears of glare. Special arrangements were made for louvre devices to mitigate it if necessary. To the designers' surprise, however, the installation was successful beyond their expectations, and produced a very high degree of visibility with no material glare. The louvre devices were never used. For once, something useful had been found by accident.

This experience formed a starting point for full investigation into the mechanism of visibility in streets and their appearance,* which, although it had been contemplated for some years† and had in fact been tentatively begun some time before, had not progressed very far. The good results obtained were explained and the principles of their reproduction in future installations studied, and from this work a new basis was established for the technique of street lighting.

(a) New Principles of Street Lighting

The Geometry of the Problem.

The modern ideas of street lighting differ from those of a few years ago in the emphasis placed on visibility and appearance of the road, in addition to that placed on the illumination falling on the road surface. This involves a study of the road and of objects in perspective rather than in plan. The field of view of an observer in the road is made up of many different objects, but it consists mostly of very long surfaces such as the carriageway or the footway, or the surfaces of buildings, seen almost end on. Consequently, some quite small region in the field of view may represent hundreds of feet of road or building, whereas another region of equal size in the field of view may represent quite a small object. A study of the perspective of roads was an important first step towards the understanding of visibility.

The Mechanism of Visibility.

It is well known that objects are discerned almost entirely as the result of the contrast with their background. This is true for the most part even in daylight, and it is certainly true in street-lighting conditions, where colour vision and the appreciation of fine details give little help. In streets, both by night and by day, objects are almost always darker than their background and are seen in dark silhouette. This seems to be the most effective method for achieving the rapid and certain perception of objects, and is often brought about artificially even in daylight, as, for instance, in the use of a white cricket screen. The particularly good visibility of objects on the skyline is proverbial. It has consequently been found that for quick and certain perception of objects on a street the road surface needs to be made bright, and if the brightness is great enough and sufficiently uniform the driver can be confident that he can see at a glance whether or not the road is clear. The background is not formed entirely by the road surface, and the many other surfaces of which it is composed must not be neglected, for they may make a substantial contribution to visibility. In the authors' view, however, the road surface is by far the most important; for if it is adequately bright, then, even if the whole of an object cannot be seen against it, such parts as can be seen are seen very clearly and the presence of an obstruction is realized. It is rare to see the whole of a large object clearly on a street at night.

When the traffic is very heavy this argument may need some modification. The field of view is then occupied, for the most part, by the vertical surfaces of vehicles ahead, and the road surface plays a less-important part. In such installations the light sources should be arranged to illuminate the backs of vehicles as well as to render the road surface bright. Nevertheless, in such roads the brightness of the road surface is still important, and drivers rely upon it more than might at first be supposed.

Another factor which plays an important part in visibility is the motion of the background and object relative to the observer. Although the effect of motion is indeterminate and cannot at present be forecast, general observation indicates that the effect of relative motion is always to render objects much more easily visible.

Effects of Road-Surface Reflection Properties.

Any investigation of road-surface brightness involves a knowledge of the complex ways in which light is reflected by the surface, for they result in a brightness distribution and a consequent appearance of the surface quite different from that which would have been expected at first sight from the distribution of illumination.

The effect of the reflection characteristics of the road surface is that a bright area associated with each light source is formed on the road surface. This area has always the same general characteristics, but its shape and size are altered by the road reflection properties and by the candle-power distribution of the lantern. In general it is T-shaped, centred about the line joining the observer to the light source and lying always between the two. It is safe to assume that no light source will ever render usefully bright the road surface beyond it.

The "head" of the T-shaped area, as will be seen from Fig. 23 (Plate 3), lies actually just on the observer's side of the light source, though in perspective it appears to be practically at the foot of the post. If the road is matt, this "head" is wide and fairly bright and may extend well across the road, giving the appearance of a narrow cross-band. It is formed by light leaving the light source in the region of 45° from the vertical. The "tail" of the T, which is always centred about the line joining the source to the observer, varies greatly in length and in brightness, according to the light distribution and the character of the surface. On a new rough surface it may be barely distinguishable; with a highly polished or flooded road surface it may be several hundred feet in length and very bright and narrow. It is formed by light leaving the source at angles near the horizontal, i.e. from 75° to 87° to the vertical, and if the fitting has a "cut-off" the length of the tail may be very much reduced. In one extreme, therefore, the bright area may be a narrow cross-band, horizontal in a perspective view, and in the other extreme a narrow, very bright longitudinal band running towards the observer, i.e. vertically in a perspective view. The most suitable shape of the

* J. M. WALDRAM: *Illuminating Engineer* (London), 1934, vol. 27, p. 305.
† *Idem*, Association of Public Lighting Engineers, Sheffield, 1928.

bright area is intermediate between these extremes. If the shapes of the areas are at either extreme, they cannot with ordinary spacings be so arranged in the field of view that no dark areas remain between them. Fortunately, it is possible with most modern road surfaces of moderate smoothness, and with properly designed light distributions, to produce bright areas which are sufficiently long and wide to coalesce when the units are at normal heights and spacings, and thus to produce a background of sufficiently high and uniform brightness to reveal all objects with certainty.

Uniformity and Level of Brightness of the Road Surface and of Objects.

The brightness distribution of the road surface in a complete installation results from the superimposition of the various bright areas, one associated with each lighting unit, in their various relative positions in the field of view, as shown in Fig. 23 (Plate 3). Since the axis of each area is always in line with the observer and the source, the complete brightness distribution will change as the observer's position changes: the art of designing the installation lies for the most part in locating the sources, and so designing the light distribution, that large bright areas are produced which effectively cover the important part of the road surface from all relevant points of view.

If the general brightness level is too low, light objects which nearly match their background may be inconspicuous. This seldom occurs at higher brightnesses. Objects become invisible when the brightness both of the object and of its immediate background is very low, and under these conditions, which often occur in badly planned installations, especially on a wet night, objects may vanish and the lighting may become a contributory cause of accidents. Both the level and the distribution of brightness of the road surface and other backgrounds are therefore important and must be considered together.

In the authors' view it is misleading to refer to maximum, minimum, or average brightness, without reference to the brightness distribution. The effect of non-uniformity of the background brightness is probably complicated by psychological phenomena; but there is no doubt that a non-uniform background is trying and puzzling, even when so arranged that objects cannot disappear against it. The analogy of protective camouflage will be obvious. Apart from this, the importance of a dark region depends very much on its size, shape, and position in the field of view. If it is large enough to conceal a significant object in a position which is of importance to the driver, then it may be dangerous. In some positions, quite a small dark patch may be deceptive and dangerous. On the other hand, the large dark region which almost always occurs to the right of a driver and fairly close to him is of no importance, because he is not concerned with any objects concealed there. The suitability of a given brightness distribution is therefore not easily assessed, and the specification of a desirable distribution is difficult.

As a rough measure of the brightness level in existing installations the authors have taken the brightness level of the road surface which forms a background to objects 200–300 ft. distant when seen by an observer usually under one post, with his eye 5 ft. high at 10 ft. from the kerb. This has been called the "effective brightness."

The degree of non-uniformity which can be tolerated is not definitely known, but is much smaller than might be supposed. A diversity of 2 or 3 to 1, which would be inappreciable in an interior lighting installation or in floodlighting, is very noticeable in a street and would be estimated as more nearly 10 to 1, probably because it occurs over so small a region of the field of view. With a diversity of 10 to 1, detail is beginning to be lost in the dark areas, as may be expected from the known properties of the eye. Patches of unusually low brightness in an otherwise adequately bright road may be very deceptive indeed and can be a cause of accidents. In centrally lighted roads of usual proportions the brightness of the surface near the kerb is often about one-tenth the brightness at the centre, and objects tend to disappear near the kerbs. A diversity not exceeding 5 to 1 over the significant part of the road surface may be taken as a rough limit.

As objects are discerned by contrast, the brightness of objects is also important. Evidently the object should be left as dark as possible if it is to be seen against the bright road or other surface. The object brightness is a maximum when light is incident upon the object at about 45°, and increases as the mounting height diminishes. The conditions for visibility are improved up to a point as the mounting height is increased, for the bright areas become larger and more uniform and the objects become darker; but the general level of brightness diminishes with increase of height. Objects are almost always dark compared with the road surface, for most of them have a low reflection factor and they are not illuminated at glancing incidence where high reflectivities can be turned to account. The reflection factor of even light clothes (excluding white flannels) is seldom more than 20 per cent. At present the lowest safe road-surface brightness level cannot definitely be laid down, but the authors believe that an effective brightness not less than about 0·1 equivalent ft.-candle* is necessary in order to satisfy road users of the presence or absence of objects. The brightness distribution in a well-planned installation of h.p.m.v. lamps is shown in Fig. 24 (Plate 3).

Glare.

The effect of glare on vision in streets has been widely discussed, and a great deal of patient experiment has been directed to determine the magnitude of its effect. There is no doubt that very bright sources can cause considerable discomfort and distraction, and can sometimes be measurably deleterious to vision. There is reason to believe, however, that the deleterious effect of glare is less serious than might be supposed, inasmuch as it is confined to conditions of very low contrast which are in any event dangerous. If the contrasts are sufficiently high to satisfy a driver, then such glare as generally occurs from street-lighting units of reasonable magnification will not have any measurable effect upon the perception of objects. Thus glare is most dangerous in the badly lighted roads. When the brightness level is reasonably high, bright sources may be noticeable but

* 1 equivalent ft.-candle is the brightness of a perfectly diffusing surface of 100 per cent reflection factor illuminated to 1 ft.-candle.

will not hinder vision. Nevertheless, every effort ought to be made—by avoiding unnecessarily high intensities near the horizontal and by using as great a mounting height as practicable—even to avoid glare which merely causes discomfort.

While the above remarks apply to what has been termed " disability " glare,* it is necessary also to consider " discomfort " glare, i.e. the distracting and worrying effect of bright sources, apart from definite hindrance to vision. This is not yet measurable, but it may be a definite factor in the effectiveness of the installation. The factors which affect discomfort glare may not be the same as those affecting disability glare, but they are not definitely known. It is suspected that discomfort glare is less as the brightness of the effective light source is reduced.

(b) Arrangement of Sources and Distribution of Light

Economic Considerations.

It is not possible in such a complex problem as street lighting to specify exactly an optimum system, for any system has to fulfil a good many requirements the relative importance of which varies in different installations. Moreover, the overriding factor, from which really arise all the problems of street lighting, is the cost. Granted unlimited funds, the optimum installation would possibly use a great many entirely concealed sources consuming a great deal of power, producing high visibility over the whole of the carriageway and footway, by, as it were, brute force. Since this cannot be afforded at present, the system of using relatively few exposed sources, of small power and dimensions at comparatively great spacings, has to be retained, and all possible means exploited of producing high visibility at low cost. Although the strictly optimum system cannot be specified, it is possible to explore the several variables concerned and to arrive at an installation which is " optimum " inasmuch as it can produce adequate visibility for fast traffic at a minimum cost.

An investigation of relative costs indicates that, on the whole, an installation using high posts and spacings up to 150 ft. is less costly than one using lower posts at smaller spacings giving the same amount of light. It is more costly to maintain, since a tower wagon and two men are required instead of one man with a hand ladder and a bicycle; but since a tower wagon is required for all mounting heights of more than about 18 ft., and since the cost of posts increases in much less than direct proportion to their height, little extra cost is involved in going from 18 ft. to 25 ft.

The selection of the mounting height is important technically as well as financially, as has been pointed out above. As the height is increased, glare is decreased, uniformity of brightness is improved, and object brightnesses are decreased. There seems to be no advantage, however, in going to greater heights than about 25 ft.

Arrangement of Sources.

The most important factor in producing good visibility is the positioning of the sources.

The many possible arrangements of sources can be divided into two classes from the point of view of visibility, viz. rows of sources mounted at the sides, and at the centre of the carriageway. The bright areas from the individual units in each row combine to produce a bright lane in the carriageway. If the units are mounted in the centre of the carriageway, in a straight street the bright lane is formed in the middle of the road and the regions near the kerb are darker. If mounted on the sides, two bright regions are formed near the kerbs, with a darker region between. When the road surface is wet, these characteristics become aggravated. A dark region near the kerbs is liable to be deceptive and dangerous, and for that reason the authors consider that central mounting should not be used except on streets so narrow that they are adequately covered by a single bright lane. On straight roads the use of sources on one side only is very bad and is universally condemned.

The width between rows of sources should preferably

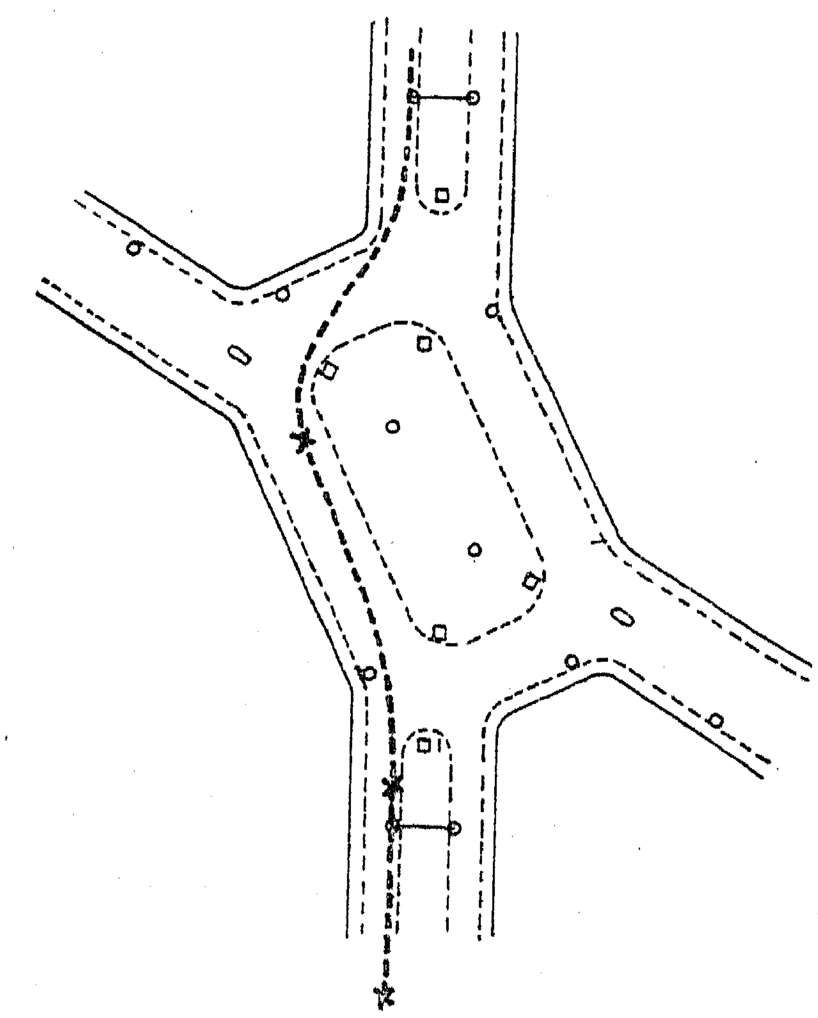


Fig. 25.—Plan of lighting scheme for intersection with roundabout.

not exceed about 30 ft., and in no case 45 ft. Thus on wide roads it is an advantage to use brackets overhanging the kerb by about 6 ft., and very wide roads require three or more rows of sources.

The spacing between sources in any one row should not exceed 300 ft. on straight roads.

The positioning of sources on bends and roundabouts is most important, and every piece of road calls for individual treatment. Straightforward uniform spacing may result in large dark areas, for the sources may subtend too great an angle in plan (i.e. they appear too far apart in perspective) and the bright areas therefore fail to coalesce. As a rough rule, for mounting heights of, say, 20–25 ft. the sources should be so arranged that from any driving position any sources more than 200 ft. distant and adjacent in perspective do not subtend at the eye an angle on plan greater than about 5°.

On roundabouts the sources should be so arranged that, as the driver rounds the island, bright areas are provided to show up the important corners and parts of the track. The plan of Fig. 25 and the three perspective

* STILES: *Illuminating Engineer* (London), 1929, vol. 22, p. 304.

views of Figs. 26(*a*), (*b*), and (*c*) (Plate 4), show a projected scheme of this kind and the way in which the sources are arranged always to come into operation without leaving a dark area in any important region. If this method of design is adopted it will be found that even in wet weather the visibility is fairly satisfactory, although it is doubtful whether any reasonable system of punctiform sources can ever be entirely satisfactory in wet weather.

On winding roads the foregoing principles lead to irregular arrangement of sources, and it is often necessary to effect compromises so as to satisfy the requirements of road intersections and of traffic in both directions.

Optimum Form of Light Distribution.

The optimum form of light distribution has been very much debated. On account of the geometry of the installation a few degrees in the polar curve in certain regions cover a large area on the road surface and a large extent in the field of view. Consequently, minor differences in polar distribution may make a very great difference to the appearance of the road. The characteristics which the authors have found to be valuable for installations of the type described above are as follows:—

(1) The ratio of the maximum intensity to that in the region of the downward vertical should not exceed about 6, in order to avoid glare.

(2) The maximum intensity in candles should, as a rough rule, not exceed one-half the numerical value of the flux emitted by the bare lamp in lumens, in order to avoid glare.

(3) The distribution between the downward vertical and the maximum should give a flat-bottomed polar curve, i.e. should follow approximately the law $I_\theta = I_0 \sec \theta$, to prevent apparent " shadows " near the post.

(4) The region of maximum intensity should extend from about 75° to $87\frac{1}{2}$°, above which angle it should diminish as rapidly as practicable. This gives long, wide, bright areas and avoids waste of flux.

(5) Adequate light should be provided upon the building, fences, etc., adjacent to the highway.

(6) In plan, the region of maximum intensity should include the direction parallel to the street axis and extend for at least 5° on the house side and 15° on the street side, in non-axial equipment; and for 10° on either side of the street axis in axial equipment.

If short spacings are employed it is theoretically possible to reduce the upper limit of the zone of maximum intensity to angles below 85°, but this is not always practicable and implies great care in installation and maintenance. A sharp upper cut-off is, in the authors' view, of rather doubtful value, as it is liable to go astray owing to poor maintenance, and they prefer a blunt distribution which can be relied upon always to work in spite of the lamp attendant.

It will be evident from the foregoing considerations that the many perennial proposals for street lighting, such as the use of fixed headlights, luminous kerbs, projectors at kerb level, etc., cannot approach the performance of an installation properly designed on more orthodox lines. It is hoped that, in view of the recent data on road-reflection properties, they will be less frequently re-invented than in the past.

(c) The Optimum Installation

The above discussion can be summarized and an installation described which is, in the sense indicated on page 262, an optimum installation. The thoroughfare considered is a main traffic route, in which a moderate volume of traffic moves at the highest permissible speed, which will generally be 30 m.p.h., or more in de-restricted roads. The road surface is taken to be one of average characteristics, satisfactorily non-skid, worn by traffic but not very polished. And further, the present economic position, which makes it imperative to keep costs low and precludes any substantial increase in energy consumption or number of sources, is considered to hold good.

It may be thought that the rapid developments of electric discharge lamps taking place at the present time are likely to affect these conclusions. In the authors' view, although the colour, shape, and size of sources may change considerably, unless there is a major reduction in both installation and running costs these recommendations will remain substantially unaffected.

The authors' specification* of the optimum installation for these conditions is as follows:—

(1) *Mounting Height.* About 25 ft.

(2) *Arrangement of Posts.* Nominally staggered, but varied where necessary for curves and crossings. The " 5° rule " given on page 262 to be followed as far as possible.

Where funds permit, much better results can be obtained if the staggered system is doubled to form an opposite system.

(3) *Width between Rows of Sources.* Not exceeding 30 ft.

(4) *Overhang from Kerb.* For carriageways not exceeding 30 ft. in width, and for sources 25 ft. high, overhang is unnecessary and in some ways undesirable. Where the width exceeds 30 ft. an overhang up to but not exceeding 6 ft. is beneficial. For good results, roads more than 45 ft. wide require three rows of sources.

(5) *Spacing.* On straight roads, the average spacing of sources in one row should not exceed 300 ft., and should preferably be less. On bends, etc., it may be desirable to transfer the sources all to the outside of the bend, in order to satisfy the " 5° rule," and the spacing may well be reduced to 100 ft. or even less on sharp bends. The spacing between any individual pair of sources in one row should not exceed 350 ft.

(6) *Light Distribution.* Approximating to the requirements in col. 1 on this page.

(7) *Light Output.* At least 3 000 lumens per 100-ft. run, and preferably twice this value, should be provided in each row of sources. (This is provided by 250-watt or preferably 400-watt h.p.m.v. lamps at 300-ft. spacing in each row.)

Cost.

The cost of an installation of this type varies considerably with local circumstances, and only approximate figures can be given. From information available, a representative annual figure, including capital charges, appears to be of the order of £350 to £450 per mile.

* Since this section of the paper was drafted, an Interim Report has been published of the Departmental Committee on Street Lighting of the Ministry of Transport, in which somewhat similar recommendations appear.

(4) CONCLUSION

This paper has been confined to present practice in a subject where rapid developments have occurred, and where future developments are, in the nature of things, to be expected. In a subject so dependent upon financial and administrative limitations, prophecy would be rash and has been avoided; and the conclusions drawn by the authors are not intended as absolute.

At present, the financial position makes safety the prime requirement; given more money, decorative effect can be added to utility, and future streets may be lighted by systems differing in some respects from those here laid down as optimum. But, for many years to come, efficiency will continue to take precedence over aesthetics.

The limit of usefulness of the high-pressure mercury-vapour lamp has not yet been reached, but for public lighting, and for traffic routes in particular, these lamps and the method of application described in this paper, seem destined to play a major part.

ACKNOWLEDGMENT

The authors acknowledge with thanks the information received from the British Thomson-Houston Co., Ltd., the Edison-Swan Electric Co., Ltd., and Siemens Electric Lamps and Supplies, Ltd.

DISCUSSION BEFORE THE INSTITUTION, 5TH MARCH, 1936

Mr. E. C. Lennox: The application of electricity to public lighting has not received from this Institution the attention which in my opinion it deserves. I would mention that another body, the Institution of Gas Engineers, has devoted special attention to the question of public lighting and has appointed a committee to deal with the subject.

In dealing with the characteristics of the lamp, the authors refer to the colour of the light, which they describe as " blue-green "; but there is no doubt that to an unbiased observer who is inside the area lighted with these lamps the effect appears to be white, whereas the light from incandescent units, gas or electrical, seems to be of a reddish or yellowish colour.

I am very pleased to see the reference in the paper to fog penetration, a statement which establishes that for practical purposes the colour of lighting does not affect its power of penetrating fog.

The authors mention that the permissible voltage-drop of their lamp is of the order of 20 to 40 volts. There are many low-voltage networks in this country, especially in semi-urban and rural areas, where voltage-drops of 20 to 40 volts may be experienced, and, especially on large interconnected networks (e.g. all those connected to the grid), a fault may cause a sudden voltage-drop over very large areas. What is the permissible sudden drop in voltage characteristic of the lamp? Two years ago I tested some lamps of this type and found that a lamp connected to a 250-volt supply failed only on a voltage-drop of 60 volts or over, which is not often experienced on supply networks. In my experience this question of voltage-drop has caused no serious concern.

In my opinion the mercury-vapour lamp with auxiliary filament in series cannot ever supplant the straightforward mercury-vapour lamp. Apart from its lower efficiency, it seems to be of much more fragile construction; it does not seem to be acceptable except in the case of promenades, where colour correction is of value. Moreover, the additional yearly maintenance cost very quickly wipes out any saving effected by the non-provision of a choke and condenser. The authors' figure for the efficiency (25 lumens per watt) is, however, the overall efficiency, and should not be compared with their figures of 45 lumens per watt for the 400-watt h.p.m.v. lamp and 36 lumens per watt for the 250-watt lamp. I have calculated that the overall figures corresponding to these should be 43 lumens per watt for the 400-watt lamp and 33 lumens per watt for the 250-watt lamp.

I am often asked about the cost of providing colour correction in a mercury-vapour lamp while maintaining a fairly high efficiency. When replying to such questions I usually remark that the colour of the light emitted by the lamp has not proved a very serious drawback. It is only by very biased observers that the colour has ever been questioned at all.

Under the heading " Current and Voltage under Starting Conditions " the authors mention an overload of 100 per cent, falling to 50 per cent within 3 or 4 minutes, as one of the starting-up characteristics of the lamp. Although this may be acceptable for separately-connected lamps, the connection of a group of mercury-vapour lamps to a separate street lighting circuit at one point, usually the tail end of a low-voltage network, raises a problem which may give some trouble to distribution engineers.

No reference is made in the paper to the question of fusing the lamp circuit of a mercury-vapour lamp. The fuses must, of course, be over-rated and this is rather a serious matter, especially in areas where earthing is a difficult problem. The earthing of a lamp column, in the North-East Coast area at any rate, is often a difficult problem, and the fixing of 7·5-amp. and 10-amp. fuses presents difficulty, especially in the case of a wiring fault to the lamp column.

Dealing with the question of lanterns, I think it is important that the brightness of the refractor is spread over the whole width of the prismatic section.

With regard to the mechanical design of the lantern, I agree with the necessity for a weatherproof fitting, and also to its being solidly fixed to the lamp column. At a height of 25 ft., lanterns are subject to considerable vibration, especially on tramway columns, and also in areas where the lanterns are placed in exposed positions. Breakage of lamp bulbs in winter storms has been due to vibration, and in the majority of cases the outer glass envelope breaks at a point near the lamp cap. I should like to ask the authors whether or not this long lamp should be supported in much the same way as the horizontal sodium lamp is supported, i.e. at a point near the end of the lamp itself. It is very important, in order to get correct results, that the lanterns should be adjusted so that the distribution of the lighting flux is in the correct

direction. Only too often one sees lanterns which are not properly fixed.

In the British Standard Specification for Street Lighting (B.S.S. No. 307), even illumination has been taken more or less as the basis, i.e. foot-candles on the road surface. The average person, however, sees not so much "even illumination" as "even road brightness." Visibility at night must be by silhouette. The road surface must be made bright, and any spill from the lantern itself is welcome and usefully employed if it falls on buildings, palings, and also on the sides of roadways, thus adding background brightness to the road brightness. The position of lamps is all-important, especially when few lamps are available, and the lighting engineer must be careful to examine the positions of lamps from all directions of traffic.

In the design of the optimum installation, the spacing is given by the authors as 100 to 150 ft. Few lighting authorities, however, can afford this class of work, and until legislation on the subject is introduced we have to be satisfied with much greater spacing. To my mind, this is not a serious drawback. I have a case in mind of a roadway lighted by 400-watt lamps with a spacing of 260 ft. which is acclaimed by all who have seen it as very well lighted, and certainly the visibility is very good; over this stretch of roadway motor-car drivers extinguish headlights quite automatically. In this case—quite a common one—the lighting authority had been invited some time previously to change over from 75- to 150-watt lamps, but could not be persuaded to do so. On the advent of the h.p.m.v. lamp, however, this authority readily agreed to make a change.

Much is claimed by the makers of certain light equipments with regard to the visual acuity—a point hardly mentioned in the paper—given by monochromatic as against incandescent lighting. Observation shows, however, that with equal lumens output a similar positioning of lamps will result in visibility of approximately the same degree. The question of visual acuity—which I would describe as creating a keen outline of the objects seen—is, moreover, not so important as the fact of seeing an object by silhouette. Considering, therefore, the annual cost of energy and maintenance, and capital charges spread over 20 years, for similar lumens output and various types of light sources, I find that the annual costs per mile for dusk to dawn all the year are roughly as follows: mercury vapour, 400-watt, £476; tungsten filament, 1 000-watt, £704; 150-watt sodium, £711. These figures indicate that, apart from the colour question, which is not important for roadway lighting, for many years the lamp described in the paper will be the most prominent type, and certainly the cheapest form of illumination for roadway lighting.

The accident hazard at night is from 3 to 5 times that during the day, and more than 50 per cent of the accidents which take place in winter occur during lighting hours. We must therefore express our gratitude for the introduction of this lamp, which should help to make safe our roadways at night.

Prof. E. W. Marchant: I should like to say a word on a point to which Mr. Lennox has referred, namely the advantages which accrue from effective lighting, in increasing the safety of the roads, a matter which is nowadays affecting the public conscience. We are all becoming educated as regards safety, and where public lighting is going to improve safety there seems to be a strong case for using some of the money in the Road Fund for this purpose. A great deal of money is spent on the improvement of road surfaces with a view to increasing safety, but surely the lighting of the roads is just as important from a safety point of view.

The second point in connection with public lighting to which I should like to refer is the fact that the public-lighting load continues during the night time when other industrial loads are not usually very heavy, and therefore there is a strong case for the supply of electrical energy for this purpose being given at a relatively low rate.

With the h.p.m.v. lamp we have now reached an efficiency almost as high as that of the flame arc lamp, which is the most efficient lamp that has so far been produced. I made a number of tests on this type of lamp many years ago, and obtained an efficiency of about 8 candle-power per watt for the lamp by itself; the lamp described by the authors has almost reached that figure. The flame arc is, of course, of little value for practical use, because it costs too much to maintain.

I am interested in the authors' statement that the presence of oxygen in the outer bulb of the lamp has been found to reduce the formation of an absorbing film on the inside of the inner bulb. Can they give us a little further information regarding this effect? There must surely be some sort of leak between the two, or there may be some cooling action due to the gas in the outer bulb.

The use of a ballast resistance is also a matter which is of interest. In the Nernst lamp, produced about 30 years ago, a ballast resistance was used in the form of heated iron wire, which, in a hydrogen-filled bulb, has a very high positive temperature-coefficient; thus, a relatively small voltage-drop was sufficient to compensate for the negative temperature-coefficient which exists in the lamp itself. In the authors' lamp the filament itself is used both as a source of light and as a ballast resistance; this is a very interesting development. The iron-wire ballast resistance would not do that so well, but it occurred to me that it might be possible to get as high an efficiency in the lamp *as a unit* by something of that kind, as by the use of these other devices.

Reference has been made to the colour given by the authors' lamp; I do not think that colour matters at all at night. I saw an installation equipped with these lamps when I was attending a meeting of the Association of Public Lighting Engineers at Margate, which is a place where one might expect criticisms to be made on the score of colour; but I did not hear any adverse comments.

I have driven over a good many miles of streets lighted by mercury-vapour lamps, and I find that that advantage which is gained by this type of continuous lighting is enormous; the clearness of the objects on the road is very greatly enhanced by their use, and the silhouette is much stronger than that given by other types of public lighting.

Mr. W. J. Jones: One of the important points to bear in mind in connection with the introduction of this new lamp is the fact that it has fired the public imagination,

and that as a result we have achieved remarkable results in the conversion of gas lighting to electric lighting; during the last few years we have converted many hundreds of miles of street lighting from gas to electricity. In fact, in this particular matter I think that we are ahead of the rest of the world, because we have now nearly 1 000 miles of street lighting carried out by this means, which is rather more than is to be found in all the rest of the world put together.

I am sorry to find that the authors do not give consideration to the economic aspects of their subject. If, for example, the annual costs to a street-lighting authority of the 1 000-watt tungsten-filament lamp with a reasonable lantern are compared with the annual costs of one of these new lamps with a reasonable lantern, it will be found that considerable savings can be made. In the case of street lighting which is on only from dusk to midnight it will be found that with this new system it is possible to pay the interest on the capital expenditure with a surplus of 25 per cent, and with all-night street lighting that capital expenditure can be met over a reasonable period of years with at least a twofold cover.

The optimium installation to which the authors refer is of the greatest importance to all interested in street lighting, particularly in view of the interim report of the Ministry of Transport on this subject. I am very glad to note that, while the authors mention the light output as being 3 000 lumens per 100 ft. of road, they do not really recommend this figure, because frankly it does not seem to me a satisfactory amount of light to use on main traffic routes. I would urge that the 250-watt lamp should not be used for this purpose, but that the 400-watt lamp should be employed in order to provide reasonable illumination. A committee of the British Electrical Development Association has been giving consideration to these problems for a number of years, and I give in Table A the recommendations which they have made for various classes of streets. The primary requirements in street lighting are to be sure of having a sufficient amount of light and to be able to distribute it effectively.

Table A

	Total light emission of bare light source, lumens per sq. ft. of carriageway	Minimum mounting height, ft.	Minimum illumination anywhere on carriageway, ft.-candles	Maximum spacing/height ratio
(1) Main traffic routes in towns, and shopping areas in suburbs	3	25	0·2	8
(2) Traffic routes where vision of distant objects is of primary importance	1·75 — 2	25	—	10 — 8
(3) Secondary traffic routes passing through roads of a residential character	1·2	20	0·1	8
(4) Residential roads	0·3	15	0·01	12

Mr. Howard Long: I notice that the authors make a passing reference to the application of the mercury-vapour lamp to industrial lighting. Such experience as has been gained in this field already shows the remarkable potentialities that exist. The drawbacks which were expected in the early days—stroboscopic effect, colour, etc.—have not shown themselves. The authors refer to the fact that the colour characteristics have not proved a drawback, as it was originally thought that they might, for street lighting. A similar remark applies to the use of these lamps for interior lighting of an industrial character. When one goes into a building lighted with these new lamps one has the impression that the sun has come out.

Mr. Paterson, in his recent review of progress,* mentioned that there are two schools of thought on this matter of public lighting. One school permits a certain degree of glare, due to high-angle rays which are necessary to obtain uniform brightness; and the other considers that one should screen the light source to, say, 5° or 10° below the horizontal. I rather support both schools of thought. I think that in the streets of cities, where there are buildings for backgrounds, the surroundings are cheerful, and brightness of the streets is of importance, no cut-off is essential; but for arterial-road lighting I am not yet convinced that the importance of the brightness factor is greater than that of not viewing bright sources against a black background.

Mr. T. N. Riley: In discussing the principles of street lighting the authors state that objects in the streets " are almost always darker than their background and are seen in dark silhouette." Instead of " seen " I would prefer to say " discerned," as the authors do in an earlier sentence, because one cannot really see an object unless the eye can be focused upon it, and the brighter the background the more difficult does this become. By day, the contrast in brightness is small, and it is quite possible to focus the eye on the object except when it is on the skyline. Then, although the object is quickly discerned, it is not seen in the strict sense of the term, and it is extremely difficult to judge its size, distance, or relative motion.

In street lighting it is most important that one should be able to judge the position and relative motion of other traffic. By arranging the light sources to get the maximum specular reflection from the road surface, real

* *Journal I.E.E.*, 1936, vol. 78, p. 171.

vision is surely made more difficult. A cricket screen is useful only to locate the ball direction quickly; if one is to hit the ball, the eye must be focused on it, and a screen of too great brightness relative to the other background would make this impossible.

A further objection to the bright road surface is that it accentuates contrast with the road boundaries, and so makes it more difficult for one to see objects approaching the road laterally. The authors state that a diversity of more than 5 : 1 is undesirable on the road itself, and yet they advocate a method of lighting which may make the difference in brightness between road surface and grass verge of the order of hundreds to one. A study showing photographs of static objects on the road is unconvincing, since a large part of the traffic risk is due to other objects moving towards the road from the dark verge, and the driver is under continual eye strain in trying to see them against the glare of the road surface. It is more comfortable and safer to drive in moonlight with a fraction of the road brightness the authors advocate because the eye has no sharp contrasts to deal with and can become adapted to the low level of illumination.

To provide the desired specular reflection the authors design their lanterns so as to get the maximum light projection at an angle of about 80° to the vertical. I am surprised that they do not refer to the practice, which seems to be most favoured on the Continent, of cutting off the light beyond an angle of about 60° to the vertical. At long distances the source cannot then be seen by the driver of a car. As he approaches the light, the hood of his car cuts off the direct view of the source. Admittedly this gives less road brightness than the authors' arrangement, but it may give better vision because the light-source glare is absent.

It is suggested that a more practical trial of the merits of different systems than that afforded by photographing static objects, would be to make measurements of the reaction time of a driver's response to objects made to move laterally across his field of view from a point well away from the road. The test might be made after (*a*) short and (*b*) long periods of exposure to the particular lighting system under test. Suitable methods of test have already been developed by the National Institute of Industrial Psychology and similar organizations.

(*Communicated*) Many speakers in the discussion referred to the headaches which some car drivers feel under mercury-vapour discharge lamp lighting, and suggested various explanations. In my opinion the eye strain caused by the effort to focus dark objects on a bright background, and by the effort to see in a field involving extremely high contrasts in intensity, is quite sufficient to account for such headaches. That is to say, they are due to the direction of projection and intensity of the light rather than to its quality as was suggested, although the quality may have a small bearing on the matter in that the mercury-vapour lamp is strong in those wavelengths to which the eye is most sensitive.

Such eye fatigue could easily be tested by the psychological methods I have referred to above.

Mr. L. J. Davies: I have recently had the opportunity of seeing discharge-lamp installations both on the Continent and in America, and although I saw excellent installations in both of these places, on the whole, as far as I could judge, those in England seem to be much the best. This is due, no doubt, to the fact that from the beginning we have always used discharge lamps in lanterns specially designed to take the linear light source that is characteristic of the discharge lamp.

There is one sentence in the paper that I do not quite understand; in Section (2)(*a*) the authors say "The precise way in which the light can best be distributed is now known. . . ." Surely they do not mean to imply that we know all that there is to be known about the distribution of light? There are no doubt many things to be learned about the intensity, distribution, and angle, of the light.

I should like to endorse what the authors say about the correct planning of installations. Road-lighting installations cannot be planned on the drawing board; it is essential that every individual piece of road should be viewed, and be viewed, as Mr. Lennox said, from every possible angle. It cannot be said that a main road has been lighted adequately when, by not lighting a side road which joins it, part of the background of that main road is left completely dark. In many cases the lamps used on the main road should be carried some distance down the side road, even though that side road does not carry a large burden of traffic.

Prof. J. T. MacGregor-Morris: In the film that was shown by the authors, it was specially pointed out that the lamps were placed on the outside of the curves. I remember one of my colleagues saying to me how foolish this practice is because when one is motoring along near the inner edge the car hides the edge of the road from the source of light on the outside edge, and therefore it is not possible to see the inside edge; particularly is this true when it is misty.

My second point concerns the physiological action of these gaseous discharge lamps on the eyes. I have taken many opportunities of asking people what they think of the illumination of roads lighted by these lamps, and in the vast majority of cases the reply is that they are excellent, that the car headlights can be turned out and that everything is in their favour; but there are just a few people whom I have met who tell me that they get a headache when they come to a section of the road thus lighted. What is the explanation? I suggest that possibly this may be that the source of light is confined largely to two lines in the spectrum, and that a man tries to focus an object first with one line of the spectrum and then with the other, so that the eye is tired by changing the focus back from one line to the other. This is the only suggestion that I can make.

Mr. R. Maxted: With reference to horizontal operation, on page 253 the authors refer to a series magnetic deflector; I would point out that this arrangement gives the reverse compensation from that required by the mercury lamp. A number of circuits are known and in practical use which give automatically the precise compensation required under greatly varying conditions of operation.*

Turning to the question of lantern optical design, I suggest that it is scarcely correct to say that a flat-bottomed distribution cannot be obtained when hori-

* R. Maxted: "Horizontal Discharge Lamps," *Electrical Review*, 1936, vol. 118, p. 309.

zontal prisms are applied to a vertical lamp. It is my experience that a flat-bottomed distribution is, if anything, more nearly achieved in a lantern where horizontal prisms have been correctly incorporated than in the more orthodox refractor lanterns. For example, the lantern shown in Fig. 19 incorporates a number of prisms at top and bottom, and in this case the flat-bottomed distribution is quite good.

The authors pass over a little lightly the question of flexibility of control and high coefficients of utilization in lantern design. With the more compact filament-lamp source a high coefficient of utilization with flexible control of the distribution is more readily obtained than in the case of the long discharge-lamp source, and as much as 40 per cent of the output from a filament lamp can be put on the road surface, either with or without a cut-off; so that, unless at least 13 per cent of the output from the discharge lamp can be applied under similar conditions, the higher efficiency of the discharge lamp is not realized in practice. I should like to ask the authors whether their reference to the 10 per cent gain in utilization attributed to the horizontal lantern [Section (2)(*a*)] refers to the bare lamp output or to the output from a vertical lantern. It has been my experience that the average vertical lantern will put about 14 per cent of the bare lamp output on a 30-ft. carriage-way, whilst with the horizontal unit shown in Fig. 21 the proportion is 24 per cent, representing a gain of 70 per cent over the performance of the vertical lantern. The amount of light reaching the carriageway is not, of course, the absolute criterion, and in fact my colleagues and I have stressed the importance of the lighting of other backgrounds in a recent paper.* At the same time, unless there is something abnormal about the distribution—too much light to the side of the road, and so on—I agree that the light reaching the carriageway is a good basis of comparison of the general efficiency of lanterns.

Referring to the distributions obtainable from horizontal lanterns, I cannot agree with the authors that it is necessary to rob any part of the distribution in obtaining high efficiency. I agree, however, that it is in the vertical plane parallel to the road axis that good control of the distribution is required. The horizontal lamp gives almost a point source in this plane, so that very complete control of the output is thus obtained, and any desired distribution is available. With the exceedingly flexible control and the unusually high coefficient of utilization obtainable, ample quantities of light are available for all angles between the downward vertical and the horizontal, so that there is no need to rob any part of the distribution. The location of the maximum beam is entirely a matter of choice, and by means of the horizontal lamp very high intensities can be produced, together with a flat-bottomed distribution. One point of interest with regard to horizontal lanterns is that the utilization is increased by raising the cut-off, as less light then has to be handled by reflectors or refractors.

Mr. H. Duckworth: On page 242 the authors, in dealing with their lamp, refer to "a stick of rare-earth oxides," and, lower down, to "rare gases, such as argon." I should be glad to have a little more information on these points.

Prof. Marchant has already referred to another point, namely the curious effect of the oxygen in hindering the formation of an absorbing film on the inside of the inner bulb; perhaps the authors would also deal with that.

Mr. T. Wadsworth: Have the authors any information as to how the atmospheric conditions in different districts affect the efficiency of the lantern?

Mr. A. C. H. Frost: At a bend on a road where the buildings are fairly high, the important thing in my opinion is to define clearly the near-side kerb, and not the off-side kerb. Placing the lamps on the inside of the bend may achieve this more readily than does the practice usually adopted.

Mr. G. S. C. Lucas: With regard to the use of electric discharge lamps in drawing offices, probably the most serious difficulty in this respect is the stroboscopic effect of the lamps in interfering with the instruments which the draughtsman uses. This can be serious and, unless it is overcome to some extent by supplementing the lighting with tungsten lamps, complaints are constantly raised against the lighting.

Turning to the question of headaches suffered by road-users where electric discharge lighting is used, is not this a question also of the repetitive glare produced as the road-user passes the lanterns in turn? The amount of light entering the eye rises rapidly as one approaches each standard, and falls immediately one passes under it. This effect can, I think, lead to headache, and, unless the installation is carefully planned to cut down the repetitive glare as far as possible, some headaches are to be expected.

In referring to the control circuit, the authors state that the current-limiting device may be either a choke or a condenser. At first sight it would appear that the condenser would control the current in exactly the same way as the choke, except that the power factor would be a leading instead of a lagging one. By using chokes and condensers on alternate lamp standards one could perhaps maintain unity power factor. The difficulty is, however, that if any attempt is made to limit the current by a series condenser only, the lamp circuit becomes quite unstable. This is probably due to the non-uniform control of the current from one cycle to the next, because the condenser does not limit the peak value or the rate of the charge, but only the quantity of the charge entering the condenser. The result is that, as the lamp runs up, the peaks of the current increase and there is a longer time-interval between the successive peaks of current, until the point is reached when the lamp will no longer strike. The difficulty can be overcome by certain circuit arrangements, but these show very little advantage in regard to cost over the present method of the choke and compensating condenser.

(*Communicated*) Referring to the optimum distribution for modern road-lighting conditions, it appears to me that the authors tend to over-emphasize the importance of the concentration of light at given angles near the horizontal, and that they have determined this optimum distribution from a consideration of long, straight roads with tarmac-surfaces polished by traffic. Under these conditions, and where the traffic is not too heavy to intercept the long-

* L. J. Davies, R. Maxted, and G. S. Lucas: "The Importance of Kinematical Factors in Roadway Illumination," *Illuminating Engineer*, 1935, vol. 28, p. 381.

distance brightness streaks, such a distribution has much to be said for it; but at bends, turns, and road junctions, such as are shown in Fig. 26 (Plate 4), the undue concentration of light near the horizontal can add little or nothing to the brightness of the road and tends only to reduce visibility by adding to glare.

It seems to me that what we must look for is a means for controlling the quantity of light near the horizontal without fixing definite distributions, so that optimum conditions can be maintained for various road surfaces and traffic densities, and particularly at points where the view is restricted by bends and curves. Furthermore, the possible development of concrete roads should not be lost sight of, because the light-coloured surfaces, whether wet or dry, will be less dependent upon the extreme specular-reflection methods of producing high and even road brightness. It may therefore be possible to reduce the intensities between 80° and 90°, thereby reducing glare and enhancing the general effect.

The authors gave a good illustration of the importance of curved roads and bends by their comparison of the still photographs of the straight roads and the moving film, taken from a car, of a stretch of road consisting primarily of curves and bends. In the moving film it was apparent that on the curves and where the letter-box and the pedestrian crossing the road were pointed out, local lighting from a single lantern was playing a major part and the long-distance streaks were for the most part absent. The clear view ahead was restricted and was certainly much less than 200 ft., and the curb and footpath backgrounds were of considerable importance. The general impression of the road as seen on the motion picture was different from that of the still photograph; in the film, where two or three lanterns are in view at any one moment the observer is conscious of the bright points of the lantern to a greater extent than with the still studies.

It is perhaps unfair to make rigid comparisons between the still photographs and the moving film, owing to the extreme difficulties presented in the latter case. Photographs are proverbially unreliable in street-lighting work, even when care is taken to calibrate the density scale. Apart from the control of the density, the photographer, no matter how well-intentioned, will automatically station himself on the road where the lighting conditions and the road traffic are optimum for the point he wishes to demonstrate. With the moving film a far better general impression is gained, but the difficulties of taking and calibrating the film are increased.

The motion film is a useful aid to a study of certain problems in street lighting where relative densities are not important, as, for instance, the position of the background relative to the road traffic, but great care must be taken when brightness distribution is in question. One important point that the motion film establishes, however, is the fact that the impression of the road lighting obtained by a moving observer is different from that obtained by a stationary observer, a point which is often overlooked and does not receive the attention it deserves.

On page 263, under the heading "Overhang from Kerb," the authors state "For carriageways not exceeding 30 ft. in width, and for sources 25 ft. high, overhang is unnecessary and in some ways undesirable." It seems to me that the overhang is determined very largely by the condition of the road surface, and where specular reflection from the road surface forms the major brightness background on the road some overhang is necessary. On the other hand, concrete roads with light-coloured surfaces will not need the same overhang. The authors appear to have determined their light distributions for polished tarmac surfaces, but to have departed from this basis in their specification of overhang on page 263.

Lieut.-Col. K. E. Edgeworth (*communicated*): Referring to the question of the use of an external reflector and to the "pool of light" difficulty which was observed when it was tried, it would appear that the difficulty can be avoided if the mirror is so designed that the whole length of the light column is visible in it from the direction in which maximum illumination is desired, say 85° to the vertical. Thus a mirror 12 in. long could be used at an inclination of 100° to the vertical to give an increase of 34 per cent in the critical direction, or a mirror 24 in. long at an inclination of 95° to give an increase of 62 per cent.

Table B shows various arrangements for the mirror, correct and incorrect, and also polar diagrams indicating the amount of added light obtained in each case. The light scale is the same as that of Fig. 17 of the paper. The pool of light which is observed with a short horizontal mirror appears to be adequately accounted for, as the direction of maximum added light is at 75° to the vertical [case (*b*)]. If the mirror were shorter the effect would be accentuated. With regard to case (*f*), it would probably not be worth while to use a mirror whose length was as great as 48 in., but a 24-in. mirror would be worth considering. There is an interesting parallel between this problem and the design of a searchlight mirror. The concentration of light which is possible in the beam of the searchlight depends on the ratio between the size of the mirror and the size of the incandescent source of light.

Finally, with regard to the estimates of cost which were put forward during the discussion, in which the system described in the paper was compared with a system employing 1 000-watt incandescent lamps, several speakers overlooked the gain due to the optical system of the h.p.m.v. lighting, which appears to be about 2 : 1, making an overall gain of 6 : 1 (as no mention was made of any optical system being in use with the 1 000-watt lamps). In other words, the system described in the paper was debited with the cost of the optical system without being given credit for its advantages.

Mr. R. Rigg (*communicated*): With reference to the 250-watt lamp with its "choke losses," etc., this lamp appears to have no economic advantage over a 500-watt metal-filament lamp when energy costs 0·67d. per unit.

The authors state that a relay has been used for the purpose of switching in a metal-filament lamp in the event of a surge extinguishing the h.p.m.v. lamp. My knowledge of the performance of these lamps in a street installation over a period of 2 years would lead me to state that the fitting of such a relay is an expensive and unnecessary precaution.

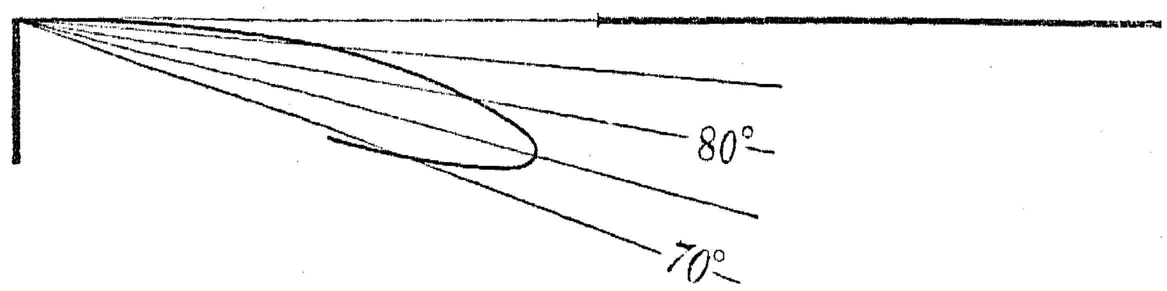
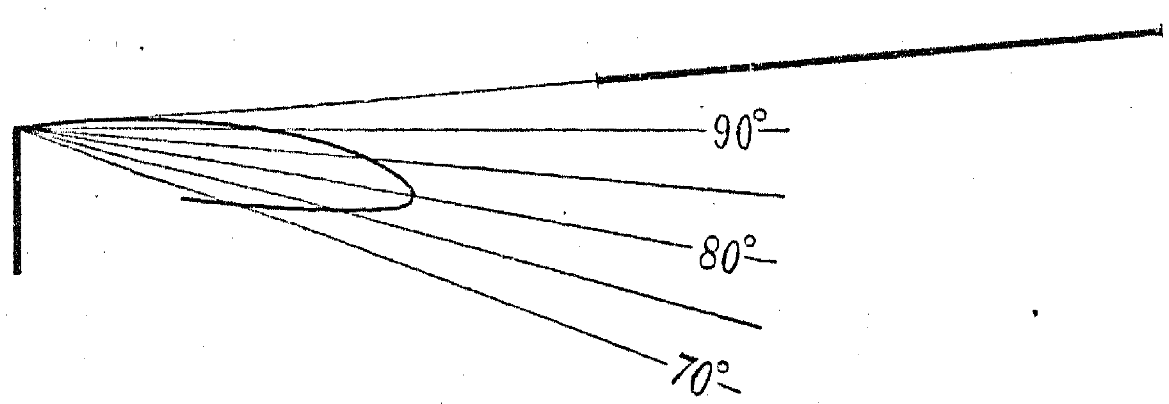
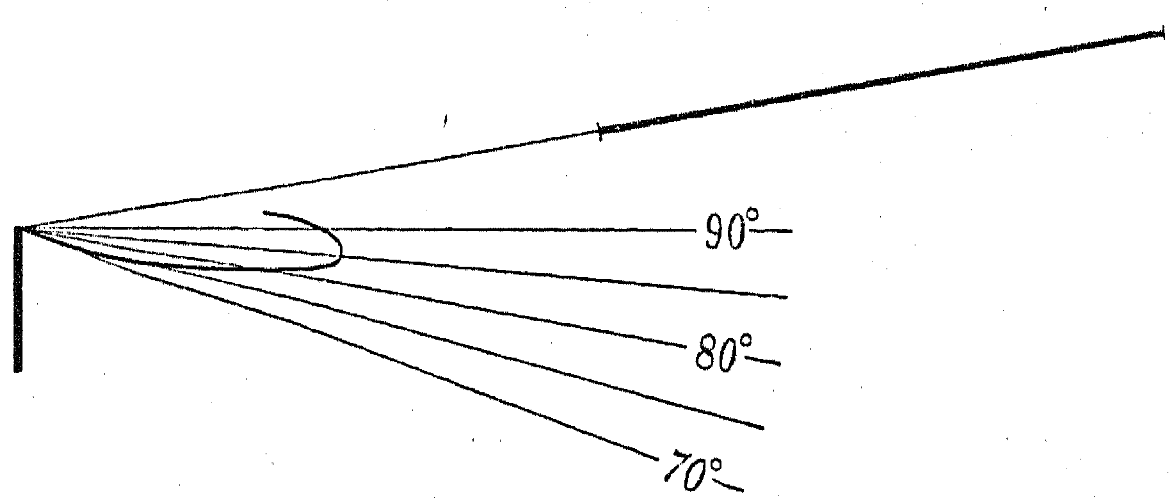
Fig. 14 shows both a time switch and a fog switch fitted in the base of the lighting unit. These, of course,

would be superfluous except at the control point of a complete installation.

A representative annual figure, including capital charges, quoted by the authors for their optimum installation, is £350–£450 per mile. Taking a mean value of, say, £400, with energy at 1d. per kWh the energy cost alone will amount to about £200. Thus local tariffs will be of major importance in computing costs. I should be obliged if the authors would supply us with an analysis of their estimate.

[The authors' reply to this discussion will be found on page 275.]

Table B*

Arrangement of reflector	Angle with vertical	Added light	Total light
	degrees	candles	candles
(*a*) Without reflector	90	—	2 700
	85	—	2 900
	80	—	3 100
	75	—	3 000
	70	—	2 800
(*b*) With horizontal reflector 12 in. long 	90	0	2 700
	85	1 800	4 700
	80	1 500	4 600
	75	1 700	4 700
	70	1 300	4 100
(*c*) With reflector 12 in. long inclined at 95° to vertical 	90	700	3 400
	85	1 100	4 000
	80	1 300	4 400
	75	1 000	4 000
	70	700	3 500
(*d*) With reflector 12 in. long inclined at 100° to vertical 	90	900	3 600
	85	1 000	3 900 (+ 34%)
	80	700	3 800 (+ 23%)
	75	300	3 300
	70	—	2 800
(*e*) With reflector 24 in. long inclined at 95° to vertical. This arrangement satisfies the condition that the light column shall be completely visible at 85°	85	—	1 800 (+ 62%)
(*f*) With horizontal reflector 48 in. long. This arrangement satisfies the condition that the light column shall be completely visible at 85°	85	—	2 500 (+ 86%)

* Throughout the Table the light column is assumed to be 3 in. high.

NORTH-EASTERN CENTRE, AT NEWCASTLE, 9TH MARCH, 1936

Mr. F. L. Cator: I notice that the inner lamp is now held within the outer glass envelope by means of two wire spider supports. Formerly in this particular lamp the inner bulb was held in position by means of spring clips attached to the lead-in wires. Has it now been generally decided that the method of using wire spiders is the more satisfactory?

I also note that the axis of the tungsten spiral forming the upper electrode is in the horizontal plane, while with the lower electrode this axis is in the vertical plane. Why is this? I should have thought that it would have first been decided that either the horizontal or the vertical position was the better, but that, this decision having been made, both electrodes would have been mounted in the same plane.

The authors state that on a very cold morning the

lamps may take 5 minutes longer to run up than they do normally, while we know that when the lamps have been alight for some time and are switched off, they have to cool down before they will restrike. This period of cooling down and then running up to full brilliance will take up to 10 minutes. Have the authors a curve showing the optimum starting temperature for a lamp? What I mean by the optimum starting temperature is the temperature of the surrounding air to give the lowest striking voltage for the lamp and at which the lamp will most rapidly run up to full brilliance. I believe that I am right in saying that if a curve is plotted for a typical h.p.m.v. lamp, with striking voltages as ordinates and ambient temperatures as abscissæ, a curve will be obtained which falls and then rises again, the lowest point of the curve showing the optimum starting temperature to be about 30° C.

As regards the choice of a lantern from the optical point of view, most street-lighting engineers and others interested in the installation of public lighting would like some information as to how to choose, from among the many lanterns which are offered them, the lantern with the best optical characteristics. I suggest that they should fix their attention on three considerations: (1) The polar curve of the lantern. (2) The iso-foot-candle diagram of the lantern when mounted at the recommended mounting height. (3) The coefficient of utilization of the lantern.

From the polar curve of the lantern the engineer is able to see in what manner the lantern redirects the light from the lamp in a vertical plane; he can therefore form an opinion as to the amount of glare that will be experienced from this particular lantern, and to what extent the lantern will make use of the reflection properties of the road to produce the brightness streaks which are so important.

The iso-footcandle diagram for the lantern and the coefficient of utilization are of course closely connected. After all, the main job of a street-lighting lantern is to redirect as much of the light as possible on to the road, and not to floodlight the surrounding buildings or fields. The coefficient of utilization of the lantern measures the efficiency of the lantern in that it tells the engineer what percentage of the light given out by the lamp actually leaves the lantern, and what proportion is absorbed by reflectors, refractors, etc. The iso-footcandle diagram further tells him how efficiently the light has been directed on to the road.

The authors emphasize the fact that objects are seen by reason of the light which leaves the road, i.e. against the brightness streaks produced, but it must be remembered that the amount of light which leaves the road, which decides the degree of brightness and the extent of the streaks, is itself dependent on two things—the reflection factor of the road and the amount of light reaching the road. The public lighting engineer has no control over the former, but he can increase the latter by installing the lantern whose iso-footcandle diagram indicates that it has been carefully designed to redirect the maximum possible amount of light on to the road.

Turning to Plates 1 and 2 of the paper, I think I am right in saying that, as a commercial proposition, a refractor cannot be made more than 60 per cent efficient, whereas a good reflector may quite easily be 80 per cent efficient. I should like to ask the authors whether, in their view, a lantern using refractors only can ever be made to be as efficient as a lantern using both reflectors and refractors.

As regards the T-shaped brightness streaks, what methods are there for broadening these streaks? Does installing the h.p.m.v. lamp in a horizontal position give a wider streak? From the optical point of view the horizontal position seems to be the more natural one when a road has to be lighted from posts placed at the edge.

The authors recommend that the lanterns should always be placed on the outside of a curve, but some curves are so gradual that the road appears practically straight when driving along it. What, in their opinion, is the maximum radius of the curve beyond which the ordinary staggered formation of units should be adhered to? I assume that this depends upon the width of the road.

Many people have remarked that under h.p.m.v. street-lighting they can not only see objects much more clearly defined, but they can judge distance more easily and accurately. The difference appears to be similar to the difference between looking at an ordinary photograph with the naked eye, and looking at a photograph through a stereoscope. I should like to know the authors' view as to the reasons for this effect. I suggest that it is due in the main to reduction of chromatic aberration and diffraction blurring. Since the light from the h.p.m.v. lamp is radiated at two or three definite wavelengths only, the lens of the eye is able to focus the image brought by these wavelengths much more sharply on the retina. Since diffraction blurring increases with increasing wavelength, it is natural to assume that it will be much less pronounced under light from which the red wavelengths are absent.

Mr. E. C. I. Macdonald: A number of references are made in the paper to the sensitivity of the h.p.m.v. lamp to change of voltage, and the extinction of the arc due to sudden voltage-drops. This points to the necessity for seeing that any motors supplied from the same network will not adversely affect the operation of the lamps, particularly at the end of long distributors some distance from the substation. In many cases the presence of motors on networks supplying public lighting cannot be avoided, but, provided adequate precautions are taken to restrict the motor starting current, trouble on the lighting circuits due to voltage fluctuations can be eliminated.

In connection with the question of cost, it must be borne in mind that even a modest layout of h.p.m.v. lighting frequently replaces a very scanty incandescent layout supplied by equally scanty distributors in roadways where there is no convenient means of reinforcing the public lighting network from the local distribution system. The reinforcement of the public lighting mains to deal with a full-scale h.p.m.v. layout then becomes an appreciable item in the total cost. On page 249, at the top of column 2, the reference to overload is ambiguous, and I suggest that the overload referred to should be related specifically to the load due to the lighting rather than, as might be inferred, to the rating

of the distributor. This will avoid any suggestion that an overload can when convenient be treated as a normal rating, a state of affairs as dangerous as it is undesirable. I agree that in general the normal current rating of the distributor will be ample to deal with the load changes which will occur with h.p.m.v. lighting. During the last 18 months I have had to lay out more than 400 networks of all types and sizes, and of the thousands of distributors comprising those networks the cross-section of less than 5 per cent—probably nearer 1 per cent—was settled by considerations of current loading rather than of voltage-drop, the distributors being rather larger than would be necessary on account of current alone.

On page 252 the authors refer to the switching surge. I should be glad if they would give some indication of the magnitude and duration of this surge.

I notice from page 253, col. 2, that when testing a lamp it is necessary to allow it to reach full brilliance before switching it off. According to the data given in the paper this may take 5 minutes or more. Can the authors state what effect this requirement will have on maintenance costs? The time occupied by a given round of testing will apparently be greatly increased thereby.

Mr. I. J. St. A. Crawshaw: The authors' proposals for the lighting of a "roundabout" (page 262) are interesting, and the example chosen shows that scientific principles are being applied very carefully to the problems of street-lighting. They make it very clear that objects on lighted roads are best seen when they appear dark against a much lighter background, and in this connection I should like to know whether they have experimented with h.p.m.v. lamps and white kerbs, the latter preferably bevelled. It is my experience that a white kerb appears, under reasonable lighting conditions, as a continuous broad ribbon, and helps to bring into prominence dark objects on the roadway because such objects break the continuity of the ribbon. The bevelled kerb appears to be brighter than a kerb with vertical edges, possibly because more incident light is reflected from it towards the observer. It has also the advantage that it is not so likely to cause a skidding car to overturn when the wheels come in contact with it, as a vertical kerb which arrests the motion suddenly.

I should like to know whether the new lamp has any features that would make it particularly useful for other public lighting, for example the lighting of aerodromes and lighthouses, or for floodlighting purposes.

Mr. D. Balmain: The paper seems to deal with street lighting chiefly from the point of view of the motorist, and I should like to know whether the h.p.m.v. lamp gives any help to the pedestrian in crossing the road. Can he see approaching vehicles as well as the motorist can see him? Secondly, it appears to me that it is not so much the lamp adjacent to the motorist which is of importance, but those farther along the road; for example, the lamps $\frac{1}{4}$ mile away are far more effective and accentuate the presence of anything on the road more than those nearest to the motorist.

Dealing with the question of the efficiency of glassware, glass for street-lighting purposes can be designed, and indeed has been manufactured, such that quite a large portion of green light and violet light is passed through, and the absorption factor is low; but it is more costly than the glass normally employed. I think most of the glassware for this purpose is a compromise between what we should like and the cheapest form in which it can be made.

As regards the effect upon the retina of bright and dark patches on the road, I would point out that the bright patches appear, to anyone travelling in a car or walking down the road, to be proceeding along the road with the traveller.

Mr. N. Elkington: I should like information upon the following points dealing with the lamp as an energy-consuming appliance to which supply has to be given without interference with other appliances fed from the same distributors.

In the first place, when the lamp is equipped with the usual condenser and choke how closely can the resultant power factor of 0·15 at starting and 0·85 while running be considered to be vectorial at the fundamental frequency? For instance, if the voltage-drop in the conductors up to the control point for the lamps is calculated for sine waves at a power factor of 0·15 or 0·85, will the result require some correction to allow for wave-distortion? What would the correction be for (*a*) a line consisting practically wholly of resistance, (*b*) a line where the reactance slightly exceeds the resistance? A harmonic analysis of the current wave would be of interest.

Secondly, assuming a group of lamps that would be extinguished during normal running by a sudden voltage-drop of 20 volts, and for which a starting kick, after the initial surge, of more than 20 volts is produced in the general network, what would be the behaviour of the lamps? Would the lamps build up continuously or would they go in and out owing to extinction by the sudden voltage-drop caused by the starting current? A curve elucidating the steady characteristics is given in the paper; a similar curve and explanation for instantaneous conditions would be appreciated.

Thirdly, what is the pressure in the h.p.m.v. lamp, and how does it vary during the running-up process?

Finally, I would mention that to labour the advantage of chokes with variable tappings over graded lamps almost detracts from the effect of the statement of the outstanding advantages. The switching-in surge in metal-filament lamps does occasionally give trouble. Consequently, if with h.p.m.v. lamps any line of research promises to reduce the surge without appreciably increasing the cost of the lamp, it is worth following up.

Mr. James Dickinson: Fig. 9 appears to me to be a little misleading. Perhaps what is actually happening could be brought out more forcibly if the power factor, defined as the ratio of the active power to the total or apparent power in the circuit, were to be plotted against phase angle as abscissa (see Fig. A). Thus the phase angle of the circuit represented by Fig. 9 would appear to be suddenly jumping from about 23° lagging to 23° leading when the capacitance in parallel with the lamp increases to a certain value represented in Fig. 9 by approximately "100 per cent capacitance."

The power factor does not pass steadily through the value unity, as the current in the supply leads changes

from lagging to leading, owing to the shape of the current and voltage waves, one of which is discontinuous.

A recent paper* giving fresh significance to the conception of reactive energy, discusses the problem of the

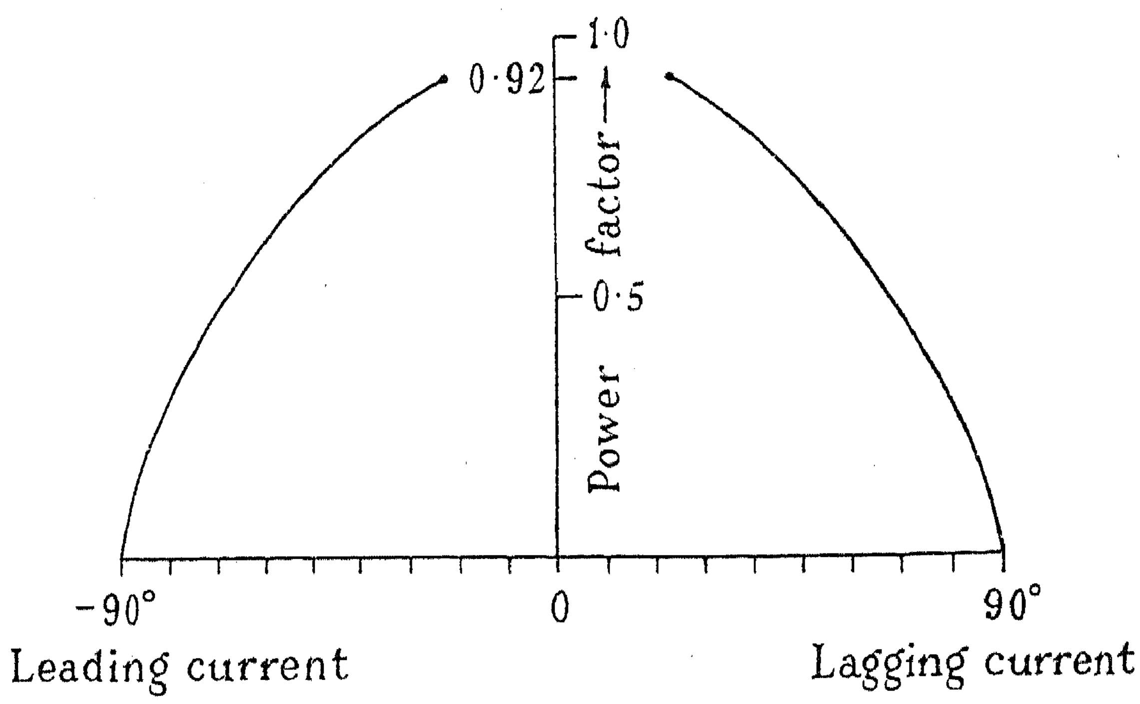


Fig. A

power factor of a mercury-arc rectifier, which, with the neon lamp, shares the characteristic of the h.p.m.v. lamp in that the currents and voltages supplied exhibit non-sinusoidal wave-forms. With such irregular wave-forms, there is a real distinction between power factor and the cosine of an angle of phase displacement, which is fundamental.

If e represent the instantaneous value of the p.d. across a circuit, and i the instantaneous value of the current in it, then the mean value of ei over a whole period gives the rate at which work is being done in the circuit, or the power being expended in it. Assuming sinusoids,

$$\int_0^T eidt = EI\int_0^T \sin\omega t \sin(\omega t - \phi)dt$$

where E and I are maximum values, and ϕ is a "certain auxiliary angle of great use in graphical calculations."* Integrating over a $\frac{1}{4}$-period, the *reaction* term in the above expression is $\frac{1}{2}EI\,\frac{\sin\phi}{\omega}$. This expression has not the dimensions of power, but represents the "reactive energy" per period.

Mr. E. C. Lennox also took part in the discussion. The substance of his remarks will be found in the report of the London discussion (see page 264).

[The authors' reply to this discussion will be found on page 275.]

NORTH-WESTERN CENTRE, AT MANCHESTER, 24TH MARCH, 1936

Mr. J. L. Carr: Some five or six years ago, at the International Illumination Congress at Edinburgh, the subject of a proposed British Standard Specification for street lighting was discussed: this was criticized by the American delegates and described as unsatisfactory. It was realized at that time that the proposed Specification was incomplete; but it represented an initial step based upon current experience and knowledge. The advances made in the knowledge of the mechanism of street lighting since then have been striking; and some of the recommendations made by the authors towards the end of their paper appear to be suitable for assisting in the framing of an improved street-lighting specification. A considerable improvement in practice must result from the work already done; and it would be interesting to compare progress in street-lighting practice in the United States, and to learn whether investigations have been taking place there on parallel lines.

The electrical discharge lamp represents a very notable improvement in the efficiency of light production: but I think the greatest advantage is in the resultant improvement in the knowledge of using artificial light on roadways. The improved efficiency, although valuable in many cases, is offset to a large extent by the high capital and maintenance costs of the equipment. Where electrical energy is supplied at a very low price for street lighting, the improved efficiency does not always represent a very substantial financial gain. The type of lamp itself is rather different from what we have been accustomed to, and the fact that it is unstable under certain conditions gives rise to a certain amount of apprehension. Now that most supply undertakings are connected electrically to a bigger scheme, they are not so immune from sudden changes in voltage as previously; and it is rather disturbing to consider that important stretches of road may be thrown into sudden darkness because of a breakdown at a considerable distance, for which the local supply undertakers are not responsible. The somewhat complicated relay system mentioned by the authors is, in my view, an undesirable complication. Serious consideration should be given to methods calculated to improve this somewhat undesirable characteristic.

A few years ago many electrical engineers advocated central suspension of street lamps, an arrangement of which I have always disapproved. It is interesting to note that, almost from the inception of the discharge lamp, central suspension has fallen into disfavour, and the advantages of side suspension, from the point of view of illumination, have become widely appreciated.

Mr. A. E. Jepson: At the bottom of page 244 the authors refer to the introduction of cadmium and zinc into the mercury lamp; has rubidium ever been tried? This element gives a very strong red ray. What is the effect on the colour of the illumination when the bluish-green light has to pass through red-hot glass?

On page 247 the authors refer to the use of a filament producing colour modification "by the admixture of white light." I suggest that this should read "by the admixture of a reddish light," for the filament has a much greater percentage of red light than the sun, and when one is observing these lamps through the outer diffusing globe the lower part of the globe certainly seems to be yellowish in colour, owing to excess of red from the filament.

I am glad reference is made to the use of tails instead of terminals for the chokes, because in certain districts in this area verdigris was formed round the terminal bases on the first chokes supplied, and resulted in breakdowns.

Mr. W. Ballard: A year or two ago several designs of lanterns were introduced for use with h.p.m.v.

* H. Rissik, *Journal I.E.E.*, 1933, vol. 72, p. 435.

* A. Russell: "Alternating Currents," vol. 1, p. 274.

lamps, giving twice the intensity of light in the direction of flow of traffic compared with the opposite direction, and I should like to know whether it is felt that there is any advantage in this system.

I read in the technical Press not long ago that a patent had been taken out for a discharge lamp which, instead of having one electrode at the top and one at the bottom, would have a group. I find that whereas the average life of the mercury-vapour electric discharge lamp is guaranteed at 1 500 hours by the manufacturers, in practice it is considerably higher. I believe that in consequence of introducing these extra electrodes an even longer life can be expected.

With regard to the large diffusing refractor exhibited by the authors, I believe it is felt that in industrial atmospheres the ribbed exterior surface tends to collect dust and dirt. I am pleased to say that, where we have tried it in this area, it has kept reasonably clean.

Mr. J. Sellars: I understand that whereas the light from the mercury-vapour lamp is objectionable from the point of view of colour, it is capable of improvement in this respect; whereas the sodium lamp does not seem to present that possibility. I should like to have the authors' confirmation on this point.

On page 253 they suggest that the use of a specially designed box, presumably to be erected on the pole, is "in many ways preferable." In Manchester we began with special boxes but have since abandoned them in favour of properly-designed bases. Since it is possible to have a base so designed as to give proper ventilation and convenient clearances, I shall be glad to learn of some of the "many ways" in which an excrescence on the pole is to be preferred.

The ability to counteract voltage-drop by the use of a suitable tapping on the choke is very valuable, particularly when comparing different fittings or installations.

Whilst I am in general agreement with the authors regarding the "road surface brightness" theory, I think it is a little misleading to state baldly that no light source will ever render usefully bright the road surface beyond it. That is quite true, but the source may light the object one wishes to see, and actually one or two of the authors' slides showed that the light beyond the lamp had increased the visibility not on the roadway itself but on the footpath. In this connection I would refer to motor-car headlights, which are situated beyond the driver and illuminate the road beyond themselves.

The authors' point about the reduction of brightness near the kerb where fittings are centrally suspended, is an excellent one. In addition, with fittings of this type, driving is done, much of the time, towards the shadow of the driver's own vehicle, so that the illumination of the danger zone near the kerb is still further reduced.

With regard to the figures for annual charges. I should be glad if the authors would state over what period the repayment of capital charges is spread.

Mr. L. H. A. Carr: I regret to note that no serious attempt appears to have been made towards the solution of the fog problem. The authors' only remedy appears to be to increase the intensity of the light sources, which must necessarily increase dazzle. I suggest that it is only the fact that most motorists use saloon cars, which in effect carry their own dazzle shields for all rays coming from above the horizontal, that has prevented considerably more criticism with regard to dazzle.

The ribbed type of glass shade recommended by the authors must, one feels, increase the difficulties of cleaning, a very necessary operation in the dirty atmospheres of industrial areas.

I cannot agree with the authors that "experience has shown that in practice the colour has proved no drawback," as I have met serious criticisms of the colour of the mercury lamp. I feel that there is a certain amount of complacency about the paper. It gives the impression that here is a new development—complete—perfect! Surely this paper is only to be taken as a stepping-stone towards further improvements in both efficiency *and* colour?

Is there any prospect of combining the neon light with the mercury light, as the two together cover most of the visible spectrum?

As a result of spectroscopic experiments during the past few days, I can confirm the authors' statement that the yellow rays are proportionally stronger in the new high-pressure street lamp than in the old low-pressure lamp. I should be glad to know the reason for this difference. It is certainly bound up with the degree and proportion of ionization, gas pressure, etc., but is particularly interesting in being contrary to the general rule for radiation from a *solid* body, where increased energy is accompanied by increased emission towards the violet.

With reference to Fig. 4, it appears that the "relative energy" diagram is multiplied by a curve representing the eye's response to various wavelengths in order to obtain the diagram of "relative visual intensity." It is well known that this curve varies for different strengths of illumination, and also that the curves published by independent authorities differ considerably. I should be glad to know what curve has been used in the preparation of this diagram, and whether any one particular curve has been adopted as a standard by illuminating engineers.

Mr. T. E. Dransfield: A very important point which has to be taken into account when the application of the mercury-vapour lamp for use in busy city streets is under consideration, is the frequency with which lamps may be extinguished for some 10 or 12 minutes in the case of a momentary drop in voltage, due to any disturbance which may occur within the large area served by a number of interconnected supply undertakings.

Mr. C. A. M. Thornton: May I ask the cause (or causes) of failure of h.p.m.v. lamps? It is not obvious why they should fail, in view of the fact that they have no filament.

How is the life of the lamps affected by vibration? I do not refer to the small vibration that may be present in street-lighting standards, but to the severe vibration met with in works equipped with rotating machinery. What progress has been made in the last 3 years in increasing the life of h.p.m.v. lamps?

What physical changes occur during the period of extinction prior to restriking? This period is stated by the authors to be 12 minutes, but I believe it is more like 7 minutes.

Mr. G. F. Freeman: With regard to Fig. 2, I take it that the radiation is opaque to its own wavelength, and that the variation across the stream is due to the fact that the radiation derived from the arc is refracted in varying degrees by the curved glass envelope.

Motorists sometimes complain of the "muzzy" effect produced by mercury-vapour lamps; has this anything to do with the stroboscopic effect associated with these lamps, which is due to the discontinuous nature of the discharge? If so, can it be cured by using a higher frequency for street lighting than for ordinary lighting?

I should be glad if the authors could give some information as to how the life of these lamps is affected by variations in the operating voltage.

Mr. H. L. Jones: I am interested to see that these new lamps are being fitted with standard goliath screw holders. There are numerous non-standard plugs and sockets on the market, and a similar lack of standardization applies to almost every electrical accessory with the exception of lamps. Will the lamps described in the paper eventually be made to a British Standard Specification, and is the globe made by one manufacturer going to be interchangeable with the globe made by another? I notice that twelve different types of lanterns are shown in the paper; surely one or two could be picked out as the best and some sort of standardization agreed upon.

Apparently the position of these lamps is chosen so as to get reflection from the lamps behind the object to be illuminated, e.g. the pedestrian or the cyclist. If that is so, apparently the motorists' headlamps are becoming rather more of a nuisance than they have been in the past, inasmuch as they are conflicting with the illumination provided by the street lamp.

THE AUTHORS' REPLY TO THE DISCUSSIONS AT LONDON, NEWCASTLE, AND MANCHESTER

Mr. G. H. Wilson, Commander E. L. Damant, and **Mr. J. M. Waldram** (*in reply*): Before replying to the remarks of the various speakers, we wish to give data for the latest rating of h.p.m.v. lamp, which was added to the range on the 1st April, 1936.

Whilst this lamp will probably be used more for industrial than for public lighting, its dimensions and other data are given in order to complete the series.

The dimensions (see Fig. 1, page 242) are as follows:—

$$A = 230 \pm 15 \text{ mm}$$
$$B = 43 \pm 4 \text{ mm}$$
$$C = 140 \pm 10 \text{ mm}$$
$$D = 85 \pm 3 \text{ mm*}$$

An Edison-screw cap is used.

The initial efficiency is 32 lumens per watt, and the average throughout life can be taken as 80 per cent of the initial value. The intensity distribution can be obtained from Fig. 3 in the paper by multiplying the ordinates by 0·267. A 10-μF condenser is usually used for power-factor correction, and the losses in the auxiliaries, for one manufacturer's products, are 13 watts.

* Dimension of one manufacturer's lamps.

The electrical characteristics are given in Table B.

Table B

Rating of lamp in use	Mains voltage (actual) at unit	Installation without condensers: Maximum starting current, in amps. (excluding switching surges)	Installation without condensers: Average running conditions: Current in amps.	Installation without condensers: Average running conditions: Power factor	Installation with 10-μF condensers: Maximum starting current, in amps. (excluding switching surges)	Installation with 10-μF condensers: Average running conditions: Current, in amps.	Installation with 10-μF condensers: Average running conditions: Power factor
200/210 volts; 150 watts ..	200	$2\frac{1}{2}$	1·7	0·45 to 0·5	$1\frac{3}{4}$	1·2	0·7
	210	$2\frac{1}{4}$	1·7		$1\frac{3}{4}$	1·1	
220 volts; 150 watts	220	$2\frac{1}{4}$	1·5		$1\frac{1}{2}$	1·0	0·8
230 volts; 150 watts	230	$2\frac{1}{2}$	1·4	about 0·5	$1\frac{3}{4}$	0·9	0·85 to 0·9
240/250 volts; 150 watts ..	240	$2\frac{1}{4}$	1·4		$1\frac{1}{2}$	0·8	
	250	2	1·3		$1\frac{1}{4}$	0·7	

The reply to the discussion can conveniently be dealt with in the order of subjects in the paper. Several speakers have referred to the applications of the h.p.m.v. lamp outside the field of public lighting. We did not describe these other uses, in part because of limitations of space, but primarily because, up to the present, the application to public lighting has been so much more important than any other. In reply to Mr. Crawshaw's question, however, the lamps described are also in use for flood-lighting of buildings and for general lighting on aerodromes. In these applications the increased efficiency over the filament lamp can be used to the full.

So far as we are aware, the lamps have not been in stalled in lighthouses. One probable reason for this is that on a hazy night the colour of the lamps appears greener than on a clear night, and some confusion might be caused on this account.

The question of standardization has been raised by Mr. H. L. Jones. The dimensions and efficiency figures given in the papers have been standardized by the four principal manufacturers of these lamps in this country, but the lamps are not yet the subject of a British Standard Specification.

Several speakers have asked for details of the cost estimates given in the paper. It is difficult to give rigid cost figures, as they are affected by many variables. For example, as a street-lighting installation is frequently fed from an ordinary supply network, it is difficult to know what proportion of the cable costs should be included. Mr. Lennox gives an overall cost figure which he himself has found; the figures quoted by us were taken from data supplied by various engineers. A typical estimate for a 400-watt installation with the most expensive type of fitting is made up as follows:—

Capital cost of installation of 400-watt lamps, 35 per mile	£1 200
Writing off capital in 10 years gives annual charge as	£120
Energy for 35 lamps at $\frac{3}{4}$d. per unit (all-night lighting 4 000 hours per annum)	£186
Lamp replacements at 1 500 hours' life (all-night lighting 4 000 hours per annum)	£140
	£446 per mile.

The capital cost includes the cost of fittings, poles, service cables, erection, wiring and jointing, together with a proportion of the cable cost. No charge has been included for maintenance, which would be from £50 to £100 per annum.

The relative costs of light as produced by h.p.m.v. and filament lamps will depend on the price of energy, as Mr. Rigg suggests. Comparing the 250-watt discharge lamp with the 500-watt filament lamp, we agree substantially with Mr. Rigg's figures if they are based on the list price of the lamps. The price of lamps to lighting authorities is usually below the list price, and the cost of energy needs to be down nearly to $\frac{1}{2}$d. a unit for the cost per lumen from the two sources to be the same. This calculation includes, as an annual charge, $\frac{1}{7}$th of the capital cost of the choke and condenser in the case of the discharge lamp, and assumes a common all-night street-lighting burning period of 4 000 hours per annum. It is still the exception rather than the rule for the energy cost for public lighting to be below $\frac{3}{4}$d. a unit, and so the h.p.m.v. lamp usually shows a marked economy.

Many speakers have referred to points of design, and to the characteristics of the new lamps. On the question of efficiency, we have not ourselves found such high efficiency figures for street-lighting arc lamps as those quoted by Prof. Marchant. Our own tests on a modern flame-arc street-lighting lantern complete (such an arc cannot be burned effectively in the open) showed it to have an efficiency of 22 lumens per watt (1·75 candles per watt), neglecting losses in the series impedance, and this compares unfavourably with the figure of about 30 lumens per watt for a complete h.p.m.v. street-lighting unit, again neglecting losses in the impedance.

On questions of manufacturing technique, the use of oxygen in the outer bulb is referred to by Prof. Marchant and Mr. Duckworth. The process by which the oxygen reduces the blackening on the inner bulb is not understood. It is of interest, however, that many other gases do not produce the same valuable result. Mr. Duckworth asks for some further information on construction. We were not specific on these points, as details of construction are likely to be varied from time to time. For the electrodes, sintered rods of the alkaline-earth silicates, particularly barium silicate, are usually employed and the rare-gas filling is commonly argon. The pressure of the gas at starting is only a few millimetres of mercury, but when fully run up is about an atmosphere. This answers the question raised by Mr. Elkington. During the period of extinction prior to restriking, the lamp cools, mercury condenses, and the pressure falls. When it has reached the order of a few millimetres of mercury, the lamp restrikes and the running-up cycle is repeated. The time to restrike for a bare lamp in air at normal temperature was given in the paper as 3 minutes, not 12 minutes as Mr. Thornton suggests.

Reference has been made to the expense involved in waiting for the lamp to run up before extinguishing when testing. This is a just criticism. It is probable, however, that the time allowance necessary will be reduced as the factors affecting this phenomena are better understood.

The arrangement of one electrode with its axis vertical and the other with its axis horizontal, as shown in Fig. 1 of the paper, is that used by one manufacturer. The conditions at the top and bottom of the lamp are not necessarily the same, and the arrangement of electrodes shown is sometimes preferred, although it is not universally adopted.

Mr. Cator asks about the support for the inner bulb. Although the wire spider is probably not more effective than the earlier "carriage-spring" support, it is the more satisfactory when ease of manufacture and economy are taken into consideration.

In reply to Mr. Thornton, the end of lamp life results from failure to strike or to run up. The principal aim of the manufacture during the past three years has been not so much to increase the average life of the lamp as to improve its uniformity. It is a common experience with any new lamp for the proportion of early failures to be much higher when the lamp is first developed than when it is has been established for a year or two. The h.p.m.v. lamp has proved no exception to this rule. In reply to Mr. Lennox and Mr. Thornton, vibration has not generally caused trouble with the h.p.m.v. lamps, which are, if anything, stronger than filament lamps of the same output. It has been necessary to locate the far end of the lamp in certain horizontal-burning lamps in floodlights, but only to ensure accurate positioning relative to the magnet.

With regard to the characteristics of the lamps, such data as are available on the effect of variations in operating

voltage on life, a point raised by Mr. Freeman, indicate that the effect is less than with filament lamps. In an extreme case of overload, catastrophic failure can occur as with filament lamps, but it is assumed that Mr. Freeman is referring to normal voltage fluctuations.

It does not appear to be possible to define an optimum starting temperature in the way Mr. Cator has done. Although we have not obtained a curve connecting ambient temperature with striking voltage, it would undoubtedly have the form suggested by Mr. Cator. But the lowest point of this curve has no connection with the time to run up. On a normal circuit the voltage available for striking the lamp is constant, and the most rapid run-up will occur when the ambient temperature is such that the corresponding striking voltage is equal to the voltage available.

Two questions have been asked on the radiation from the luminous column. In reply to Mr. L. H. A. Carr, increasing the pressure of the mercury vapour increases the proportion of changes over the energy-levels which correspond to the emission of the yellow-green lines. The phenomenon of radiation in these lamps is different from that of thermal radiation, and would not, therefore, necessarily change in the same way with increased energy input. The chief reason why the brightness of the arc is greatest at the centre is that the excitation is greater there than at the edges. The arc width is a small proportion of the diameter of the inner bulb, and the refraction effect mentioned by Mr. Freeman would scarcely be noticeable.

The question of the colour of the light often invites discussion: we are glad to have the confirmation of several speakers that it is unimportant in street lighting and even, as Mr. Long remarks, in industrial lighting. In this connection we would refer Mr. L. H. A. Carr to Mr. Lennox's remarks. In reply to Mr. Jepson, rubidium is unsuitable for correcting the colour, since, although its spectrum shows strong red lines, it would not be excited under the conditions occurring in the lamp, and the efficiency would be low. For a similar reason, neon cannot be used in the lamp as Mr. L. H. A. Carr suggests. The question has been explained by Ryde.*

While the question of colour correction by different means is being actively studied at present, in our view it is unlikely that corrected lamps will show a real advantage over the uncorrected lamps, for street lighting, for some time to come. The methods so far attempted either reduce the luminous efficiency or make lanterns less efficient, or both. We agree with Mr. Sellars that there is little hope of materially improving the colour of sodium lamps.

In reply to a further question by Mr. Carr, the relative visibility curve used in deriving the "relative visual intensity" from the "relative energy" is the mean curve which has been internationally agreed for some years.†

The question of fog is dealt with in the paper to the extent to which it is relevant. The light from the h.p.m.v. lamp penetrates fog neither more nor less than that from any other lamps, and it is unlikely that any startling solution to that problem will be forthcoming, for no visible radiation will effectively penetrate dense fog. Good reports have been received of h.p.m.v. installations in fog, as Mr. Lennox confirms.

Several speakers refer to the possibility of peculiar visual or physiological effects being brought about by the radiation from the lamp, and suggest explanations. For the most part the suggestions made cancel out; this is to be expected, since experiment* and experience both indicate that there is no material difference in ability to see in streets between light from tungsten, mercury, or sodium lamps. Most of the effects mentioned, such as change of visual acuity, chromatic aberration, diffraction, etc., are insignificantly small under street-lighting conditions.

Mr. Riley's suggestion that it is difficult to focus objects seen against a bright background seems contrary to all experience; it is only necessary to instance the visibility of black print on white paper, or of objects seen against the sky. We are unable to follow his suggestion that objects on the skyline cannot be "focused." Perhaps his suggestion arises from some confusion of terms. While it is difficult to adduce definite evidence, it seems likely that many of the reported effects mentioned are due to the distribution or intensities employed, as Mr. Lucas suggests, coupled with atmospheric effects. In comparing two installations it is almost impossible to separate effects due to the intensity and brightness distribution, etc., from those, if any, due to the physical properties of the radiation. There have been very few complaints of glare from h.p.m.v. installations of the type described in the paper: where they have occurred the glare has usually been traceable to local circumstances which had resulted in a very low road surface brightness, or to the few individuals who are unusually susceptible to discomfort glare.

Some discussion has centred round other methods of stabilizing the discharge than by means of a choke.

Prof. Marchant suggests the interesting possibility of using a ballast resistance consisting of an iron wire whose high temperature-coefficient of resistance would give an almost vertical voltage/current characteristic. Consideration of Fig. 6 will show that, although this type of curve gives good stability at either the starting or the running point, it cannot pass through both the points R and S. In practice, therefore, two such resistances with a change-over switch would be necessary. This arrangement is, in fact, similar to that used in the composite h.p.m.v. and tungsten-filament lamp referred to on page 247. Another suggestion which has been made from time to time is that a condenser should be used instead of the choke. Mr. Lucas mentions reasons why a condenser cannot be used successfully in this way. The reason is interesting, although economic considerations rule out the practicability of using this arrangement.

Many points have been raised on those characteristics of the lamps which affect their performance on a street-lighting network.

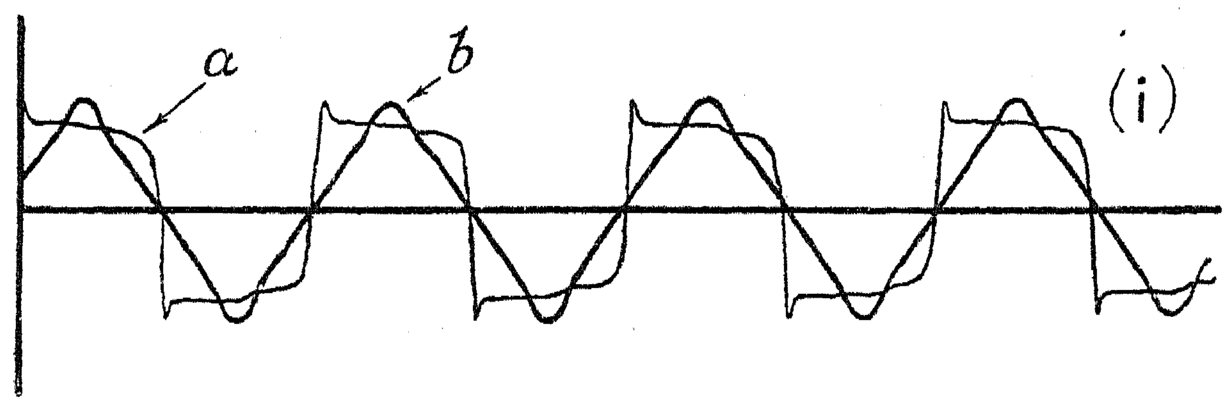
With regard to the switching surge, it is not possible to add any more useful data to that already given on page 252. The important practical point is that the switching characteristic of the modern h.p.m.v. lamp has no deleterious effect on switchgear of the sizes recommended.

* *Journal of the Royal Society of Arts*, 1933, vol. 82, p. 624.
† *Proceedings of the International Commission on Illumination*, 1924, p. 67.

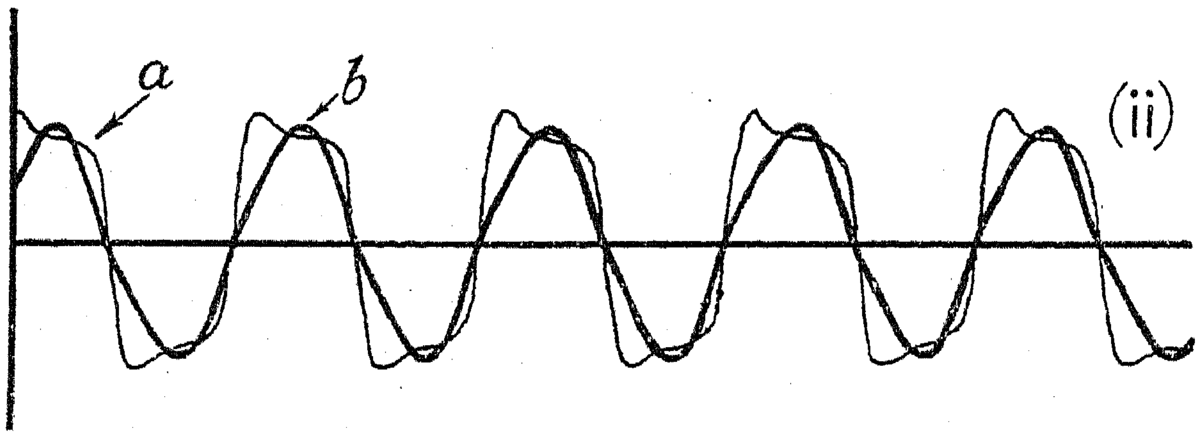
* K. M. Reid and H. J. Chanon: *General Electric Review*, 1935, vol. 38, p. 580.

Some criticism is levelled by Mr. Macdonald at the suggestion that it was in order to load the supply cables for a short period to a value in excess of the full-load current rating. The normal full-load rating is usually for continuous operation, and is based primarily upon temperature considerations. A cable has an overload capacity like most other electrical apparatus, and it seems reasonable to suppose, therefore, that the suggested overload on starting-up h.p.m.v. lamps, taking place once a day for a matter of a few minutes, would not have a deleterious effect upon the cable.

Various speakers have raised the question of voltage-drop in distribution networks. It is pointed out by Mr. Macdonald that in the vast majority of cases the size of cables is limited by the permissible voltage-drop at the farthest unit rather than by the current-carrying capacity of the cables. For this reason an advantage may be claimed for the h.p.m.v. lamp as compared with the tungsten-filament lamp, because the wattage of the



(i) During run-up: $v_t = 30$V.



(ii) When fully run up.

Fig. B.—Current and voltage wave-forms of 400-watt h.p.m.v. lamp.
(*a*) Lamp voltage.
(*b*) Lamp current.

h.p.m.v. lamp may be adjusted to the rated value by means of the adjustable choke. The limitation is the minimum voltage at which the h.p.m.v. lamp will strike with certainty. The values given on page 254 may be taken as safe conservative values, although higher values as suggested by Mr. Lennox are possible in some circumstances.

In extreme cases, where existing cables of high resistance must be used, the use of an h.p.m.v. lamp of low voltage rating will overcome the trouble (for example, a 220-volt lamp could be used on a supply varying from 230 volts at the supply end to 200 volts at the farthest end), but the disadvantage of a lower power factor will then appear.

In reply to various questions on the calculation of voltage-drop, it can be said that calculations based on the assumption of a sinusoidal wave-form at 50 cycles per sec. and a power factor under running conditions, as given in Table 2, will give values quite sufficiently accurate to use in the choice of cable size. As pointed out on pages 254 and 255, the low power factor under starting conditions results in a voltage-drop less than that under running conditions. The suggestion that hunting might occur on starting, owing to the starting current giving a voltage-drop greater than that permissible, may safely be discounted for this reason. In practice, a cable chosen to give, at the farthest post under running conditions, a voltage greater than the minimum on page 254, will ensure satisfactory operation as regards striking and power factor. Further, with the rating on such a basis, the normal voltage at the post will usually be within the voltage range of the choke tappings.

An harmonic analysis of the current curve has not been made, but, for the information of Mr. Elkington, typical curves are given in Fig. B for starting and running conditions.

We are unable to agree with Mr. Dickinson's suggestion that Fig. 9 could be plotted with "phase angle" as abscissa. The conception of angular displacement, as with that of vector quantities, applies only to sinusoidal variations, and the variations with which one is dealing here are not sinusoidal. The values shown in Fig. 9 as "power factor" are the true values of the ratio

$$\frac{\int_0^T vi dt}{\sqrt{\left[\int_0^T v^2 dt \int_0^T i^2 dt\right]}}$$

and cannot be pictured as the cosine of any angle.

Two points have been raised on installation practice. Mr. Lennox mentions one difficulty he has experienced in the earthing of street-lighting poles. One can fully appreciate the possibility of a pole becoming alive under faulty conditions in such circumstances, and the chances naturally depend upon the rating of the fuses used. But the h.p.m.v. lamp is in this respect no worse than a tungsten-filament lamp of the same light output. For example, a 400-watt h.p.m.v. lamp and a 1 000-watt tungsten-filament lamp must each be fused to carry about 5 amperes, in the first case for only a few minutes, but in the second continuously. The problem is one which can never be completely solved, except by a special earth continuity conductor run back to a really efficient earth, or the extravagant use of earth-leakage-trips.

The second point was raised by Mr. Sellars in connection with the comparative advantages of boxes and pole-bases as a housing for auxiliaries. From the manufacturers' point of view, a well-designed box can make the housing of these important units definitely satisfactory, whereas the design of poles—particularly if an existing installation is being converted—may make them far from suitable.

Problems of light control have been raised by Mr. Maxted and Mr. Lucas. The reference in the paper to horizontal prisms used with a vertical lamp was condensed, and applied to prisms designed to redirect flux emitted above the horizontal, as is done with refractors for filament lamps. The lanterns shown in Figs. 16(*a*) and 16(*d*) also use some horizontal prisms with good effect, apparently in the same way as in the lantern of Fig. 19, but light is allowed in both lanterns to escape

above the horizontal, and cannot apparently be redirected through any material angle without the bad effects recorded in the paper. The question of utilization of flux, mentioned also by Mr. Cator, is considered to be of secondary importance compared with the distribution of intensity, inasmuch as it is quite easy to increase the utilization and yet to decrease the effectiveness of the lighting; and conversely to improve the appearance, brightness, and visibility, while reducing the utilization. The latter is therefore of importance only when the intensity distribution has been decided on other grounds. Consequently we do not attach much importance to the iso-foot-candle diagram referred to by Mr. Cator, for a high utilization is no guarantee of a good result. It can be agreed that the horizontal-burning lamp allows much more exact control of the distribution in vertical planes, and for that reason has an advantage; but with the distribution which we prefer the horizontal lamp with suitable redirecting equipment was found to put only 10 per cent more light on the road than the vertical lamp in suitable equipment. The difference between Mr. Maxted's figures and ours is entirely due to the difference in distribution; with the type of distribution given by the lantern of Fig. 21, figures were obtained in agreement with his. The main difficulty with lanterns for horizontal-burning lamps is the extra complication, for they have to be made very accurately in order to ensure both the correct light distribution and the correct operation of the magnetic control. Mr. Maxted makes the case for the series magnetic deflector used with horizontal-burning lamps seem worse than we think it is. The series magnet, with tappings for the various lamp-voltage ratings, is in use and is perfectly satisfactory in practice. Moreover, the windings are of heavy-gauge wire and unlikely to break, and either a break or a fault to earth on the magnet, if wired on the phase side, merely extinguishes the lamp without damage. This cannot be said for other circuits which have been proposed. The flexibility of light control with the horizontal lamp is rather a mixed blessing; there are many reasons for choosing a lantern with a good distribution without sudden peaks or cut-offs, which does not require or possess focusing adjustments, and is consequently not easily deranged.

In reply to Mr. Ballard, the "bi-asymmetric" distribution was tried in the earlier lanterns, but its real advantages have not proved to be material. Mr. Davies has kindly pointed out an unintentional claim to omniscience in regard to light distributions; this has been altered for the *Journal*.

In reply to Mr. Cator, the width of the bright areas is determined (for a given surface) by the intensity near 85°, but not by the width of the distribution. A wide "peak" to the distribution ensures that large bright areas will be formed when the lantern is viewed over a wide range in plan, but it does not alter the actual width of the bright areas. He mentions also the relative merits of refractors and reflectors. Their photometric efficiency is only one of several important practical features. For the type of distributions required in street lighting, a reflector, if it is to control much light flux, must be made very large, or else must be subdivided (see Fig. 21). Furthermore, it is much more sensitive to variations from sample to sample; it is also more difficult to make to fine limits than a refractor, and it requires much more accurate assembly than a refractor. It is also rather less permanent. On the whole, therefore, good refractor lanterns are often simpler, cheaper, and more reliable, than good reflector lanterns, although their components are more complex to design, more expensive to make, and less efficient. We value particularly the remarks of Mr. Balmain, to whom, with his colleagues, the production of several of the refractors discussed in the paper is due.

In reply to Mr. L. H. A. Carr, the cleaning of a well-designed refractor is not difficult; that of the lantern of Fig. 20 is specially designed for easy cleaning and low losses due to dirt; Mr. Ballard confirms that the attendants find no trouble with them. In reply to Mr. Wadsworth, the collection of dirt by lanterns varies enormously with the state of the atmosphere, and is particularly high in foggy weather. It is much less with non-ventilated enclosed lanterns than in open or ventilated lanterns. The presence of slight mist also greatly varies the appearance of the installation in a way which is not yet understood.

Col. Edgeworth's suggestion for the use of plane mirrors is interesting, and had in fact occurred to us in our early designs.* The trouble is, as will be seen from Col. Edgeworth's sketch, that for a good distribution the fitting becomes impracticably large. The smallest of his fittings would be 4 ft. long, and the largest, following his sketch, would appear to be 10 ft. long, assuming a light column 3 in. long. The actual light column of a 400-watt lamp is 6 in. long, so that these figures would be doubled. Reasonable size can only be obtained at a sacrifice of the distribution.

The last section of the paper, dealing with new principles of street lighting, has caused much comment.

In his reference to "continuous lighting," Prof. Marchant has given us a happily chosen phrase which expresses exactly the effect which we have attempted to produce in the street. Mr. Riley queries the validity of the statement that good visibility can be obtained when objects are in dark silhouette, but practically all investigators in this field are now agreed on this point, as was evident from the discussions in Berlin at the International Commission on Illumination in 1935. Direct experiment has established beyond question that visibility of objects increases with increasing contrast, over the range encountered in vision in streets. We agree that a very bright carriageway appears to accentuate a dark footpath, but the practical effect of this on safety is not certain. If it proves to be important, the solution lies, of course, in making the footpath or other backgrounds adequately bright. Mr. Crawshaw's suggestion of white bevelled kerbs is valuable. Mr. Sellars rightly points out that the sources may render bright the *footpath* for a short way beyond them. They may also, as he says, illuminate the object which it is desired to see; but this cannot be regarded as an advantage, for the reasons given in the paper. He also instances the motor-car headlight; and a comparison of the intensity of a pair of headlights—50 000 to 100 000 candles or more—with the 3 000–6 000 candles of a street-lighting fitting is confirmation of this point. As Mr. H. L. Jones

* British Patent No. 405425.

points out, headlamps tend to conflict with street lighting by revealing objects in a different way.

We appreciate Mr. Lennox's and Mr. Davies's support of the contention that careful planning is essential, and that it cannot be done only on the drawing board; and we concur with the remark concerning the lighting of the junctions of side roads for the benefit of main-road traffic. The 150-ft. spacing referred to by Mr. Lennox is a desirable limit, but is perhaps the least important of the figures suggested for the optimum installation. We agree with him that on some roads quite a good installation can be made with greater spacing, but suggest the limit of 150 ft. for the reasons given. It has not been found that materially less overhang is required on a concrete road as compared with an asphalt road as Mr. Lucas suggests; in our experience a concrete road takes a surface sheen analogous to that on asphalt when worn by traffic, and much the same treatment seems to be called for.

Prof. MacGregor-Morris and Mr. Frost query the wisdom of placing sources on the outside of a bend. We cannot agree with Prof. MacGregor-Morris's argument, because the road ahead of a car is always bright by reason of sources ahead of, not behind, the driver. Consequently the car cannot cast shadows as his colleague suggests. Had this effect occurred, it would have been apparent in our film, which was taken from a car. If sources are placed on the inside of a bend, the near side of the road for vehicles on the outside of the bend, and the offside for vehicles on the inside, is left dark, particularly in wet weather; if they are properly arranged on the outside, both kerbs are shown up well. The film showed, among other things, the effect of too great a spacing of sources on a bend, in which case dark regions are left on the kerb on the outside of the bend. In reply to Mr. Cator, the "5° rule" will best indicate when the normal staggered arrangement should be abandoned. Roads seldom conform to a regular curvature, and, if they do, its determination is difficult; the "5° rule" is probably the most helpful line of attack.

Mr. Lucas suggests that on curved roads the importance of long, wide, bright areas is lost, and fairly high intensities near the horizontal lead only to glare in such situations. Possibly he has in mind a very severe and long bend, in which some of the units in the middle of the bend can only be seen at close range. But such bends do not occur in the street shown in the film, and are very rare in practice. In a street 200 ft. is a surprisingly short distance. In the very winding road shown in the film, the lamp which is most concealed by the bend (shown at A in Fig. C), can usefully operate for 300 ft. in one direction, and 450 ft. in the other. In Fig. 26(*b*), Plate 4, the farthest source operates usefully at 300 ft. distance, using light up to 85° from the vertical. For another source, the figures are 370 and 450 ft.: all the remainder can usefully work at much greater distances. The sources which happened to show up the pillar box and the pedestrian, have other work to do at much longer ranges. In our view it is particularly important that the bright areas on a bend should be long, because, owing to the bend, each is seen separately and is not helped by nearer sources. If there is a cut-off the bright areas are curtailed, and the effect is marred, especially in wet weather.

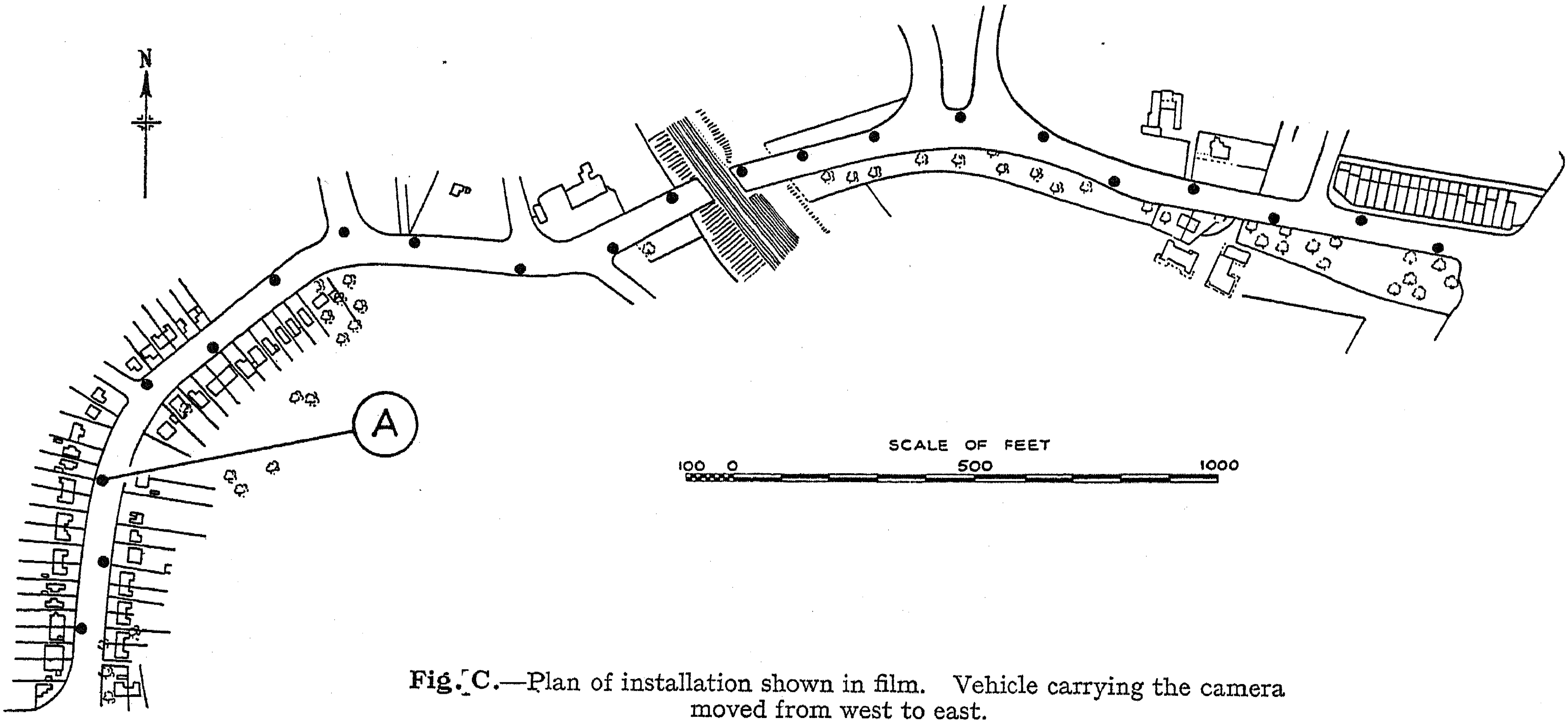


Fig. C.—Plan of installation shown in film. Vehicle carrying the camera moved from west to east.

It is true, as Mr. Lucas suggests, that still photographs cannot tell the whole story of a street-lighting installation. The film is better, but still in some ways imperfect. But controlled photographs can record very well some of the impressions gained on an inspection, and as such are of value to the experienced observer. Probably there is no branch of engineering in which a complete record of everything of importance can be obtained by a single means.

We have carefully studied the "cut-off" system, mentioned also by Mr. Long and Mr. Riley, and have seen some excellent examples of its application, both in this

country and abroad, but we remain of the opinion that, on the whole, the system which is advocated in the paper gives the better results. The cut-off system was not mentioned in the paper because it is not usually employed with h.p.m.v. lamps.

Mr. Lennox and Mr. J. L. Carr criticize the British Standard Specification for street lighting, and it must be admitted that in its present form it does not take into account the more recent developments in the art. But it is a very much simpler matter to suggest one good way of lighting a street, as has been done in the paper, than to draw up a fair and simple specification which will admit all good ways and restrain all poor ways. The care and thought which have gone into the existing specification are not obvious on reading its clauses; and the Committee is continually engaged in revision. It would perhaps be fair to say that none of the critics of the Specification has yet been able to propound a better one. Mr. W. J. Jones's suggestions—which do not of course propose to cover the same ground as the B.S. Specification—are very interesting. It is noted that the B.E.D.A. Committee has specified the bare lamp output per unit area of carriageway. In the proposals which we gave, reference was made rather to the output of each individual source, as determining in part the bright area associated with it. The specification of generated lumens per sq. ft. of carriageway might be taken to imply that if the carriageway width were doubled the proper course would be to double the power, or else the number, of the sources, which, it is felt, might well give a very bad result. In reply to Mr. H. L. Jones, we feel that to specify lantern globes at present would be to restrain progress. Since lanterns and globes are designed together, there seems little reason for interchangeability. While we can sympathize with the difficulties of the B.S.I. Committee, we look forward to the time, which may not be far distant, when it will be possible to set out, on sounder principles than rules of thumb, the exact position of each source, and specify its output and distribution, and then proceed as engineers to provide the distributions and locate the sources in those positions.

Finally, we would refer to the position of street lighting in this country, mentioned by Mr. W. J. Jones, Mr. Davies, and Mr. J. L. Carr. There seems little doubt, as Mr. Jones and Mr. Davies have said, that Great Britain can be said to have established a lead over other countries. It is to be hoped that this progress in the lighting of highways to an adequate standard will be continued.

GENERATION AND ABSORPTION OF GAS IN INSULATING OILS UNDER THE INFLUENCE OF AN ELECTRIC DISCHARGE*

By G. W. NEDERBRAGT.

(*Paper first received* 9*th October*, 1935, *and in final form* 24*th January*, 1936.)

SUMMARY

It is known that insulating oils may give off gas, chiefly hydrogen, under the influence of an electric discharge. This generation of gas is particularly marked in the case of oils which have been exhaustively refined to make them more resistant to oxidation. In this paper an attempt is made to discover to what extent the generation of gas of refined oils can be suppressed by the addition of relatively small quantities of aromatics. Insulating oils always have a more or less extensive boiling range. If it be desired to have a slight tendency to generate gas, with a relatively small percentage of aromatics, it is important that the aromatics present should be of the lowest boiling constituents.

INTRODUCTION

If insulating oils are exposed to the effects of an electric discharge through a gas, marked changes are found to take place in the composition of the oil. Owing to this phenomenon electric discharges play an important part in cable deterioration. The discharges occur in bubbles of gas between the layers of paper, in the working high-tension cable. The presence of these bubbles may be due to the fact that when the cable was filled it was not sufficiently impregnated. They can also be formed by shrinkage of the oil during cooling after the cable has been strongly charged, if the oil cannot flow in rapidly enough through the paper, so that local vacuum occurs. The effect of the discharge in a bubble of gas upon the surrounding oil gives rise to the formation of cable wax, a high molecular product found in all deteriorated cables. The formation of this wax is accompanied, however, by generation of hydrogen. This makes the bubble of gas grow larger, which finally leads to breakdown of the cable.

Mineral oils, which differ considerably in composition, also behave very differently in an electric discharge. In the literature on the subject, data with regard to several hydrocarbons may be found. S. C. Lind and G. Glockler† applied silent discharge to ethane and ethylene; gaseous, liquid, and solid hydrocarbons were formed, and also hydrogen, but ethane gave greater quantities of hydrogen than ethylene. Other investigators‡ found polymerization with the formation of little or no hydrogen in the case of benzene and naphthalene. C. S. Schoepfle and L. H. Connell§ examined cable oils by bombardment with rapid electrons, which developed a product greatly resembling cable wax; at the same time hydrogen was formed. The amount of hydrogen formed increased with the degree of refining of the oil. C. S. Schoepfle and C. H. Fellows,† continuing these experiments, found that, of pure hydrocarbons, aromatics formed the least gas and saturated hydrocarbons the most.

In view of these results it is clear that the nature of the oil greatly affects the reactions that take place in the bubble of gas. If the oil is highly refined, and hence of a strongly saturated character, then owing to the discharge a great deal of hydrogen is separated from the molecules of oil and the bubble of gas grows larger. On the other hand, the presence of aromatics and olefines can cause an oil to show a slighter tendency to separate off hydrogen.

In this paper the results are given of an investigation into the influence of slight additions of aromatic substances upon the generation of gas in an oil. As the object of the research was to examine a large number of additions and their influence on various oils, a method was evolved which made it possible to determine quickly the gas generation of a sample. In a number of determinations the quantity of gas separated off was measured as a function of the duration of the discharge, in a discharge tube in which there was only gas of low pressure over the oil. These experiments were supplemented by others, using a gas pressure of 1 atmosphere. Both kinds of determinations were necessary, as the pressure in the bubbles of gas in a cable can have very divergent values. According to Dunsheath's recent paper,‡ Hirschfeld, Meyer, and Connell measured, in a solid cable, pressures ranging from 6 atmospheres down to zero. The temperature of the cable, and the resistance of the paper to the expanding or shrinking oil, greatly affect local pressure. A discharge takes place the most easily in the bubbles of gas under reduced pressure.

MEASUREMENTS UNDER REDUCED PRESSURE

Apparatus

For the measurements under reduced pressure the discharge tube of Fig. 1 was used, containing at the bottom the oil under test (in general 25 cm^3). The discharge tube was placed in an oil bath. With a Gaede pump the pressure in the tube, exclusive of the oil vapour-pressure, was reduced to 1 mm of mercury. Next, the tin-foil coatings were subjected to an alternating potential. Outside tin-foil coatings have an advantage over electrodes placed inside the tube in that no carbonization takes place at the former.

On its way from one tin-foil coating to the other the current passes through the glass as a displacement current, then as an electron and ion current through the

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† See Bibliography, (1). ‡ *Ibid.*, (2), (3). § *Ibid.*, (4). † See Bibliography, (5). ‡ *Ibid.*, (6).

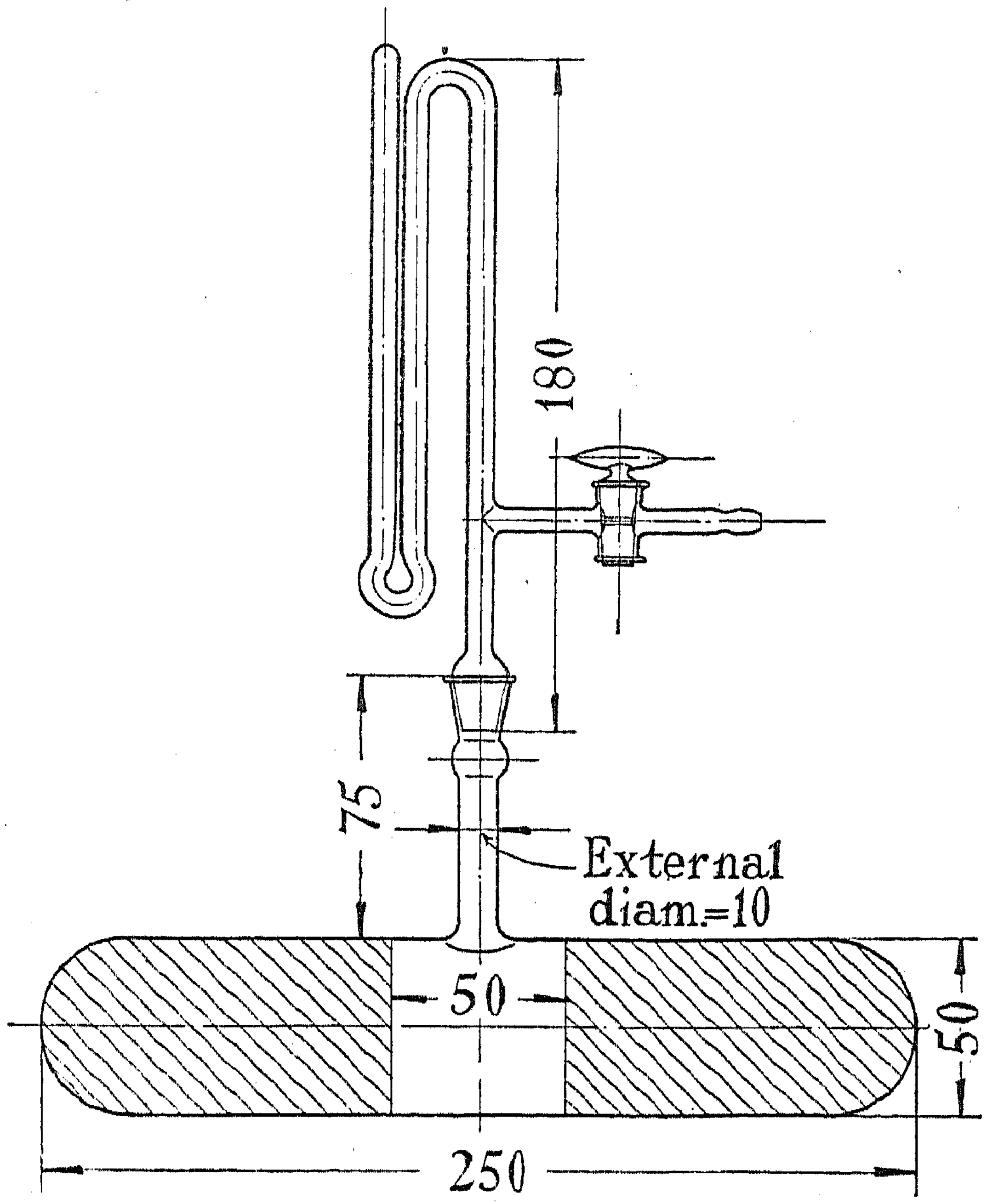

Fig. 1.—Discharge tube for investigating generation of gas by cable oil under influence of glow discharge.
Cross-hatched area is the part covered by tin-foil. Dimensions in millimetres.

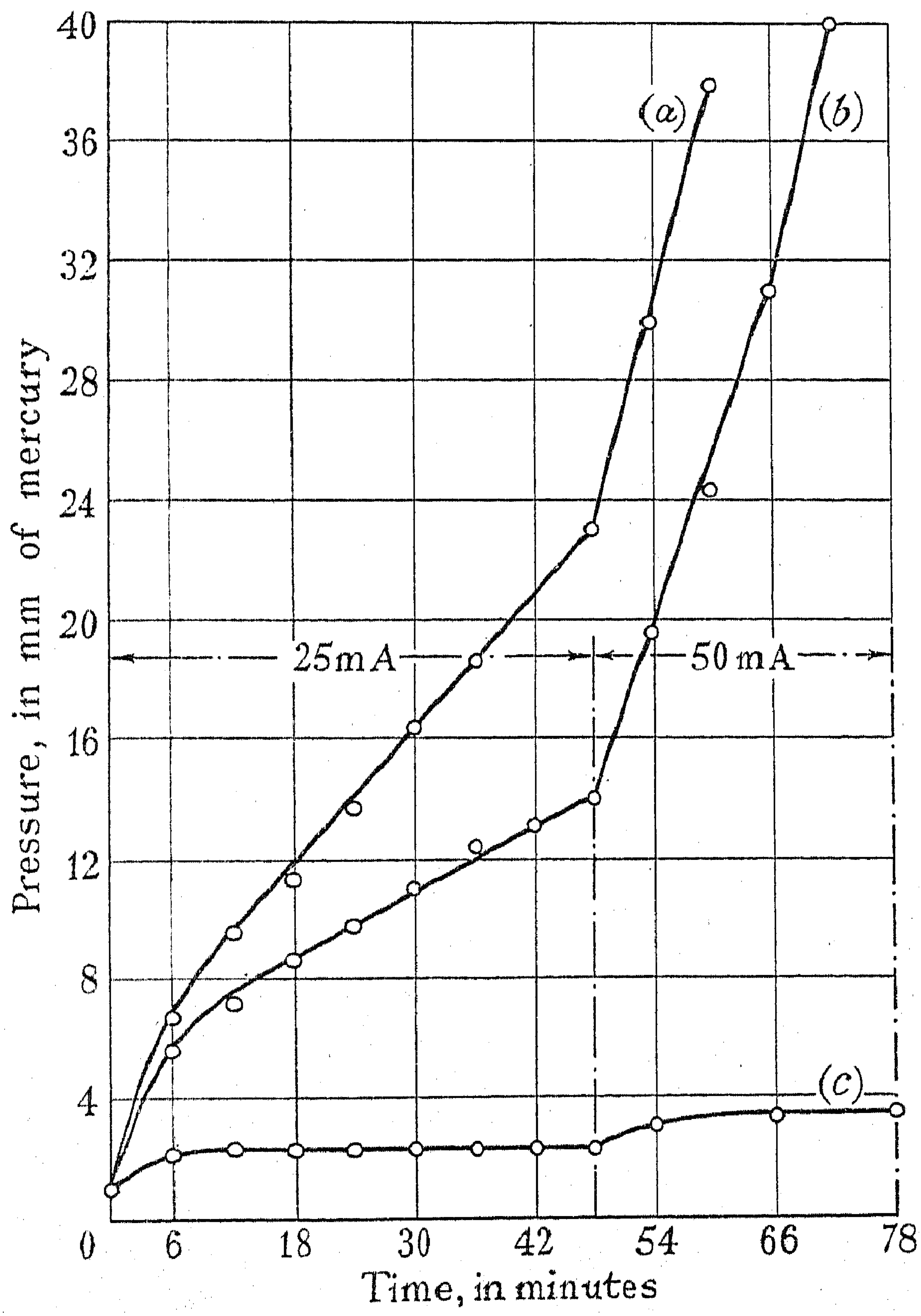

Fig. 2.—Gas generated by various oils: temperature about 20° C.
(*a*) Oil A (highly refined spindle oil). (*b*) Oil B (refined spindle oil). (*c*) Spindle-oil Edeleanu extract.

gas space, and finally again through the glass. A small portion of the current passes through the glass where this is covered with oil and therefore also passes through the oil as a displacement current.

As a frequency of 7 500 cycles per sec. was available, a fairly strong current could be maintained in the discharge tube, even with the outside electrodes. The current strength generally applied was 25 to 50 mA, at which value it was easy to make the determinations. After only a few minutes, differences in gas generation were noticeable between oils of a different nature. Above some oils the pressure rose rapidly, above others slowly, or not at all. In the part of the discharge tube not coated with tin-foil, striae were visible at the lowest pressures,

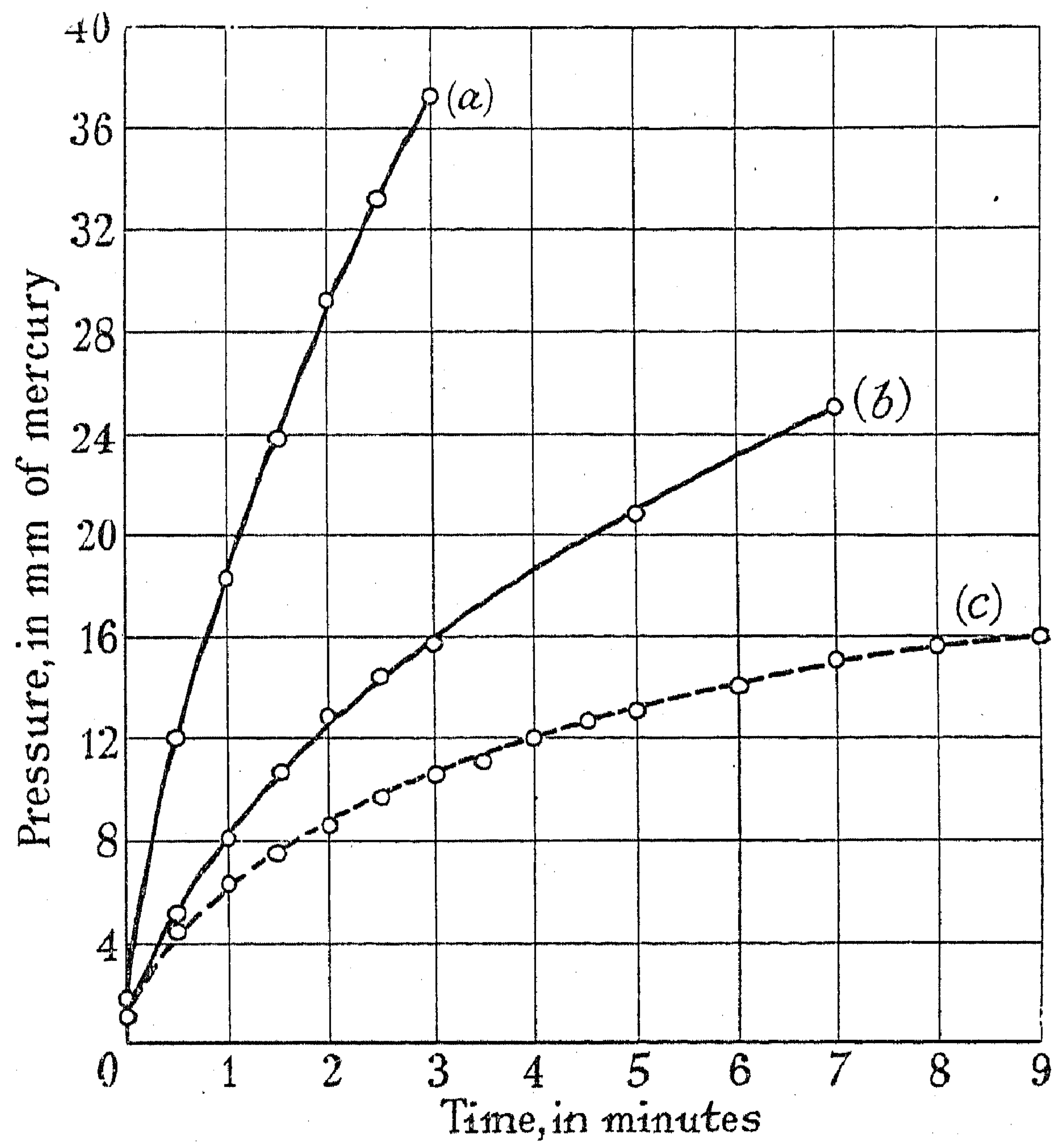

Fig. 3.—Oil D (*paraffinum liquidum*) at various temperatures and currents.
(*a*) 86° C., 45 mA, 7 500 H.
(*b*) 32° C., 60 mA, 7 500 H.
(*c*) 32° C., 30 mA, 7 500 H.
Volume of oil = 100 cm³.

perpendicular to the axis of the tube and the surface of the oil. These became fainter if the pressure increased, and they finally disappeared altogether.

In Fig. 2 the increases in pressure above two highly refined spindle-oils and a spindle-oil Edeleanu extract are compared by plotting the pressure read from the gauge as the ordinate against the time of passage of the current as the abscissa. For 48 minutes the oils were subjected to a discharge of 25 mA, and then the strength of the current was increased to 50 mA. The temperature given in the Figure is that of the oil bath.

Results

Fig. 2 shows the great difference between the behaviour of oils of different composition. Whereas above the highly aromatic Edeleanu extract the pressure scarcely increased at all, it rose very markedly over the raffinates.

Figs. 3, 4, 5, and 6, show the influence of a second factor

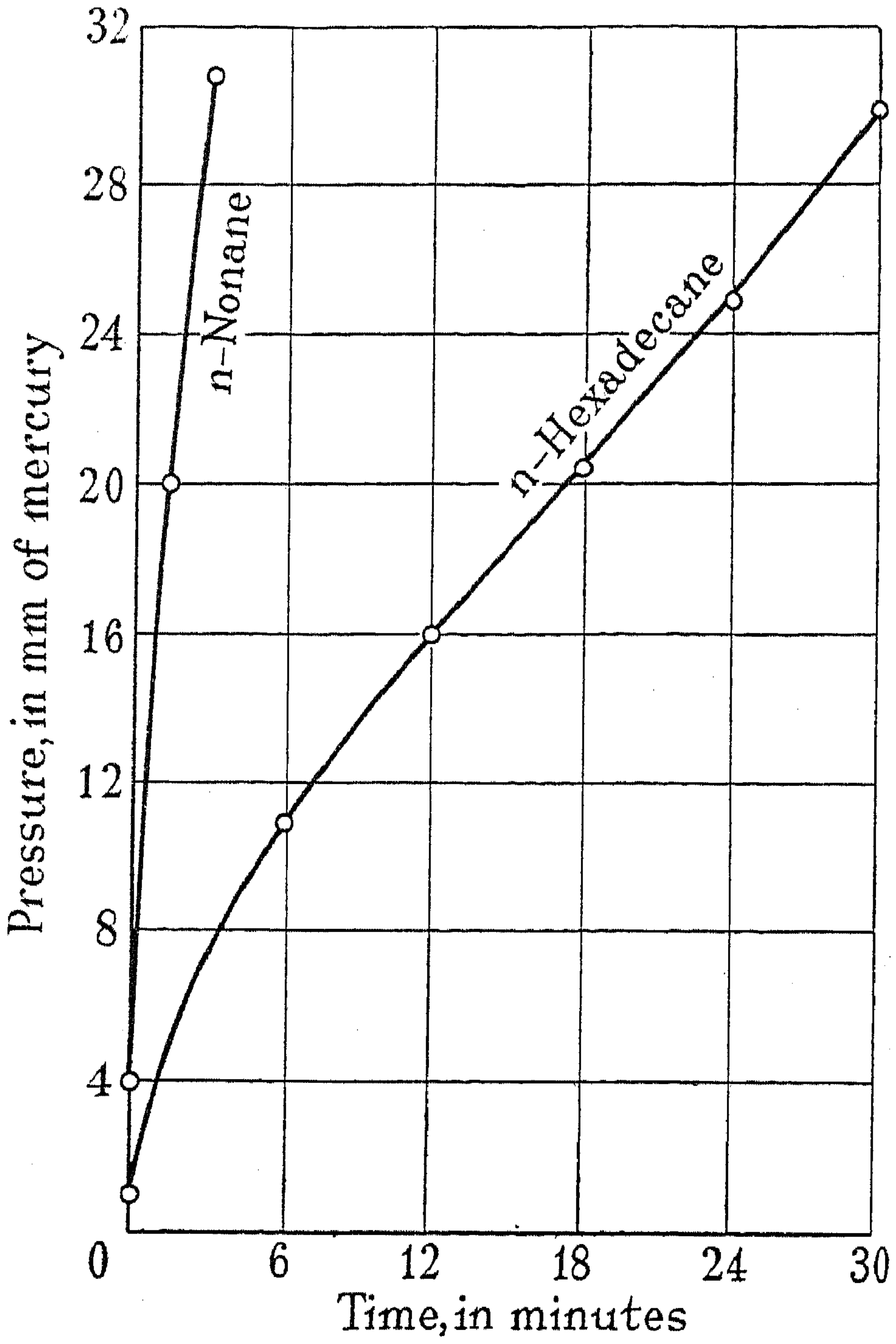

Fig. 4

Temperature about 22° C., discharge current 25 mA, volume of oil 25 cm³ in each case.

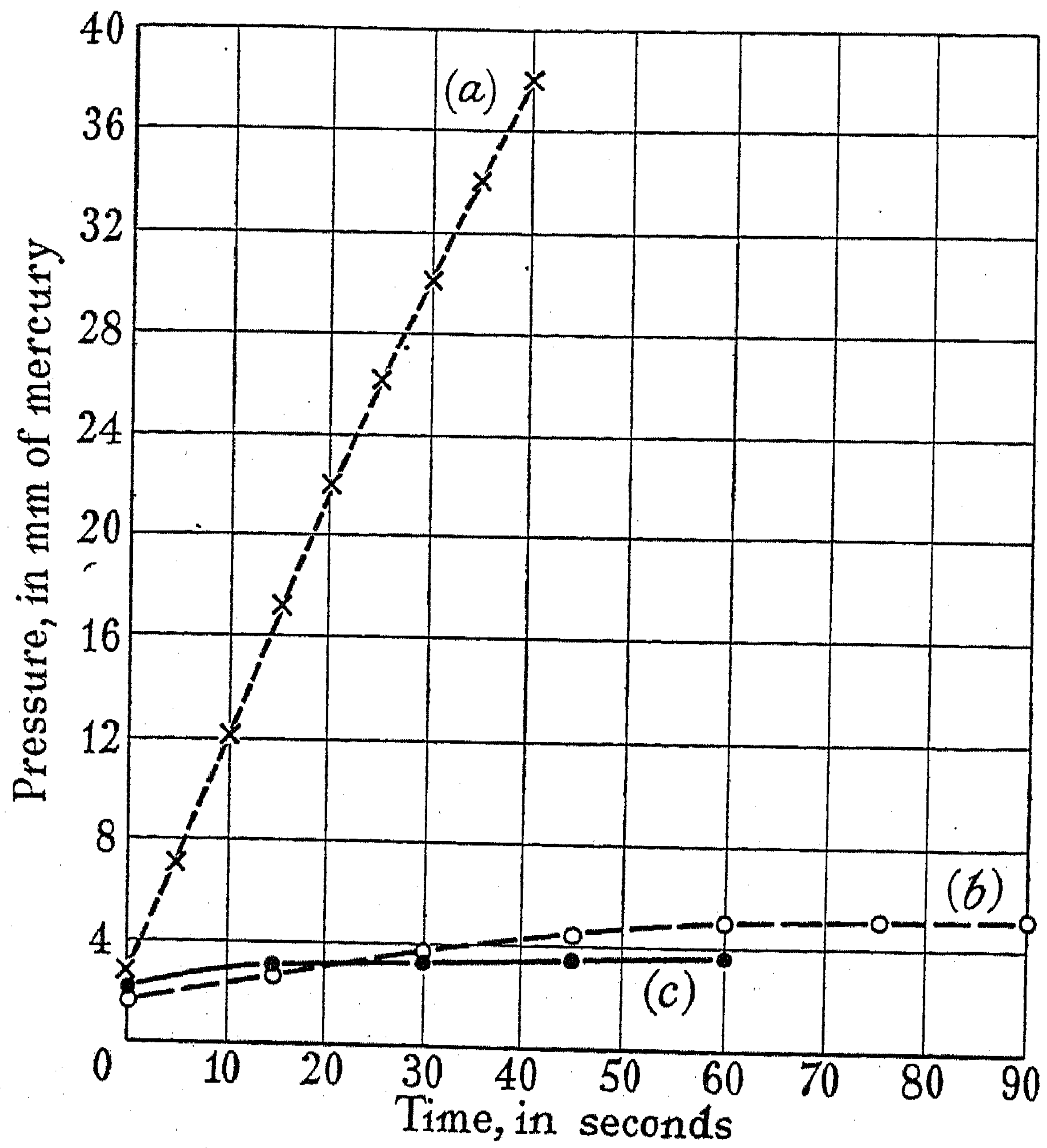

Fig. 5

(a) Edeleanu extract + 10 per cent *paraffinum liquidum* + 10 per cent gaso line (boiling point 141°–160° C.).
(b) Edeleanu extract + 10 per cent *paraffinum liquidum.*
(c) Spindle-oil Edeleanu extract.
All tests at 60 mA, about 20° C., and 7 500 H.

of importance, namely the volatility. The generation of gas may take place in two different ways: (a) Owing to the discharge, the liquid molecules are destroyed, part of the fragments being volatile. (b) Owing to the discharge, oil molecules are demolished in the gas phase. The gas phase is then no longer in equilibrium with the liquid. This will then supply further molecules to the gas phase, where they are in their turn demolished. It is clear that in process (b) the volatility of the oil is an important factor. Figs. 3 to 6 show that process (b) plays an important part in the generation of gas in the apparatus of Fig. 1, for the following reasons: (1) The gas generation of *paraffinum liquidum* is accelerated by increasing the temperature (Fig. 3). (2) The gas generation of various paraffinic hydrocarbons is greater the higher the volatility (Fig. 4). (3) The gas generation of an Edeleanu extract from a spindle-oil distillate is only slightly in-

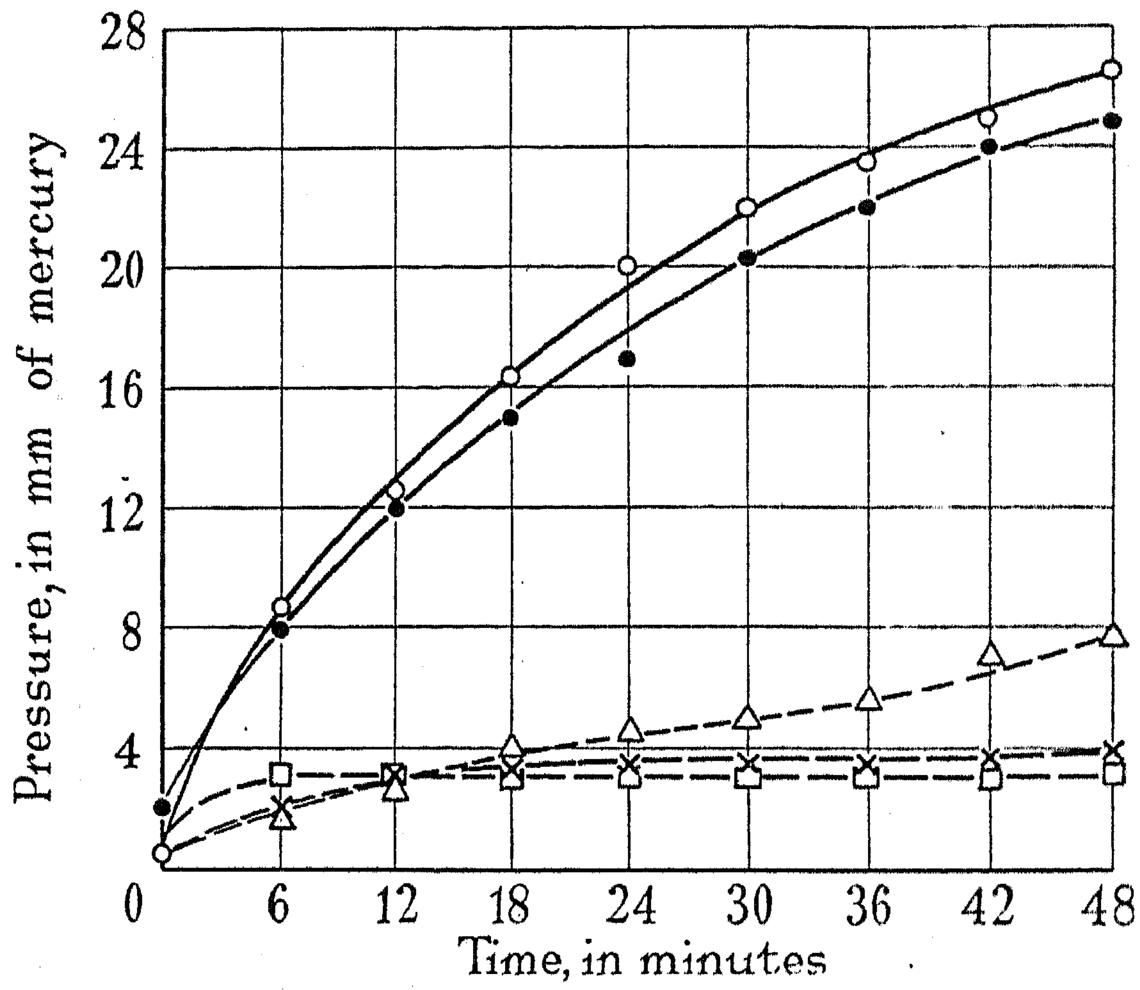

Fig. 6.—Oil C (highly refined): 25 cm³, 25 mA, temperature about 20° C.

○ No admixture.
● With 5 per cent tri-isopropyl naphthalene.
△ With 0·5 per cent benzene.
X With 1 per cent toluene.
□ With 5 per cent tetraline.

fluenced by the addition of 10 per cent of *paraffinum liquidum*, but is greatly accelerated if 10 per cent of a much more volatile product is added (gasoline B.P. 141°–160° C., see Fig. 5). The slight influence of the 10 per cent of *paraffinum liquidum* on the gas generation is attributable to the fact that neither the gas phase nor the liquid phase undergoes a radical change by this addition. When, on the contrary, 10 per cent of gasoline is added, a gas phase rich in gasoline vapours is formed above a liquid which still consists largely of the original extract. According to explanation (b), a strong gas generation may therefore be expected, which, however, is less strong than above 100 per cent of gasoline, as the vapour pressure is substantially lower. (4) The generation of gas by a non-aromatic is hardly influenced by a low percentage of an aromatic of the same volatility, but is, on the contrary, greatly reduced by the same percentage of a more volatile aromatic (Fig. 6).

By the addition of a low percentage of more volatile aromatic there is formed above the refined oil a gas phase in which the percentage of aromatic is much higher.

During the passage of a discharge a mixture is formed of aromatics, oil molecules, and demolition products. This mixture will polymerize quickly. If the vapour phase consists only of oil molecules, as is the case above the oil without the addition, the tendency to polymerization of the demolition products formed during the discharge is much smaller; further, owing to the high H : C ratio of the oil, gases like hydrogen, methane, and others, will always remain.

MEASUREMENTS UNDER A PRESSURE OF 1 ATMOSPHERE

From the measurements under reduced pressure it was found that the generation of gas by a refined oil can be lessened by adding a low percentage of aromatics more volatile than the oil. As the pressure in a cable can vary greatly, it was of importance to find out whether the effect of small quantities of aromatics would be the same in gas discharges under higher pressure. The new measurements were carried out at 1 atmosphere. They could not be made in the apparatus of Fig. 1, because excessively high voltages would be required in view of the long distance which the discharge has to cover in the gas. For this reason an apparatus with a shorter distance between the electrodes was used. The frequency was 50 cycles per sec. in these measurements. Air, nitrogen, and hydrogen, were examined as gas fillings.

During the determinations with air atmosphere a decrease in pressure was noticed, with highly refined oils; this was probably due to the reaction of the oxygen from the air with the oil. In nitrogen the pressure rose above all the oils examined, but this rise took place at different rates. Above an oil with 3 per cent of naphthalene the pressure rose a good deal less rapidly than above the oil without this addition. In the determinations with hydrogen atmosphere a great difference was noticeable from the beginning between highly refined oils and oils with many aromatics. With the former the pressure began to rise, but with the latter to drop, as soon as a current passed through the discharge tube.

As the temperature of a cable in actual use may be considerably higher than that of the surroundings, the influence of an increase in temperature on the gas generation or absorption was ascertained for oils under hydrogen pressure. Determinations were made at 20° C. and at 60° C.

Above a highly refined spindle oil subjected to a discharge in hydrogen, the pressure rose more rapidly at the higher temperature. Above a less refined oil the pressure-drop at the low temperature was accompanied by an increase in pressure at the higher temperature. Above a highly refined spindle oil with 3 per cent of naphthalene the hydrogen pressure dropped when the discharge passed through, both at 20° C. and at 60° C. Fortunately, however, the pressure dropped more quickly at the higher temperature.

CABLE OILS THAT GENERATE LITTLE GAS

After the experiments described above had been carried out, it was possible to deal with the problem of how to prepare cable oils that generate little gas. A solution of this problem would be of great practical use.

Insulating oils are often highly refined in order to obtain a high resistance to oxidation. By the refining process the tendency to generate gas is greatly aggravated, however. This shows that the two requirements, of high resistance to oxidation and little tendency to gas generation, are, up to a certain point, irreconcilable. Now, if care is taken to shut off the air sufficiently, the requirement of resistance to oxidation can be ignored and a less refined oil, with a high content of aromatics and/or unsaturated products, can be used. If, however, both requirements must be met together, then addition of 10 per cent or less of aromatics more volatile than the oil can give the desired result.

Small percentages of naphthalene, tetraline, and

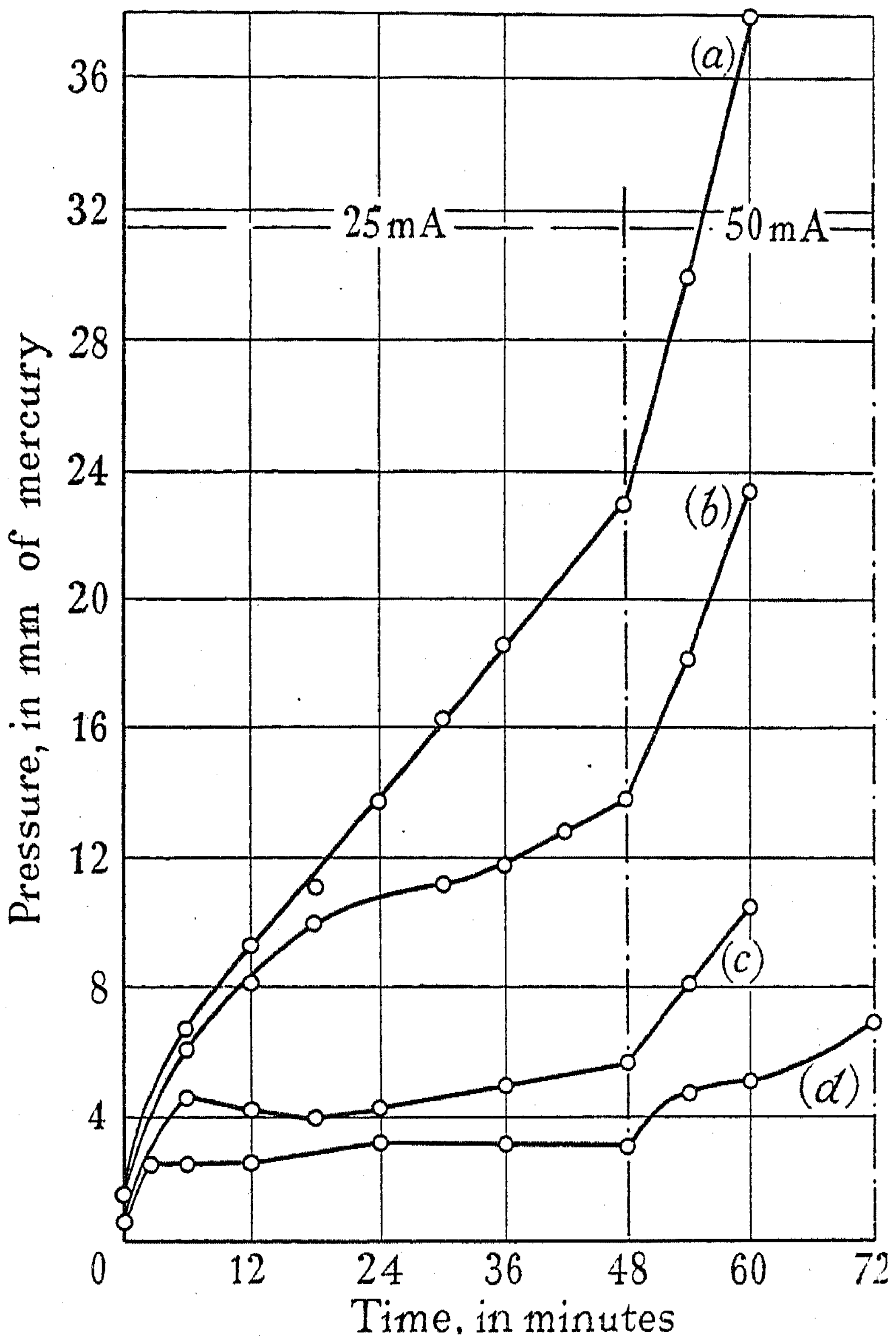

Fig. 7.—Oil A (filled cable oil) and admixtures.

(a) No admixture.
(b) 0·5 per cent anthracene oil.
(c) 0·5 per cent tetraline.
(d) 0·5 per cent naphthalene.
All tests at about 20° C., with 25 cm³ cable oil.

α-methyl naphthalene, can be added to a well-refined oil without seriously affecting the resistance to oxidation.

Fig. 7 shows the influence of 0·5 per cent of naphthalene, tetraline, or anthracene oil, on the gas generation of a filled-cable oil, while Fig. 8 shows the influence of 10 per cent of α-methyl naphthalene on the gas generation of a heavy cable oil. These determinations were made under reduced pressure. Examined under hydrogen of 1 atmosphere pressure the influence of an addition of only, say, 0·5 per cent of naphthalene is less pronounced. In the determinations under a hydrogen pressure of 1 atmosphere very favourable results were obtained with 3 per cent of naphthalene. The percentage required can vary greatly. It goes without

saying that of the oils that have the required resistance to oxidation, the one will be selected that generates least gas.

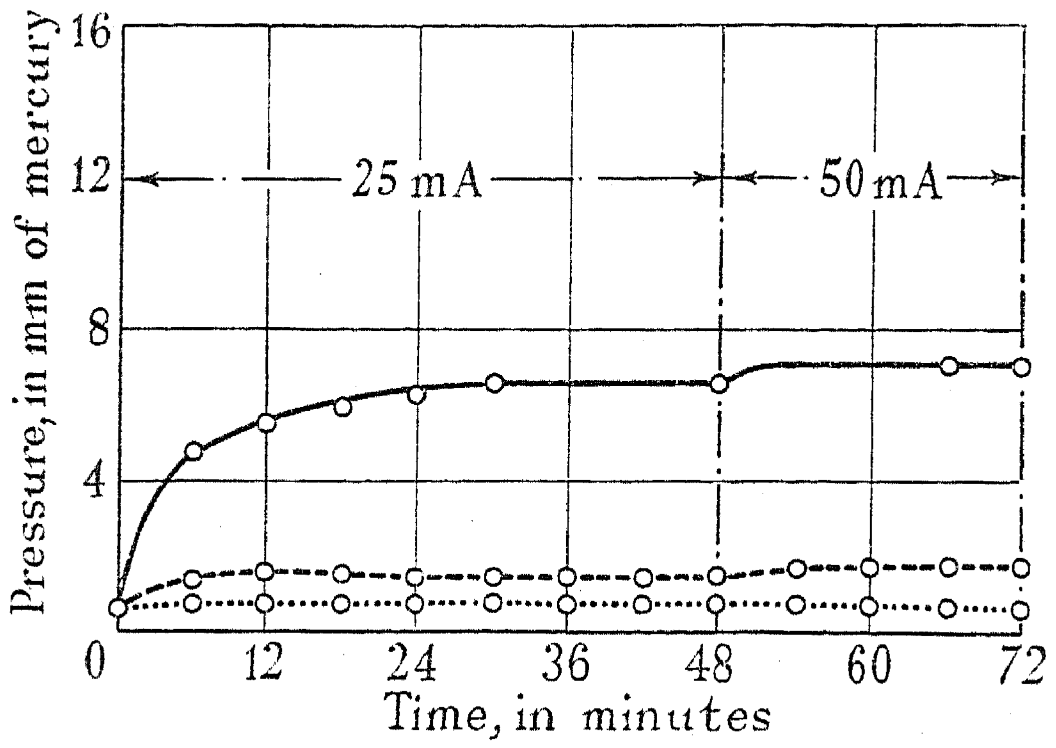

Fig. 8

—— Heavy cable oil.
- - - - 100 per cent of α-methyl naphthalene.
. . . . . Heavy cable oil + 10 per cent by weight of α-methyl naphthalene.
All tests at about 20° C., with 25 cm^3 cable oil.

Aromatics to be Added

The number of aromatic compounds which have a favourable effect on the generation of gas by a cable oil is very great. Both pure hydrocarbons and compounds with other atoms such as N, Cl, O, etc., may possess this property. From these compounds have to be chosen those which have no unfavourable influence on the breakdown voltage, the angle of loss, and the oxidation resistance of the oil. The influence on the gas generation

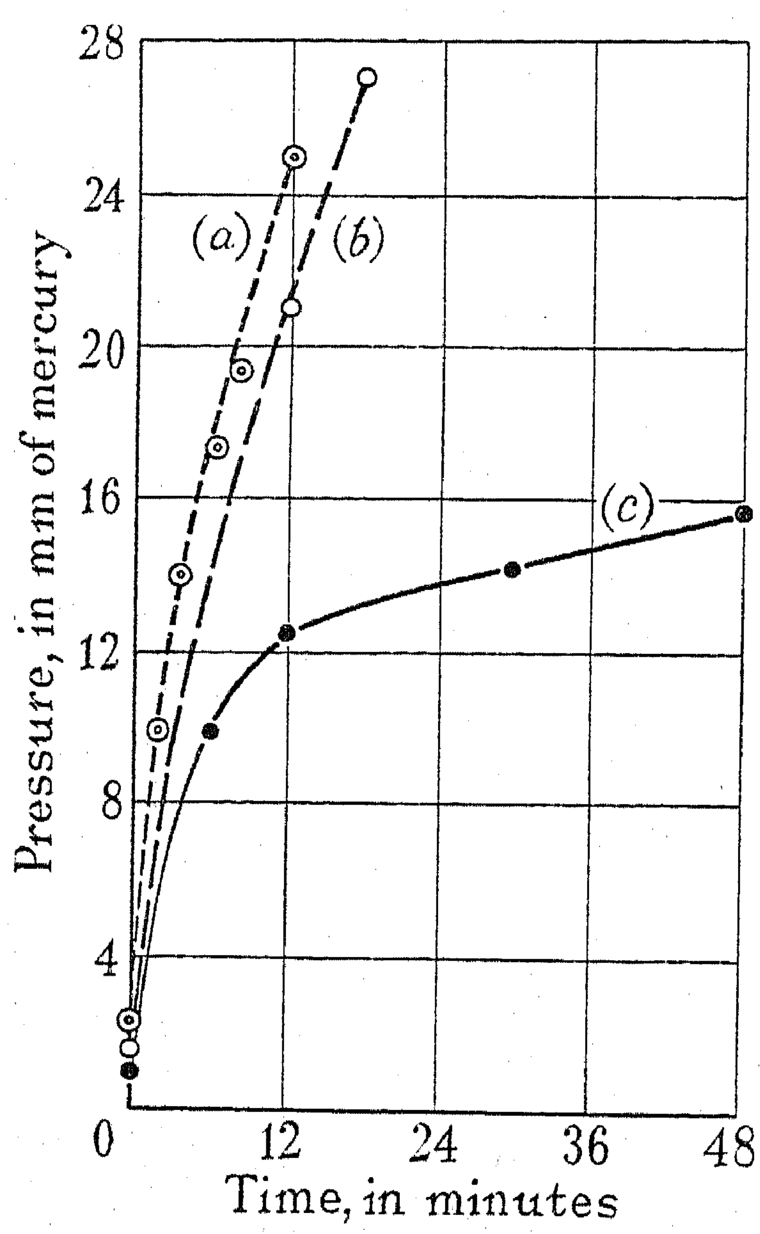

Fig. 9

(a) 25 cm^3 aniline.
(b) 25 cm^3 quinoline.
(c) 25 cm^3 tetraline.
All tests at 25 mA and about 20° C.

produced by substances of the following groups was ascertained:—

(A). Benzene, naphthalene, diphenyl, and anthracene, with their partly hydrogenated compounds and their alkyl-substituted derivatives; in which, however, the number of H atoms added should remain small and the alkyl groups,

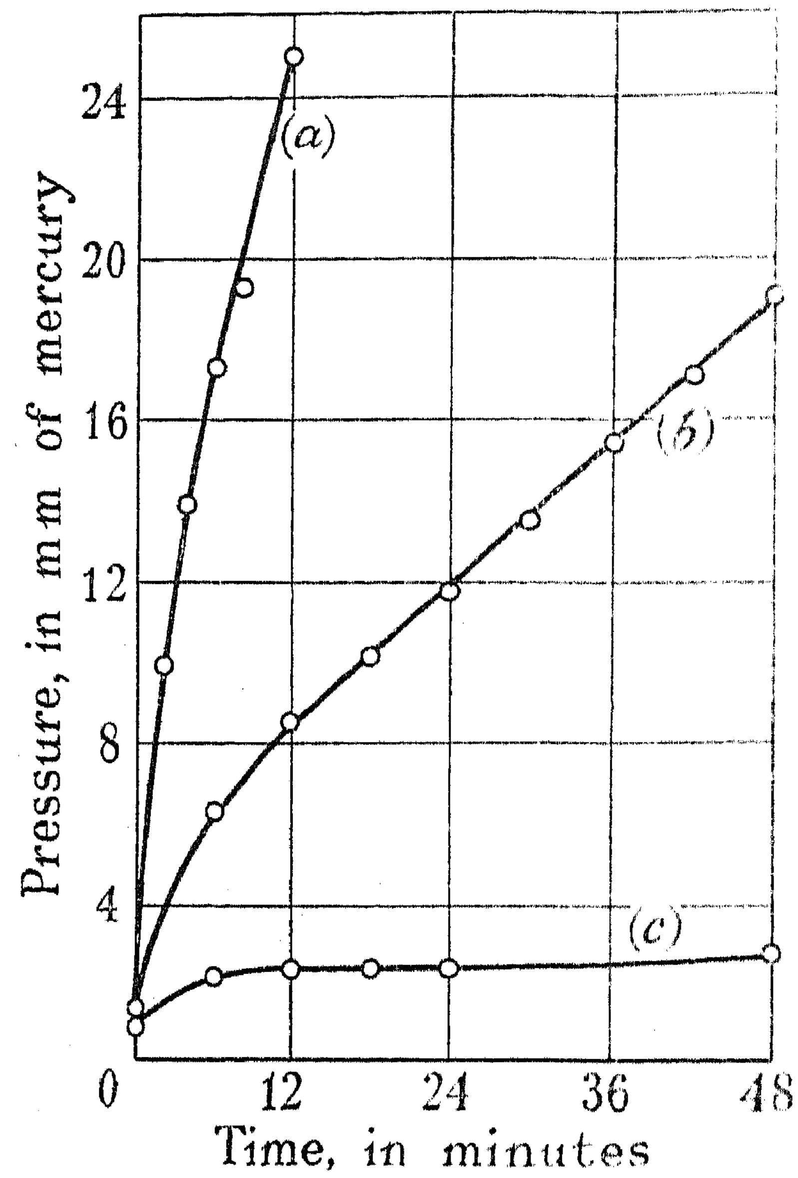

Fig. 10.—Oil E (*paraffinum liquidum*) and aniline; 25 mA, temperature about 20° C.

(a) 25 cm^3 aniline.
(b) 25 cm^3 oil E.
(c) 25 cm^3 oil E + 1 per cent aniline.

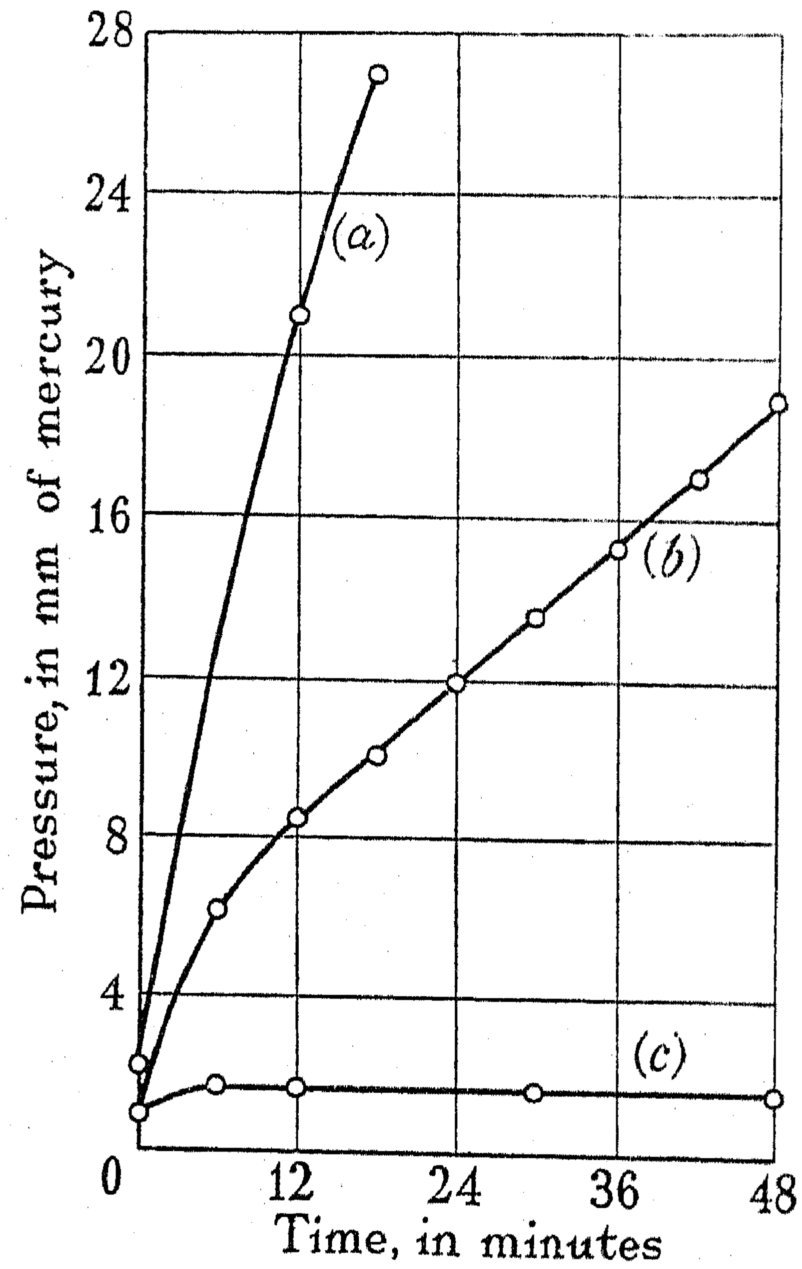

Fig. 11.—Oil E (*paraffinum liquidum*) and quinoline; 25 mA, temperature about 20° C.

(a) 25 cm^3 quinoline.
(b) 25 cm^3 oil E.
(c) 25 cm^3 oil E + 1 per cent quinoline.

if added, should remain short. For it is essential that the slight H-saturation and the compact structure of the molecule should be maintained. Besides the four

aromatics mentioned above, this group includes toluene, xylene, ethyl benzene, α-methyl naphthalene, tetraline, and others. In decaline the H-saturation will presumably be too great.

(B). The same substances as in Group A, but having in one of the aromatic rings a C–H group replaced by an N atom; e.g. pyridine and quinoline.

(C). Mono-amino compounds of aromatic hydrocarbons, e.g. aniline.

Harkins and Gans,* in their research concerning naphthalene, spectroscopically identified the following fragments in the electrodeless glow discharge: (C), (H), (C +), (C_2), (CH). These were soon united. As is apparent from Fig. 8, this formation of large molecules in the tube of Fig. 1 takes place so rapidly for α-methyl naphthalene that the pressure is not increased, although new molecules constantly enter into the gas phase.

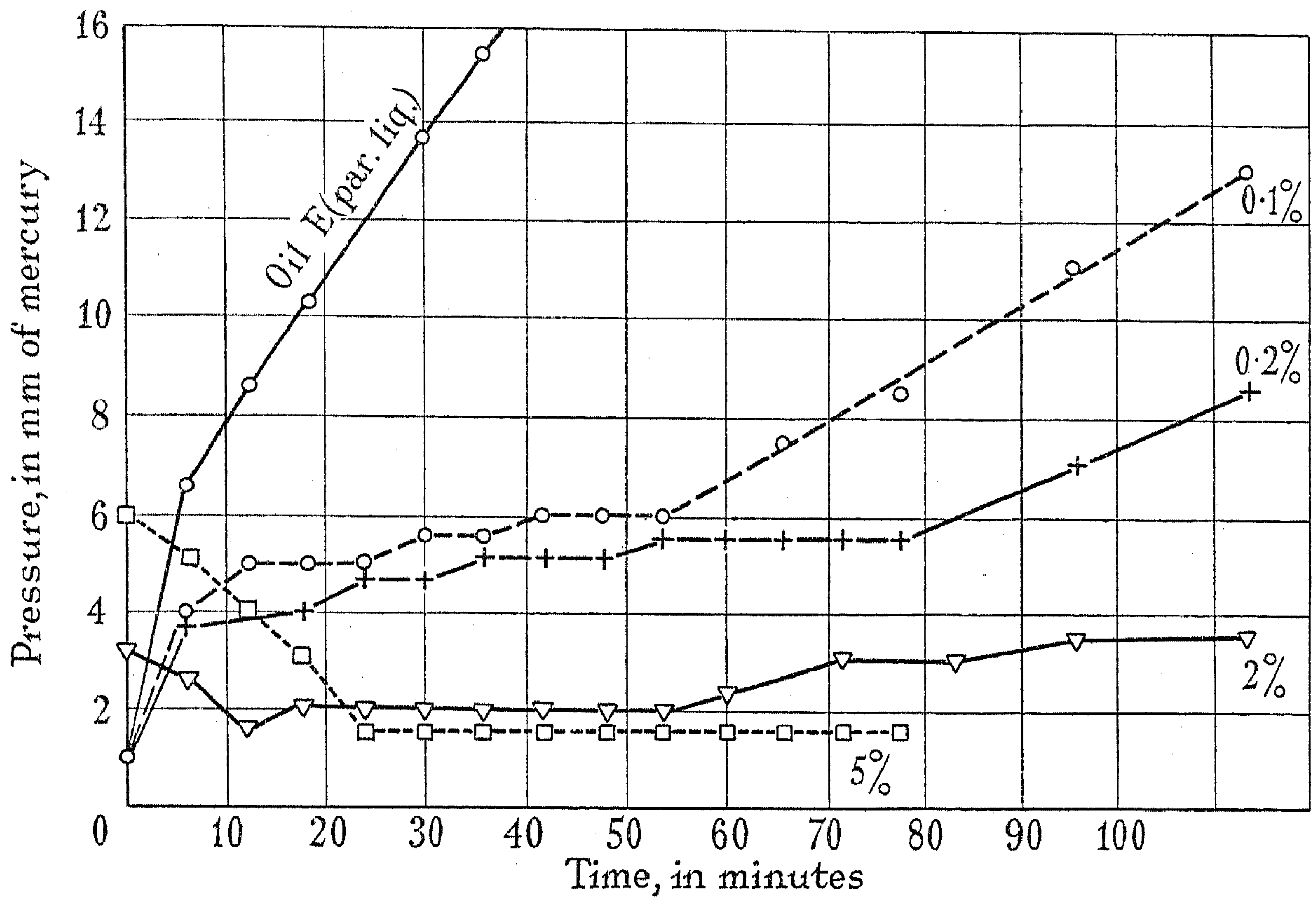

Fig. 12.—Oil E (*paraffinum liquidum*) with benzene: 25 mA, temperature about 20° C.

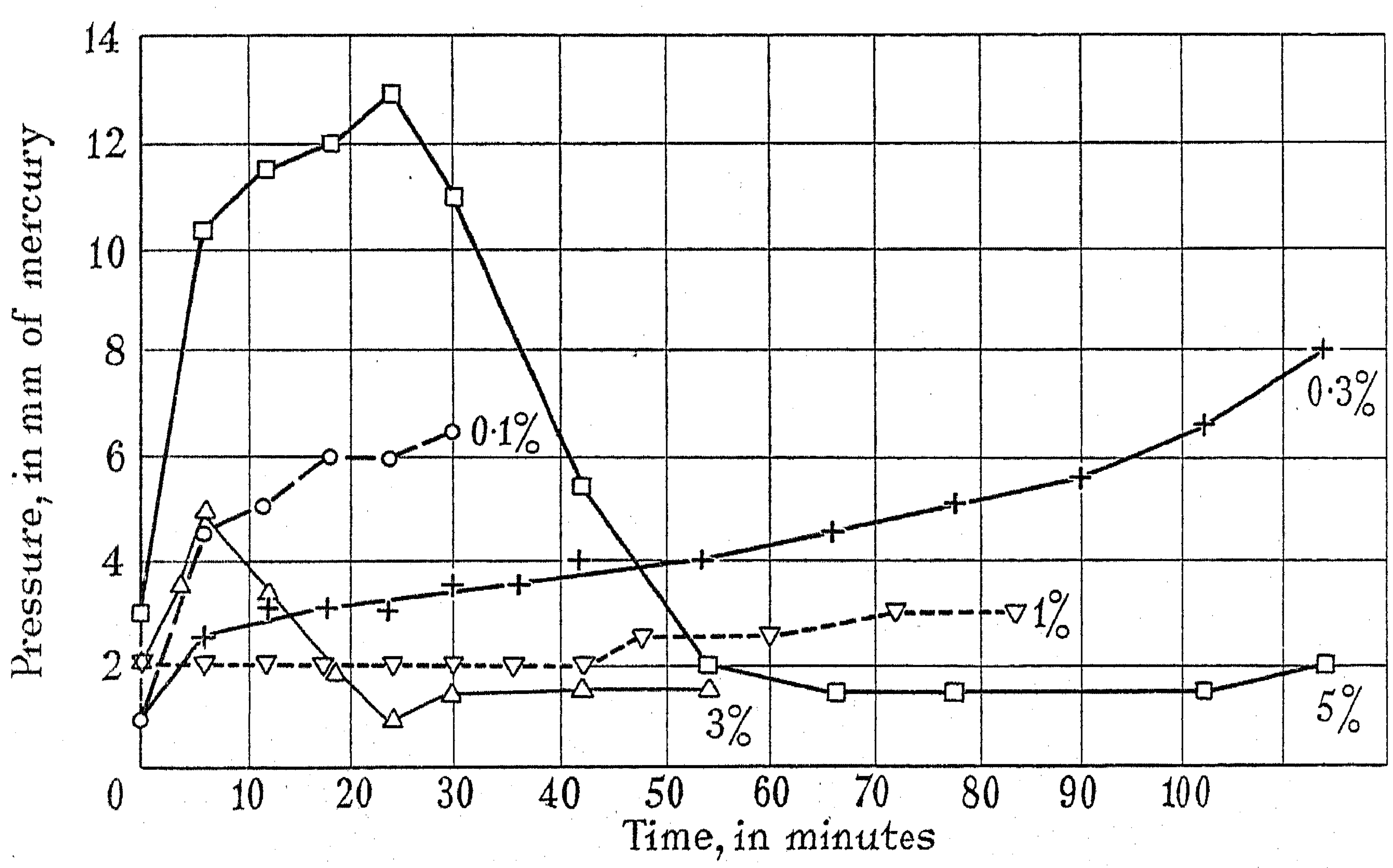

Fig. 13.—Oil E (*paraffinum liquidum*) with toluene: 25 mA, temperature about 20° C.

Fig. 9, however, shows that with tetraline, aniline, and quinoline, the pressure rises. At lower temperatures this would happen less readily, as then the volatility is smaller. The vapour pressure, however, can also be

* See Bibliography, (2).

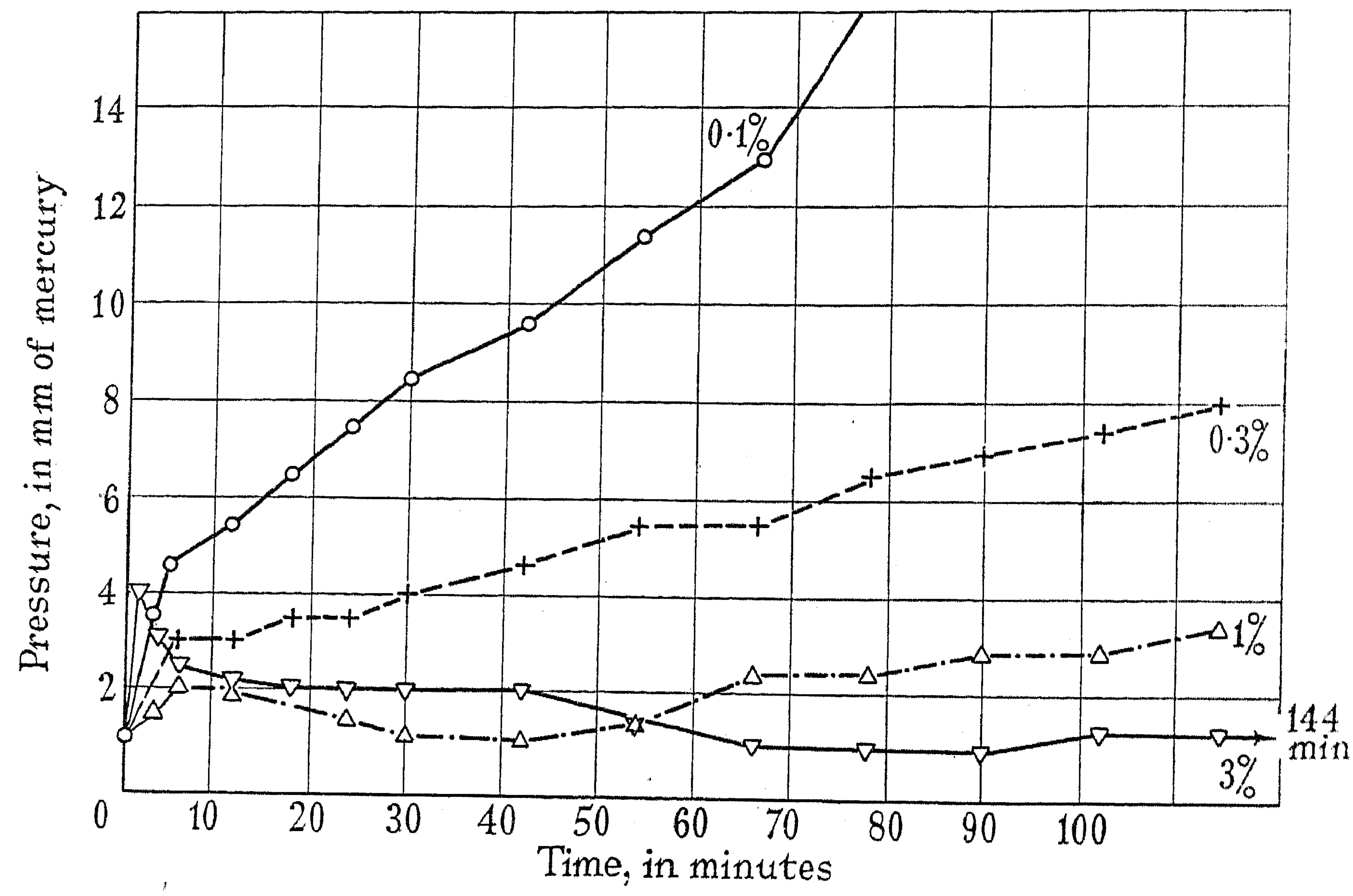

Fig. 14.—Oil E (*paraffinum liquidum*) with ethyl benzene: 25 mA, temperature about 20° C.

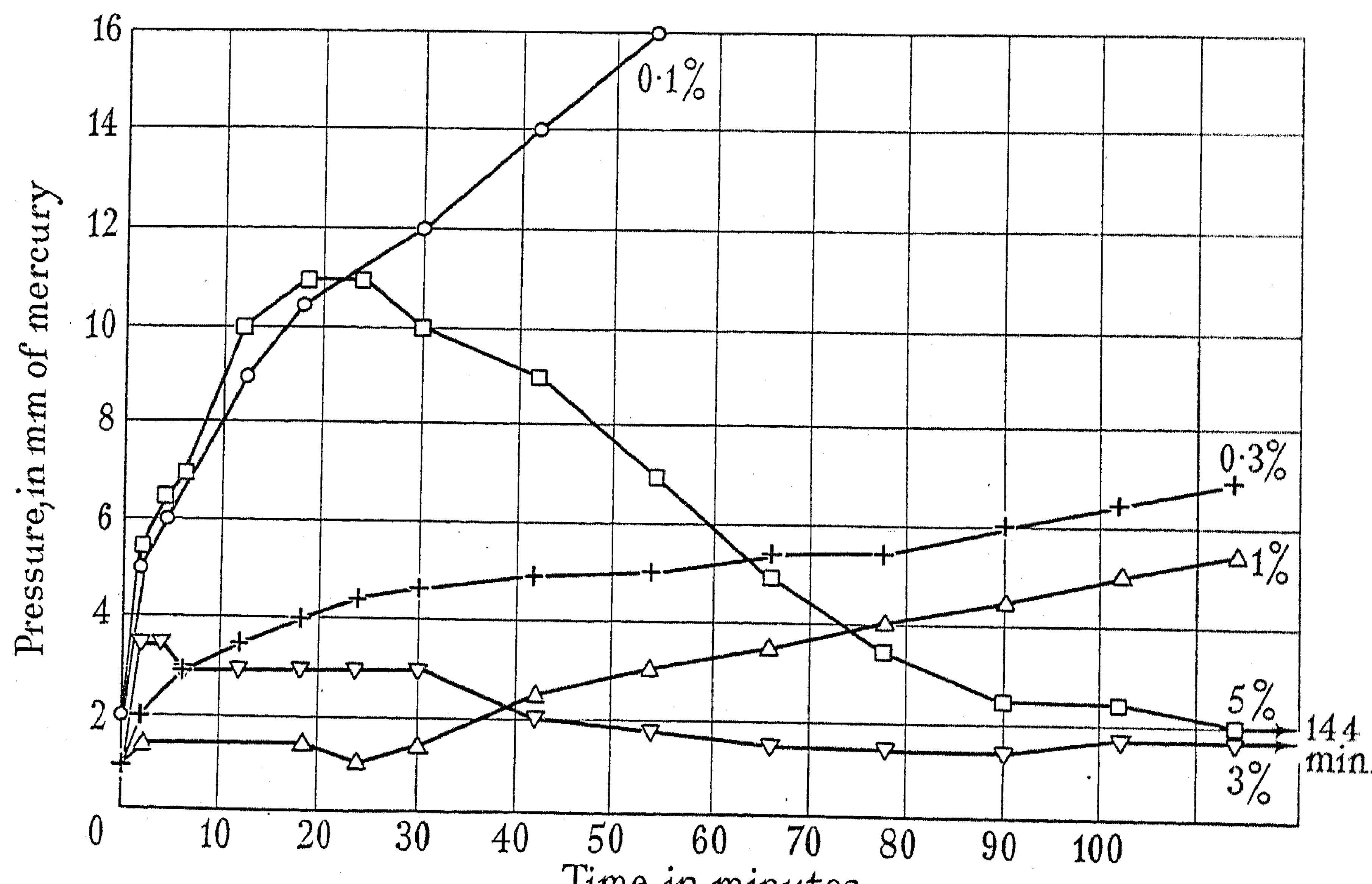

Fig. 15.—Oil E (*paraffinum liquidum*) with xylene: 25 mA, temperature about 20° C.

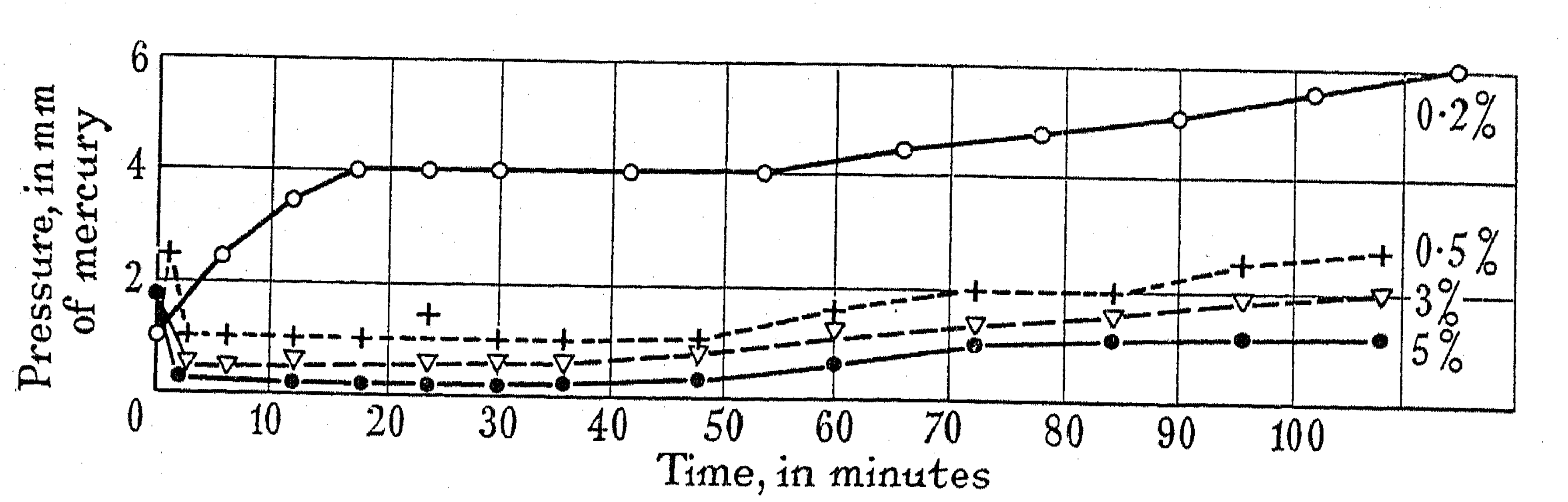

Fig. 16.—Oil E (*paraffinum liquidum*) with naphthalene; 25 mA, temperature about 20° C.

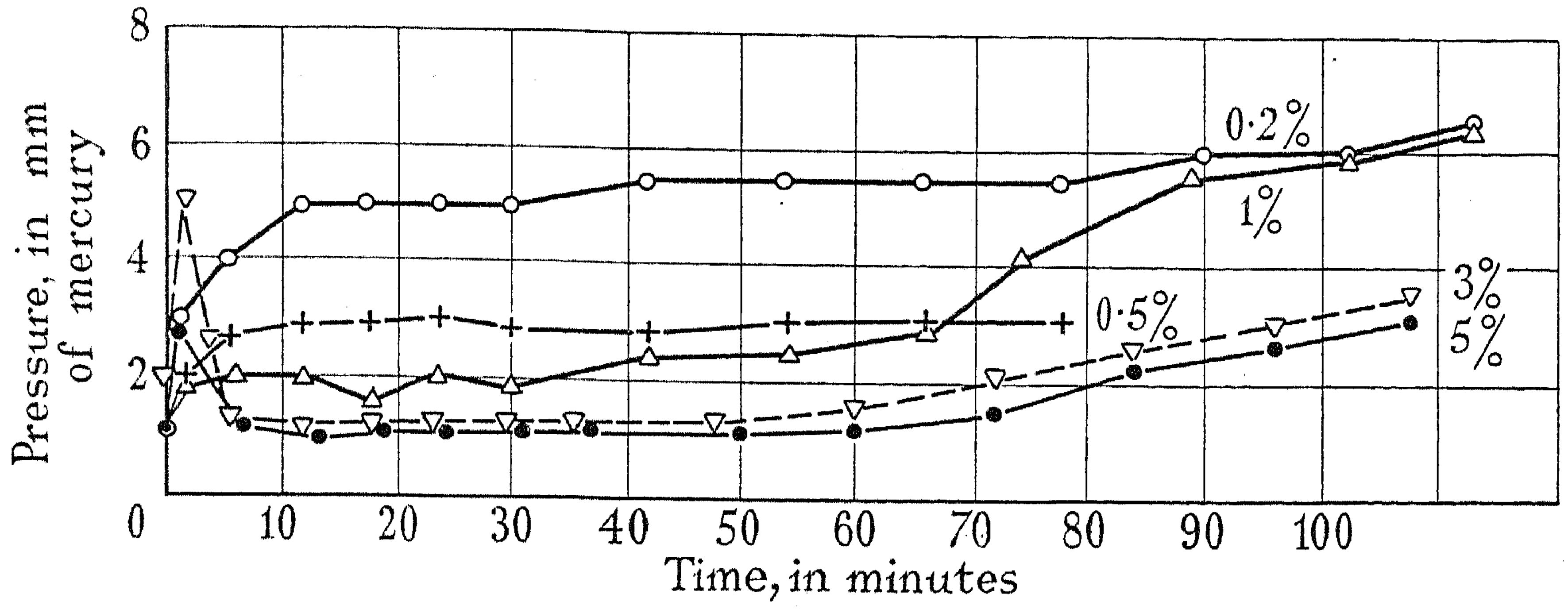

Fig. 17.—Oil E (*paraffinum liquidum*) with tetraline; 25 mA, temperature about 20° C.

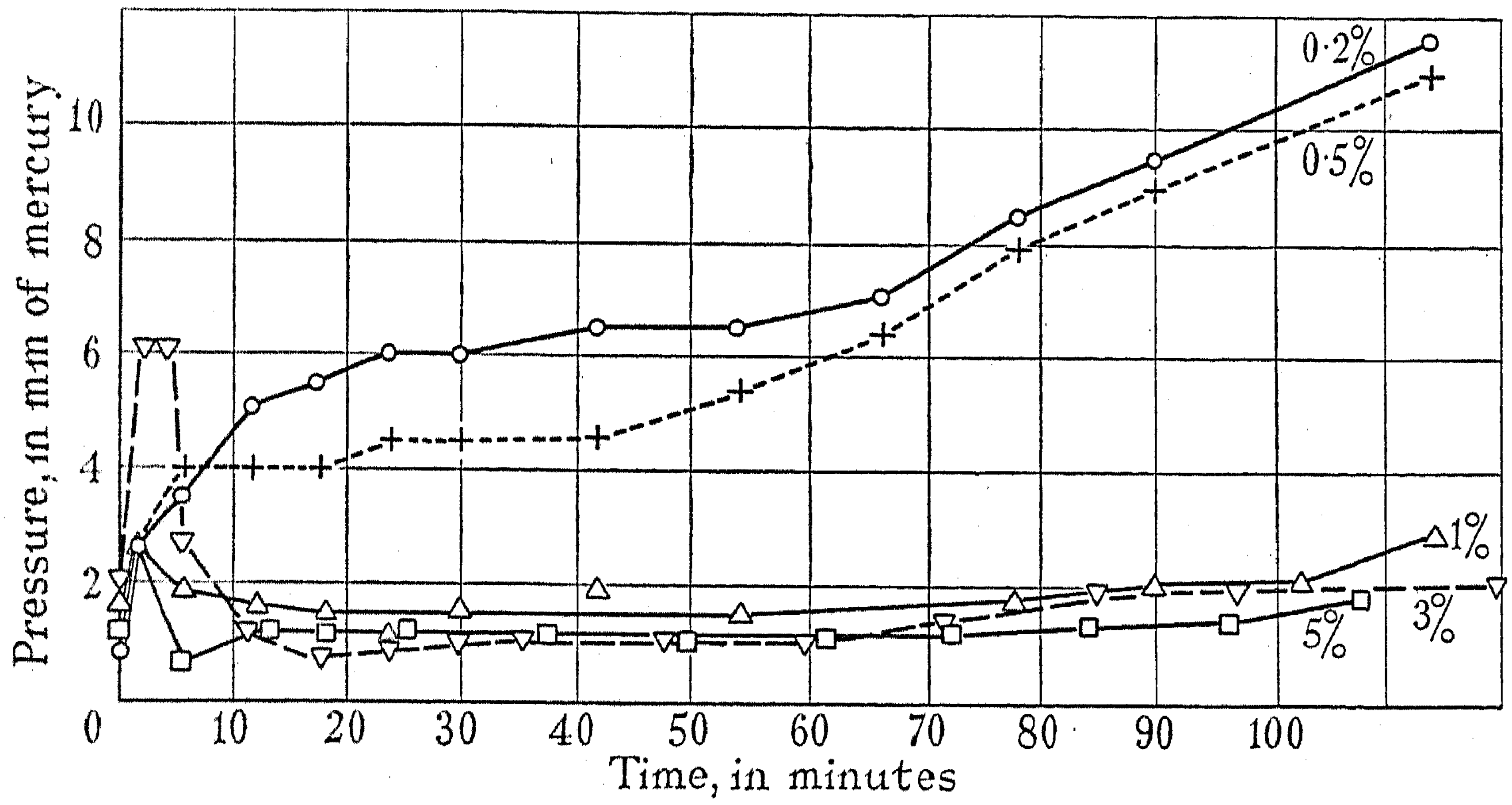

Fig. 18.—Oil E (*paraffinum liquidum*) with methylnaphthalene; 25 mA, temperature about 20° C.

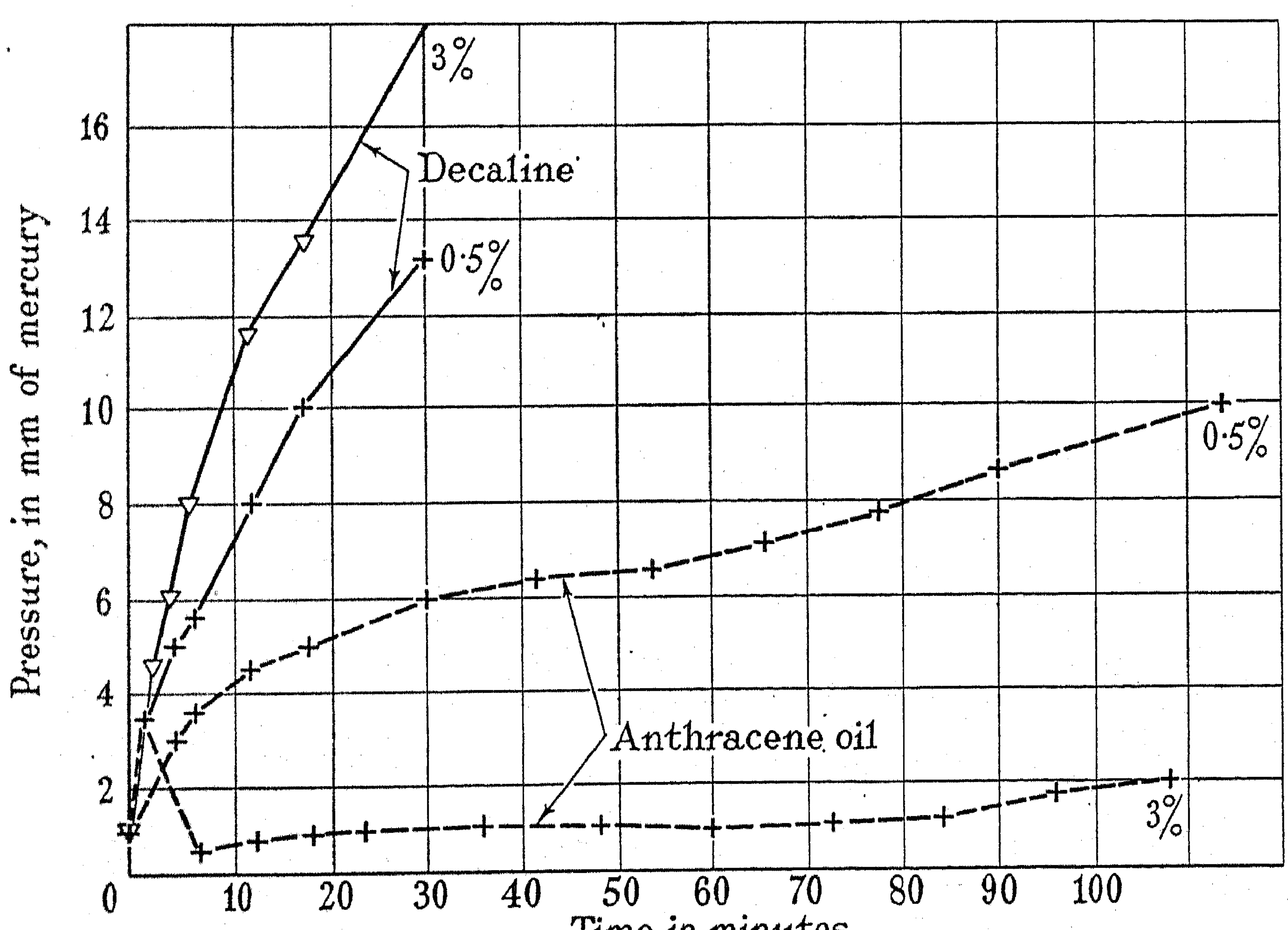

Fig. 19.—Oil E (*paraffinum liquidum*) with (i) decaline and (ii) anthracene oil; 25 mA, temperature about 20° C.

reduced in another way, namely, by dilution with a less volatile component, which is exactly what happens with volatile aromatics when used in a cable oil. It is therefore possible not only that the addition of aromatics reduces the tendency of the oil to generate gas, but that, conversely, the mixture—as yet containing but few volatile components—generates less gas than the aromatic itself.

Fig. 10 shows the generation of gas in oil E, in aniline, and in a mixture of the two. Fig. 11 refers to similar experiments with respect to quinoline. In both cases the mixture is better than either of the components.

Percentage of Aromatics to be Added

The influence of various percentages of aromatics was systematically ascertained for various substances of Group (A), by adding them to *paraffinum liquidum*. The results are plotted in Figs. 12 to 19. The generation of gas of the *paraffinum liquidum* itself is shown in Fig. 12.

In Fig. 19 the results are registered of the examination of anthracene oil and decaline. Addition of the latter was found not to have a great effect.

CONCLUSION

The results of the investigation prove that the generation of gas by an oil under the influence of an electric discharge greatly depends on the nature of the most volatile components of the oil. The gas generation of various oils can be greatly reduced by adding small percentages of aromatics more volatile than the oil. This can, in practice, lead to considerable improvement of oil exposed to discharges, as is the case with cable oils, switch oil, condenser oils, and transformer oils. The most suitable substance to add, and the best percentage addition, differ widely for various oils.

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ACKNOWLEDGMENT

The author wishes to express his gratitude to the management of the Bataafsche Petroleum Maatschappij for granting him permission to publish this paper.

TRANSIT TIME EFFECTS IN DIODES, IN PICTORIAL FORM

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[Communication from the Research Staff of the M.O. Valve Co., Ltd., at the G.E.C. Research Laboratories, Wembley, England.]

(*Paper first received* 30*th March, and in final form* 13*th May,* 1936; *read before the* WIRELESS SECTION 6*th May,* 1936.*)

SUMMARY

In order to obtain a pictorial representation of what happens in thermionic valves at frequencies at which the transit times of the electrons play an important part, electron positions and velocities, in a plane parallel diode, were calculated for a succession of times.

In the case of a saturated diode, the voltage was assumed and the current obtained from the diagram. The result shows that there is an alternating component of electron current which leads the applied voltage and appears in the circuit as an addition to the geometrical capacitance current. The valve, therefore, appears as a finite resistance in parallel with an effective capacitance greater than the geometrical capacitance.

In the case of the space-charge-limited diode the current was assumed and the voltage obtained from acceleration diagrams. The result shows that the valve behaves as a resistance in parallel with a capacitance smaller than the geometrical capacitance.

This method has two advantages over purely analytical methods. It describes the mechanism of the effects, and it shows the presence of harmonics which usually disappear in the approximations made in analysis.

(1) SATURATED DIODE

Consider a plane parallel diode the cathode of which emits a constant stream of electrons with zero velocity. Let the electron current be so small that it does not appreciably modify the field between cathode and anode. Then the force acting on each electron is known.

Let the field at all points in the space be

$$\epsilon(1 + M \cos \omega t) \text{ volts per cm}$$

Then
$$km\frac{d^2x}{dt^2} = e\epsilon(1 + M \cos \omega t) \quad . \quad . \quad . \quad (1)$$

where e = electronic charge,
m = electronic mass,
k = ratio erg/joule = 10^{-7},
x = distance in cm from the cathode,
M = constant.

Integration of equation (1) gives

$$\frac{dx}{dt} = \frac{e\epsilon}{km}\left[t - t_1 + \frac{M}{\omega}(\sin \omega t - \sin \omega t_1)\right] \quad . \quad (2)$$

and

$$x = \frac{e\epsilon}{km}\left[\frac{(t-t_1)^2}{2} - \frac{M}{\omega^2}(\cos \omega t - \cos \omega t_1) - \frac{M}{\omega}(t - t_1)\sin \omega t_1\right] \quad . \quad (3)$$

where $x = 0$, $dx/dt = 0$ when $t = t_1$, t_1 being the time of emission from the cathode.

* There was no time at the meeting for discussion on the paper. Written communications are therefore invited for consideration with a view to publication in the *Journal*.

In the example worked to give a diagram of electron distribution, the following numerical values were assumed:—

$$\alpha = \frac{e\epsilon}{km} = 10^{16} \text{ cm/sec.}^2,$$
$$M = \tfrac{1}{2},$$
$$\omega = \tfrac{1}{2}\pi \times 10^8,$$

and the steady-state transit time $T = 10^{-8}$ sec., which is a quarter of a period.

These are the values of α and T which would be obtained by assuming the distance d from the cathode to the anode to be $0 \cdot 5$ cm, and the steady voltage V_0 across the diode to be $2 \cdot 82$ volts.

It was assumed that one electron was emitted from the cathode every 10^{-9} sec., that is, 40 electrons were emitted in a complete cycle. The positions and velocities of these electrons were calculated from equations (2) and (3) when

$$t = 0, \quad 3 \times 10^{-9}, \quad 6 \times 10^{-9} \ldots 39 \times 10^{-9} \text{ sec.}$$

These positions are shown in Fig. 1(*a*). The row AB shows the positions of all the electrons in the space at the instant at which the voltage is at its maximum. The other similar rows are the corresponding pictures for later times; for instance, CD shows the positions of all the electrons in the space when the voltage is increasing through its mean value i.e., when $\omega t = 270°$.

The curves of the type AF are graphs of position against time for individual electrons. The slope at any point on such a curve gives the velocity of the electron at that point.

The contribution of each electron to the current in the external circuit is

$$\frac{e}{d} \cdot \frac{dx}{dt}$$

Therefore the current at any instant is given by†

$$I = \frac{e}{d}\sum \frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

† This expression is equivalent to

$$i = \int_{t-\tau}^{t} \frac{\psi(t_0)}{x_1} \cdot \frac{dx}{dt} dt_0$$

given by D. O. NORTH: *Proceedings of the Institute of Radio Engineers*, 1936 vol. 24, p. 108; and to

$$I = \int_0^{\tau} I_{ct_0}\frac{\dot{x}}{d}d\tau$$

given by C. J. BAKKER and G. DE VRIES: *Physica*, 1935, vol. 2, p. 683.

the summation being taken for all the electrons in the space at that instant. The current wave obtained in this way would have discontinuities due to the smallness of the number of electrons, so the curve was smoothed by interpolation. This was done by assuming a charge of 1 000e, distributed continuously between consecutive electrons, and attributing the mean velocity of these electrons to this amount.

The current wave so obtained is shown by curve I in Fig. 1(b). Curve V is the assumed voltage.

It will be seen that the number of electrons in the space is a minimum at about $\omega t = 50°$, and a maximum at about $\omega t = 260°$, the voltage having been a maximum at $\omega t = 0°$ and a minimum at $\omega t = 180°$. This is because the electrons are pulled out of the space quickly when the voltage is large, and accumulate in the space during the time when the force on them is small. The maximum current occurs at the time when the big number of electrons, which accumulated in the space during the time when the voltage was small, is being swept across and out of the space by the increasing voltage. The accumulation is drawn in to the anode before the voltage reaches its maximum, so the current maximum leads the voltage maximum. The closeness of the parabolas in Fig. 1(a) depends on both the velocity and the number of electrons, and is therefore an indication of the current strength.

A harmonic analysis of the current wave for the first three harmonics gives

$$I = 1\cdot 59 \times 10^{-7}\left[1 + 0\cdot 26 \cos(\omega t + 42°) - 0\cdot 092 \cos(2\omega t - 39°) - 0\cdot 032 \cos(3\omega t + 41°)\right] \text{ amperes}$$

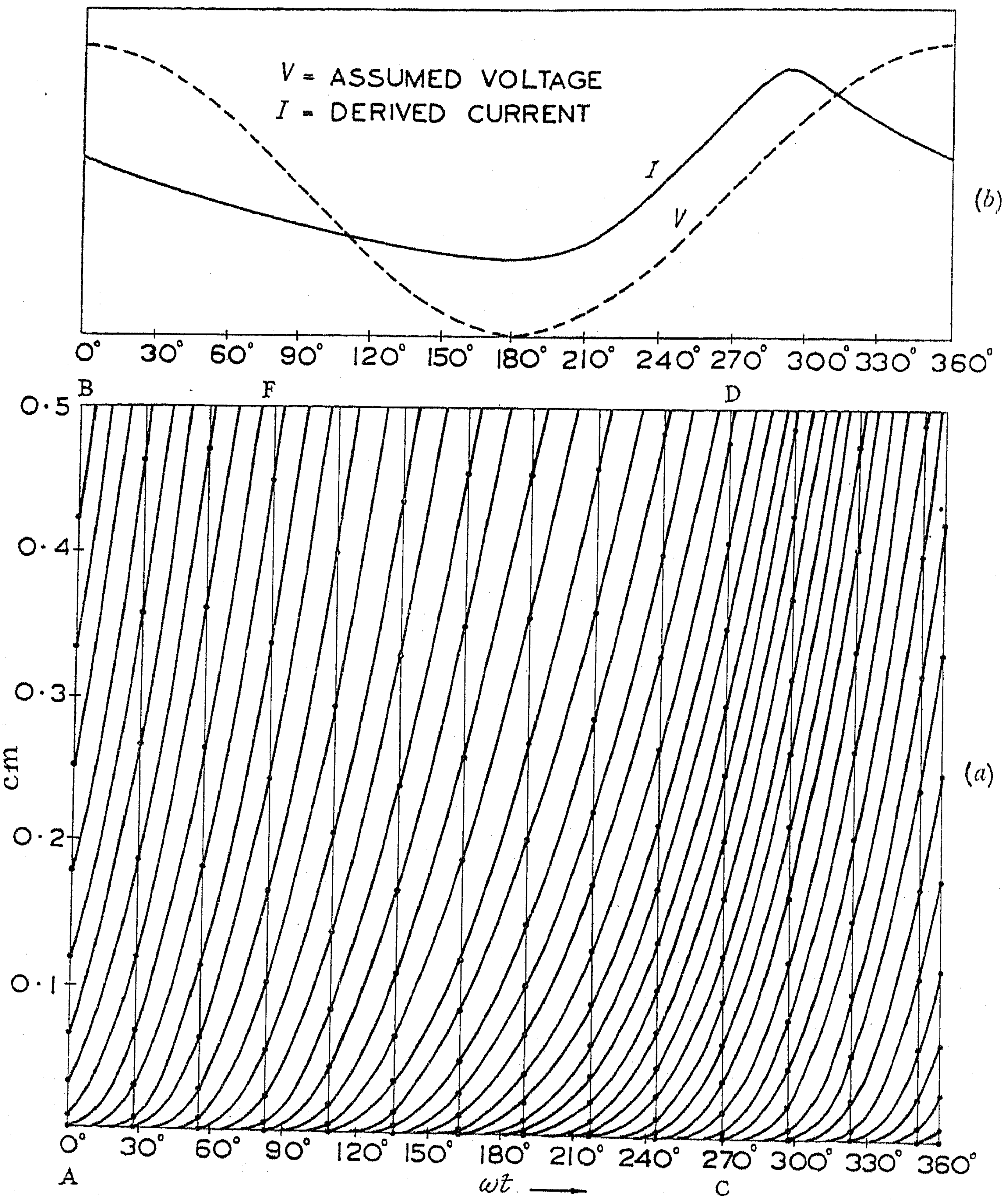

Fig. 1 (a) and **(b)**.—Distribution of charge in a temperature-limited diode.

The total current I, its fundamental component I_1, and its second harmonic component I_2, are shown in Fig. 2. The fundamental component of the current leads the voltage by 42°. Approximate calculation from equations derived by Bakker and De Vries* gives the angle of lead as 46°.

The electrons, the velocity of which we have used to deduce the current, finally deliver their kinetic energy as heat at the anode, so it is of interest to graph the arrival velocity at the anode over a cycle. This graph

* C. J. Bakker and G. De Vries: *loc. cit.*

is curve (*a*) of Fig. 3. The maximum and minimum velocities lag behind the maximum and minimum of the applied voltage. This is to be expected, because the electron which arrives with the greatest velocity was in transit when the voltage was a maximum. If, however, the time of transit of the electrons is small compared with the period of the alternating voltage, each electron will travel the whole distance from cathode to anode under the influence of a substantially constant field. The velocity of arrival will then be in phase with the voltage. This is shown by curve (*b*) in Fig. 3. Comparison of curves (*a*) and (*b*) shows that the velocity, and therefore the energy of arrival, of the electrons are not, in general, those corresponding to the potential difference at the moment of arrival. In fact the difference, shown by the uppermost curve of Fig. 3, only disappears for those electrons which have been in transit during a maximum or minimum of the potential, i.e. when the field was substantially constant during the transit.

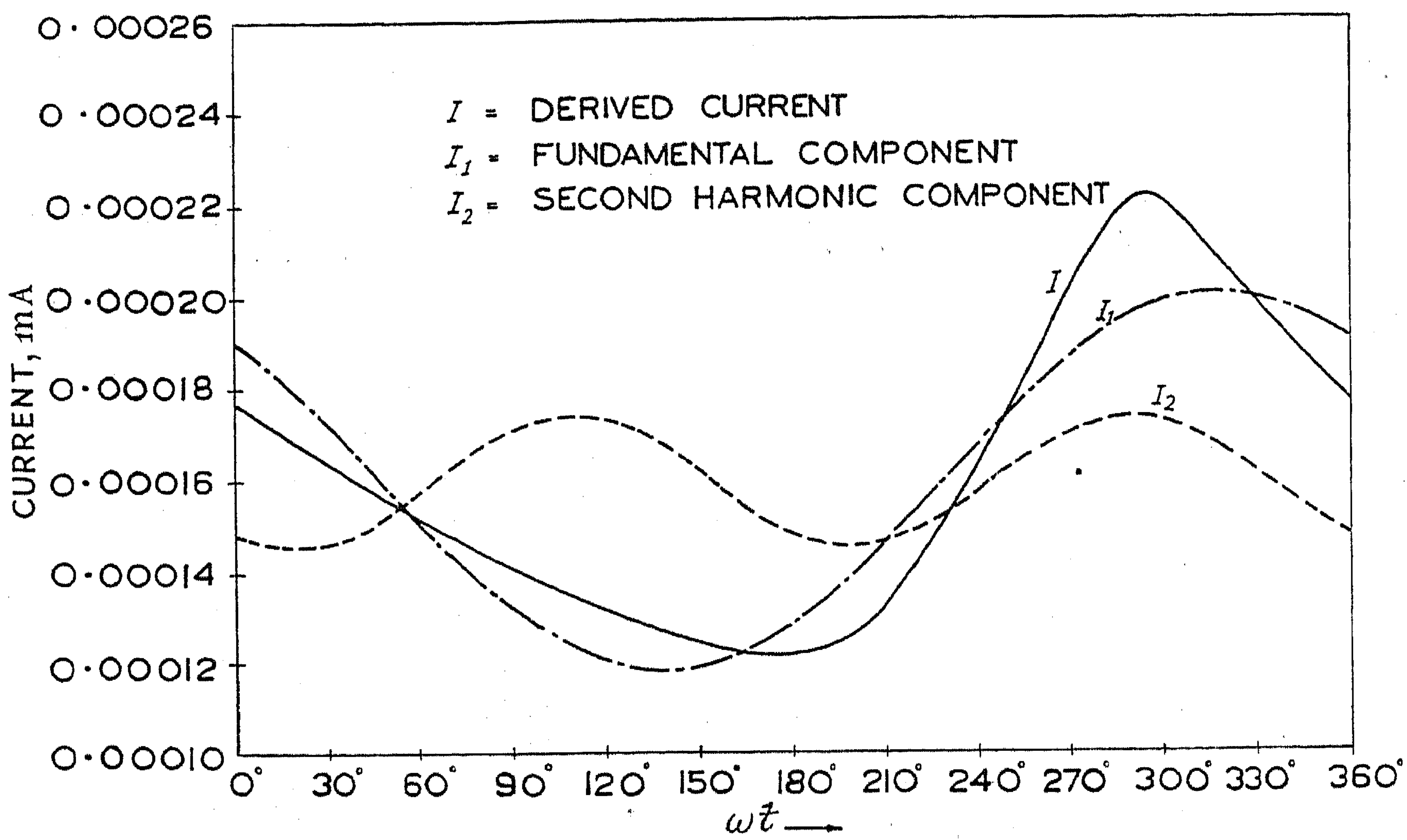

Fig. 2.—Analysis of current.

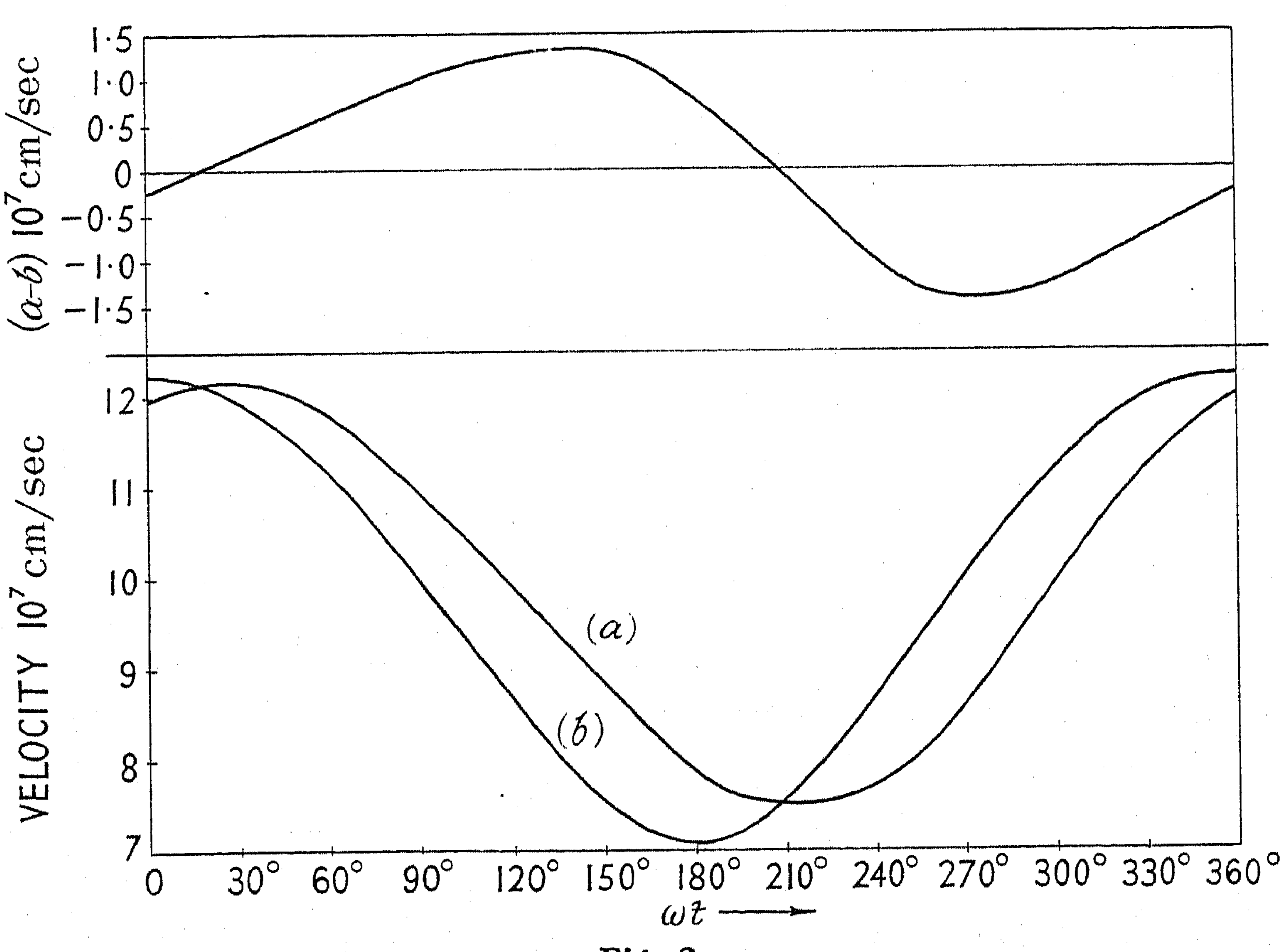

Fig. 3

(*a*) Velocity of arrival of electrons at anode.
(*b*) Velocity of arrival of electrons at anode when transit time is very small in comparison with the period.

(2) SPACE-CHARGE-LIMITED DIODE

It is not possible to treat the case of the space-charge-limited diode as a mechanical problem in the way the

saturated diode was treated, because it cannot be assumed that the electrons move unaffected by each other. Therefore, neither the distribution of field strength nor the number of electrons leaving the cathode can be deduced from the voltage across the diode.

In a space-charge-limited diode, so many electrons are emitted from the cathode that all lines of force from the anode end on electrons in the space. Therefore, the field strength is reduced to zero at the cathode. The field at the cathode is thus constant, and there is no displacement current through it, so the total current at the cathode is conduction current. The current at the anode, which is partly displacement and partly conduction current, must at each instant be equal to the current at the cathode. In other words, the conduction of electrons through the diode and the current corresponding to the geometrical capacitance current must both be accounted for by the electrons leaving the cathode.

Now, if the total current be assumed, the number of electrons leaving the cathode at any instant is known, as the current at the cathode is entirely conduction current.

The current per unit area is related to the acceleration of an electron by the equation*

$$I = \frac{km\kappa}{e} \cdot \frac{d^3x}{dt^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where e = electronic charge,
m = electronic mass,
κ = permittivity of a vacuum,
k = ratio erg/joule.

Let $$I = I_m(1 + M \cos \omega t)$$

Then $$\frac{d^2x}{dt^2} = \frac{eI_m}{km\kappa}\left[t - t_1 + \frac{M}{\omega}(\sin \omega t - \sin \omega t_1)\right] \quad . \quad (6)$$

and

$$x = \frac{eI_m}{km\kappa}\left[\frac{(t-t_1)^3}{6} - \frac{M}{\omega^3}(\sin \omega t - \sin \omega t_1) + \frac{M(t-t_1)}{\omega^2}\cos \omega t_1 - \frac{M(t_1-t_1)^2}{2\omega}\sin \omega t_1\right] \quad . \quad (7)$$

where $d^2x/dt^2 = 0$, $dx/dt = 0$, and $t = t_1$ at the plane $x = 0$, which is taken to be the cathode of the diode. The condition $d^2x/dt^2 = 0$ is the mathematical expression of space-charge limitation, i.e. the field at the cathode is zero, neglecting initial velocities.

The field strength ϵ is given by

$$\epsilon e = kmd^2x/dt^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the potential difference between the anode (the plane $x = d$) and the cathode at any instant t is

$$V = \int_0^d \epsilon dx$$
$$= \frac{km}{e}\int_0^d \frac{d^2x}{dt^2}dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Analytically, this integral can only be evaluated by successive approximation, because equation (7) cannot by solved for t_1. However, the integral may be found graphically by plotting d^2x/dt^2 against corresponding values of x for a series of values of t_1, t being constant for each graph. Two specimen $(d^2x/dt^2, x)$ curves are shown in Fig. 4. The area bounded by such a curve and the line $x = d$ is the integral in equation (9). In this way the voltage V was found for a series of values of t throughout a cycle, and is shown dotted in Fig. 5.

In obtaining the above curve, the following assumptions were made.

$$I = 0\cdot 1(1 + \tfrac{1}{2}\cos \omega t) \text{ milliamperes per cm}^2,$$
$$\omega = \tfrac{1}{2}\pi \times 10^8,$$
and $$T = 10^{-8} \text{ sec.}$$

The mean value of I and this value of T are those which would be obtained by assuming an anode-cathode spacing of $d = \frac{1}{3}$ cm and a steady voltage $V_0 = 2\cdot 82$ volts.

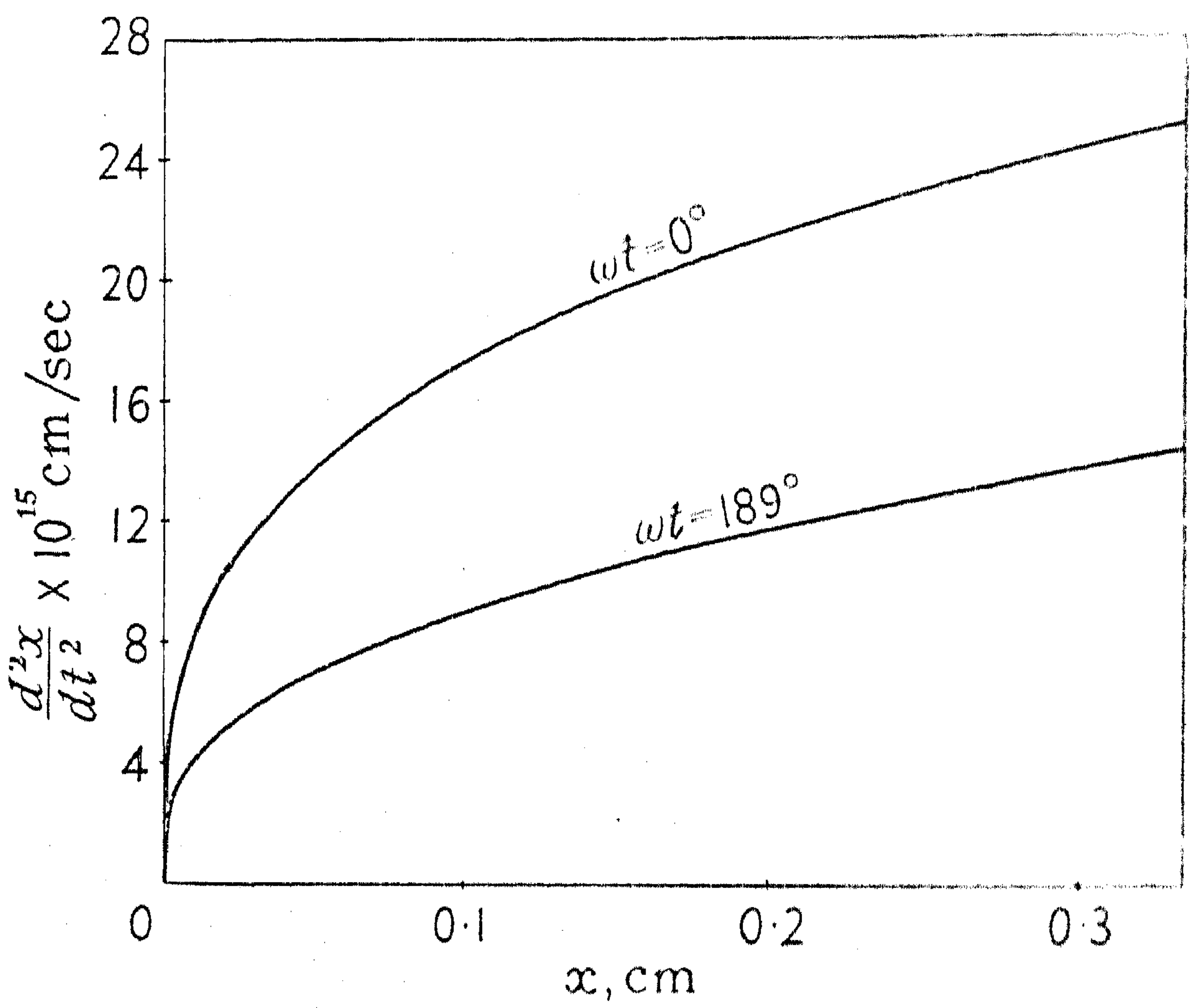

Fig. 4.—Specimen curves of acceleration $\frac{d^2x}{dt^2}$ plotted against x.

Using these values, the expression obtained by harmonic analysis of the voltage curve is

$$V = 2\cdot 78 + 0\cdot 89 \cos(\omega t - 25° \, 47') + 0\cdot 03 \cos(2\omega t + 60° \, 5')$$

The fundamental component is shown by the full line in Fig. 5; the voltage contains less than 4 per cent of second harmonic.

The currents one might expect to be produced by this voltage are compared with the assumed current in Fig. 6. Curve I is the assumed current. I_0 is the conduction current calculated from the 3/2 power law and the voltage derived from the assumed current. I_C is the geometrical capacitance current for the same voltage. The difference between I and the sum $(I_0 + I_C)$ is shown by curve I_D. This curve summarizes the result of the calculation. It can be seen from Fig. 6 that the largest component of I_D is in antiphase with I_C, showing that the total current I appears to flow through a capacitance smaller than the geometrical capacitance. The value of this apparent capacitance is 0·6 of the geometrical capacitance, which agrees with the value given by Benham.*

The apparent value of the capacitance obtained in this way is only correct to the first decimal place, because I_D is the difference between I and $(I_0 + I_C)$. I_0 and I_C

* F. B. LLEWELLYN: *Bell System Technical Journal*, 1935, vol. 14, p. 632.

* W. E. BENHAM: *Philosophical Magazine*, 1931, vol. 11, p. 472.

have been calculated from the voltage, which was derived by graphical integrations. For the same reason, it would not be justified to discuss the other components of I_D. They are of the same order of magnitude as the probable error of these calculations.

In order to obtain a picture of the distribution of charge throughout the space, it was assumed that the charge left the cathode in equal discrete quantities, Δq, each to be represented by a dot on the picture. As the current varies throughout a cycle, the times taken by these charges to leave the cathode will vary, and will be given by

$$\Delta t = \frac{\Delta q}{0 \cdot 1(1 + \frac{1}{2}\cos \omega t)}$$

The element of charge, Δq, was taken to be $2 \cdot 5 \times 10^{-14}$ coulomb. The times t_1 bounding these Δt's were used as the t_1's in equation (7) to find the positions of these charges at any subsequent time.

The distribution of space charge throughout a cycle is shown by the dots in Fig. 7, and the velocities by the gradients of the parabolas. The fact that the current leads the voltage can be explained qualitatively from Fig. 7. As the voltage is increasing through its mean value, the current is increasing, but the number of electrons in the space is also increasing and, before the voltage reaches its maximum, the charge in the space is so great that the field near the cathode is reduced. The number of electrons leaving the cathode will therefore

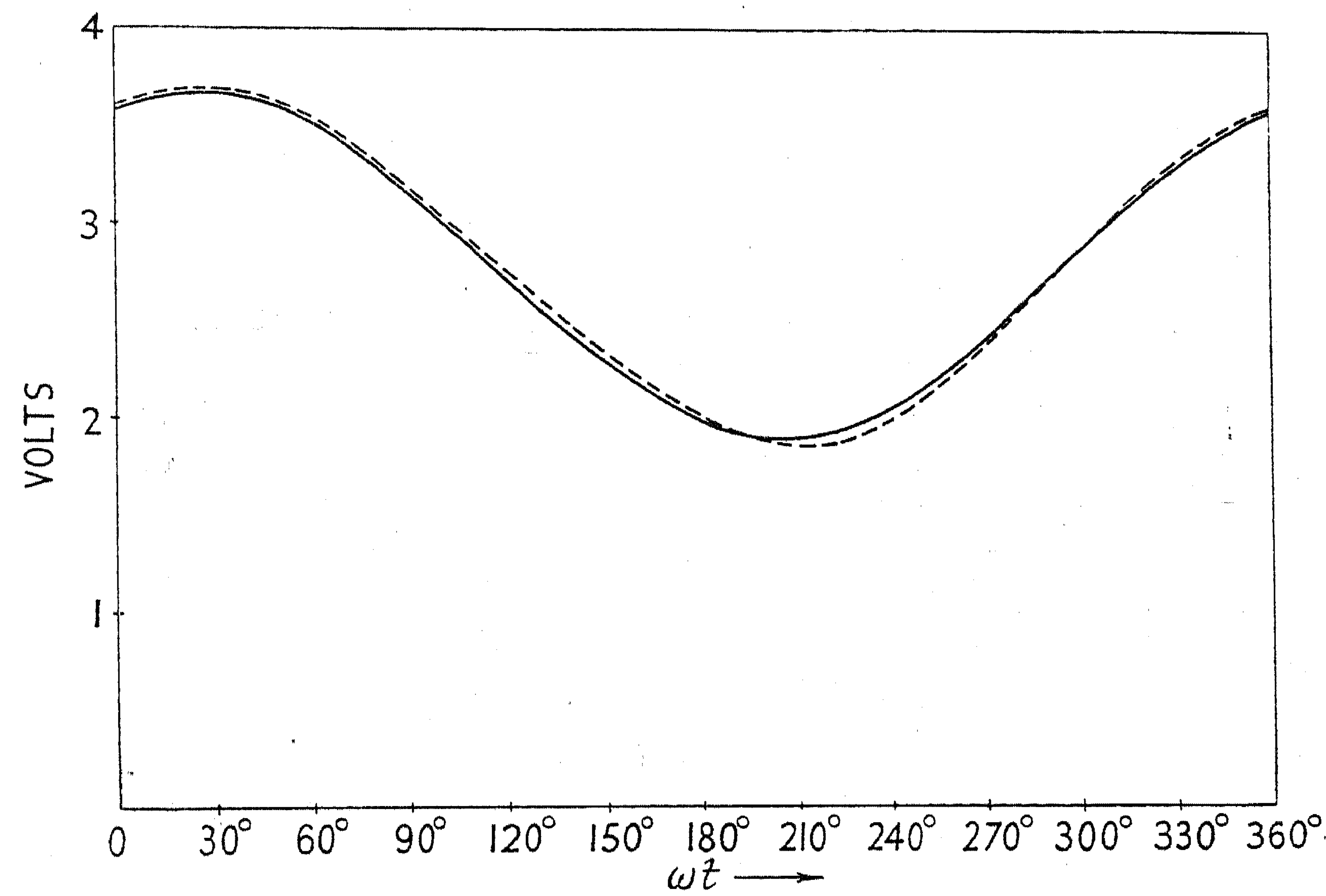

Fig. 5.—Graph of the voltage and its fundamental components.

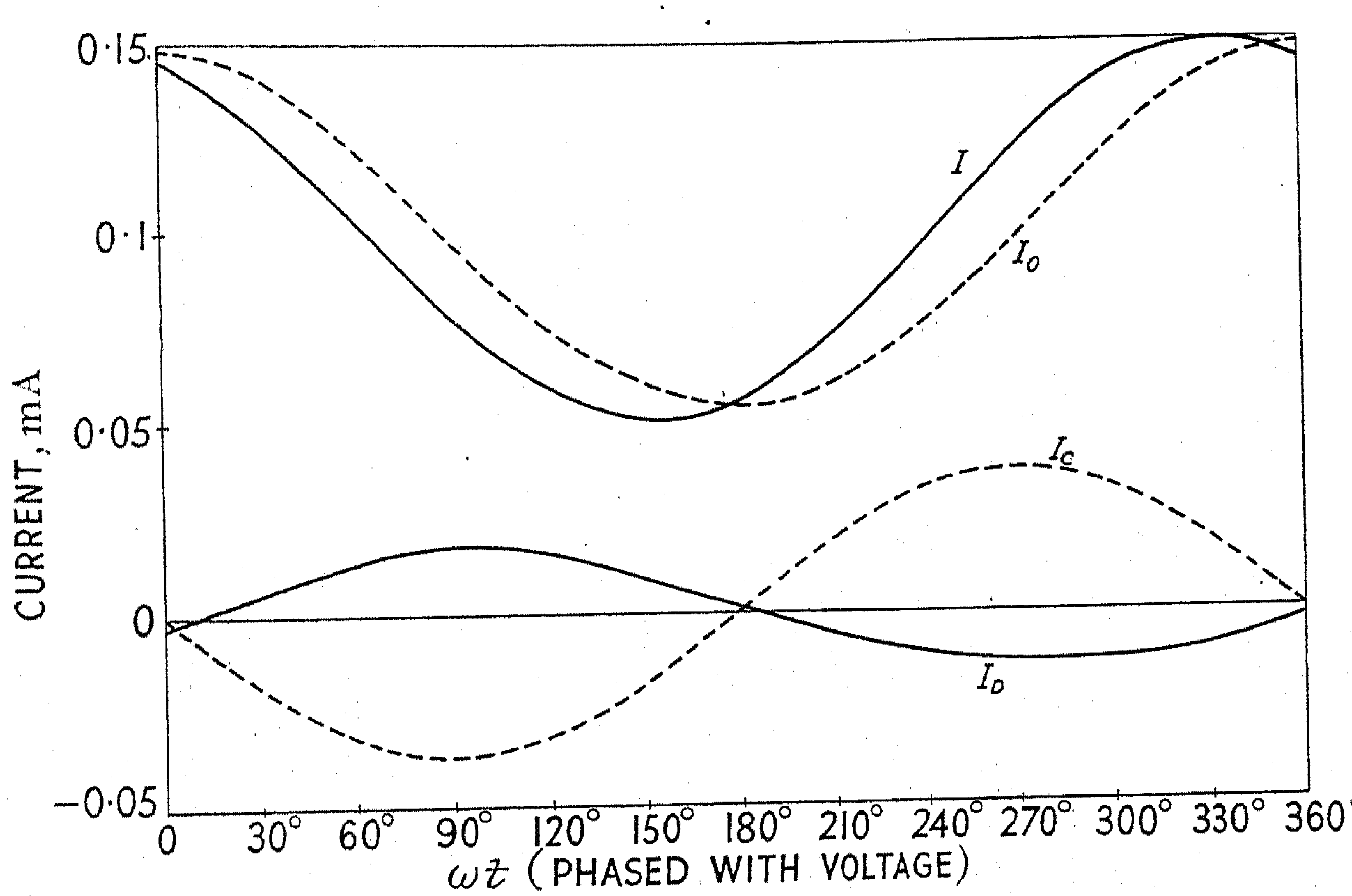

Fig. 6.—Analysis of the assumed current I.

I = assumed current.
I_0 = conduction current, calculated from the 3/2 power law.
I_C = geometrical capacitance current.
$I_D = I - (I_0 + I_C)$.

begin to decrease, and the current passes through its maximum before the voltage. Conversely, it can be seen that the retarding space-charge is so diminished, as the voltage is falling, that the number of electrons leaving the cathode starts to increase again before the voltage reaches its minimum.

As in the case of the saturated diode, the closeness of the parabolas shows the maximum of the current, the closeness depending both on steepness and on the number of electrons. It is seen that there is a much greater variation of closeness across the space from cathode to anode than in the case of the saturated diode. Because of their great number, the electrons near the cathode have a much greater effect on the current in this case.

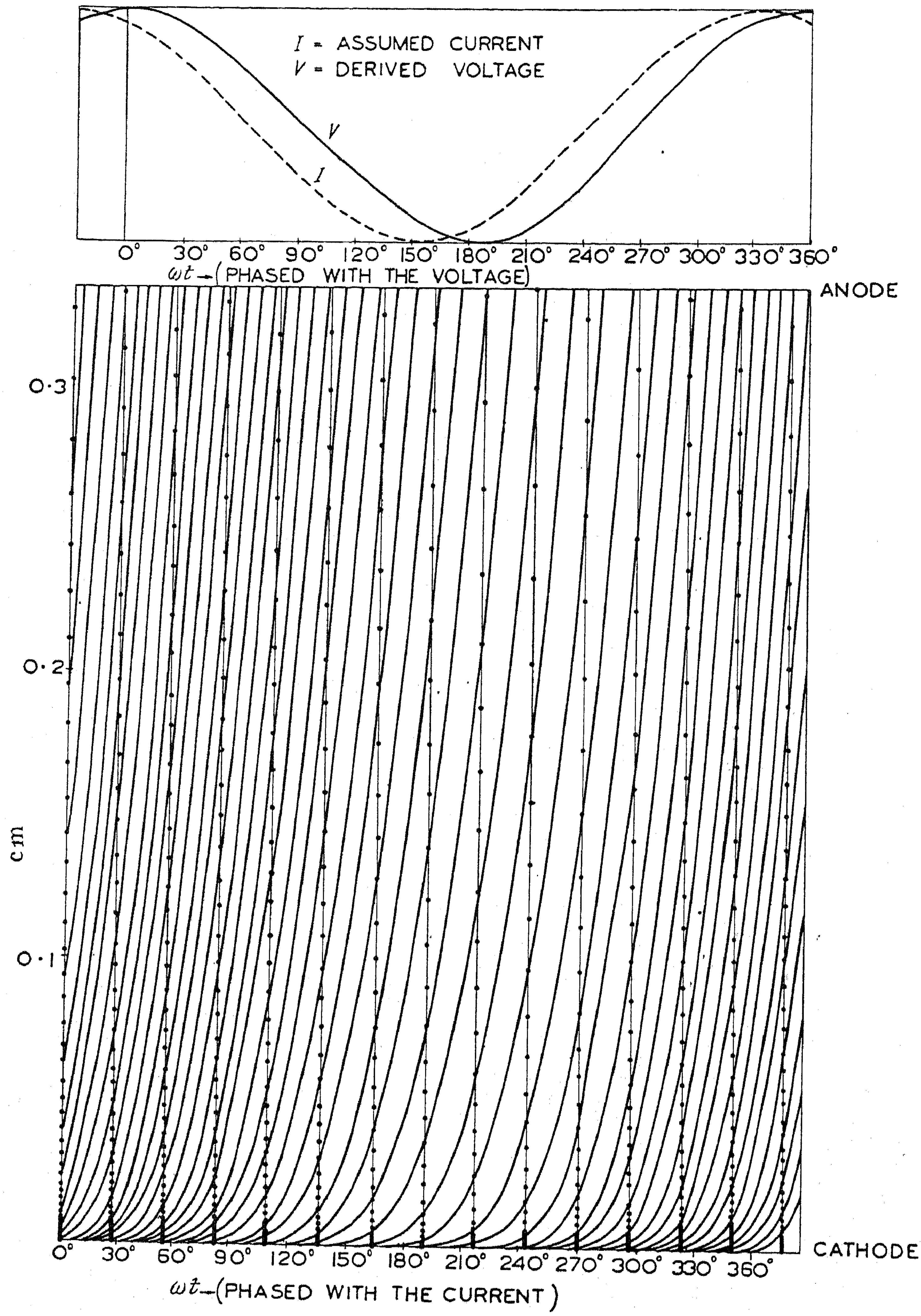

Fig. 7.—Distribution of charge in a space-charge-limited diode.

ACKNOWLEDGMENT

In conclusion, the authors desire to tender their acknowledgments to the General Electric Company, and the Marconiphone Company, on whose behalf the work was done which has resulted in this paper.

THE ELECTRICAL STABILITY OF CONDENSERS

By H. A. THOMAS, M.Sc., Associate Member.

[From the National Physical Laboratory.]

(*Paper first received* 18*th March, in amended form* 4*th April, and in final form* 6*th May*, 1936; *read before the* WIRELESS SECTION 6*th May*, 1936.)

SUMMARY

The paper gives an account of (*a*) an investigation into the nature and causes of undesired changes in the capacitance and power factor of condensers, and (*b*) means of obtaining greater constancy. The principal results and conclusions are as follows:—

The changes are almost entirely due to temperature variation. The thermal behaviour of condensers with solid dielectric is in general non-cyclic, and for a representative selection the temperature coefficients of capacitance ranged from $-1\,800$ to $+\,200$ parts in 10^6 per deg. C. The thermal behaviour of air-dielectric condensers is not cyclic in general, but more nearly cyclic than that of solid-dielectric condensers, and cyclic behaviour was obtained in some cases. The coefficients of a representative selection ranged from $-\,65$ to $+\,150$ parts in 10^6 per deg. C. Where the behaviour was cyclic, the capacitance coefficient was between 2 and 3 times that of the linear expansion of the metals used.

An analysis of the causes of abnormally large coefficients showed that the variation of capacitance with air density is about 2 parts in 10^6 per deg. C. With entirely free expansion of metal plates, the coefficient is approximately that of the linear expansion of the metal, in accordance with simple theory. Variation of "fringe" and "stray" field with temperature has no appreciable effect on temperature coefficient. Change of residual internal stress in the metal plates and members with change of temperature is not in general a very important factor. Distortion due to temperature-gradients arising from inequalities of thermal characteristics of component parts is likely to be a significant factor in many cases. Variation of elasticity and moment of inertia with temperature is not significant. Unequal expansion of different parts may be a very important factor in producing large coefficients, particularly if the one set of electrodes is not quite symmetrical with respect to the other (i.e. unequal air-gaps). Even where constraint is intended to be avoided by the free sliding of plates in grooves, the frictional constraint is likely to be sufficient to cause distortion. In air-dielectric condensers with equal air-gaps, constraints arising from inequalities of expansion of different parts suffice to account for coefficients of 2 or 3 times the theoretical value for free expansion; with slight inequality of air-gaps (a few mils only) the effect of any small distortion in the plates due to this or any other of the causes listed is very greatly enhanced.

An examination of the properties of insulating materials showed that the temperature coefficient of permittivity of solid dielectrics ranges from about $-\,700$ in 10^6 per deg. C. (high-permittivity ceramics) to about $+\,2\,000$ in 10^6 per deg. C. This temperature coefficient may be significant in variable air-condensers. Ebonites, synthetic plastics, and a specimen of fused silica, showed non-cyclic thermal behaviour. Certain ceramics were very satisfactory in respect of deformation and cyclic behaviour.

By comparing a complete and detailed analysis of the thermal characteristics (conduction, radiation, etc.) of the component parts of a condenser with the thermal behaviour of the complete condenser some insight into the reasons for abnormal temperature coefficients can be obtained, and the knowledge thus gained can be applied to design.

Minimization of temperature coefficients of capacitance requires elimination of residual stresses and of variation of mechanical constraint, uniformity of effective thermal mass in the various elements, and accurate location of a mechanically suitable insulator with a low temperature-coefficient of permittivity.

Among the methods of compensation for temperature-change, the use of a solid dielectric with negative temperature-coefficient of permittivity has possibilities, but some serious practical limitations. There are practical objections to the use of "invar" or copper-plated invar. Bimetallic systems are practicable and effective. A first trial model with bimetallic compensation, constructed without regard to cost and bulk, had a negligible temperature-coefficient of capacitance for slow changes of temperature. A second model was constructed on less expensive and more practicable lines. The temperature coefficient was adjustable to any value between about $-\,80$ and $-\,7$ parts in 10^6 per deg. C., and could be reduced to zero by a small modification of the design. The performance was very satisfactorily cyclic.

The results of the investigation lead to the general conclusion that, by the methods described, it is possible to construct comparatively inexpensive air-dielectric condensers with temperature coefficients adjustable to values in the neighbourhood of zero for slow changes of temperature. Further work is necessary before a satisfactory solution can be given of the problem of obtaining similar results for rapid changes of temperature.

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(1) INTRODUCTION

The electrical stability of a condenser may be defined as the degree of constancy of capacitance and power loss obtainable under any specified conditions of operation. When condensers are used in uncontrolled valve oscillators, the frequency stability is primarily dependent upon the electrical stability of the inductance and capacitance of the oscillation circuit. The stability of inductance coils has already been investigated,* and the present paper deals with the complementary problem of stabilizing the capacitance of the oscillation circuit.

It was found by experiment that temperature variation is responsible for the bulk of the observed variations of capacitance, and consequently the paper is concerned mainly with the effect of this variable not only upon the actual change of capacitance but also upon the rate of the capacitance change. The paper deals primarily with the stability of variable air-condensers since continuous variability of frequency requires the use of such components, but attention is given also to fixed condensers with an air or solid dielectric.

Measurements of the frequency-drift associated with the initial temperature-rise of various oscillators show that air-dielectric condensers are responsible for an appreciable portion of the observed frequency-changes. Furthermore, these frequency-changes suggest that the temperature coefficient of capacitance is often much larger than the linear expansion-coefficient of the metal elements of the condenser, to which it should theoretically be equal if all dimensions increase in accordance with this quantity.

* See Reference (93).

Published values of the temperature coefficient of capacitance of precision air-condensers* together with measurements on various types of condensers† confirm that the temperature coefficient is often much in excess of the expansion coefficient of the metal vanes. Now the construction of a typical air-condenser is such as to make it a complex mechanical and thermal system, and the determination of its properties when subjected to temperature variation involves the solution of the following four separate problems: (*a*) The effects of temperature-change on a perfect or "ideal" condenser. (*b*) The effects of temperature-change on the physical properties of the metal portions. (*c*) The effect of the variations of the physical properties of the insulating material with temperature-change. (*d*) The effect of the thermal properties (e.g. conductivity, emissivity, etc.) of each portion of the condenser.

In any actual condenser, changes in capacitance due to all these separate factors take place simultaneously. It was therefore necessary to conduct a series of experimental and analytical investigations to determine the probable magnitude of each effect and then to combine the knowledge so obtained to explain the observed behaviour in particular cases.

(2) MEASUREMENT OF TEMPERATURE COEFFICIENT OF CAPACITANCE

The first part of the investigation was the collection of data on the performance of various condensers representative of present practice in commercial and service transmitters and receivers. As in the case of the measurements on coils,‡ it was found convenient to use a self-oscillatory system incorporating the condenser under test and to observe the changes in capacitance in terms of the frequency variations of this oscillation system.

The general arrangement of the testing apparatus is shown schematically in Fig. 1. The oscillation circuit consists of a coil L and a condenser network; the coil is specially wound on a glass "former" to give good stability of inductance and is enclosed in a temperature-controlled metal box; the condenser network consists of the precision condensers C_1, C_2, C_3, and the condenser under test C_T. Oscillation can be maintained only when the capacitance of the network lies between two values, and consequently when the capacitance of the condenser under investigation lies outside these limits it is necessary to increase or reduce artificially the total capacitance of the system. This is done by means of the condensers C_2 and C_3; when the value of C_T is very small, the switch S is closed and C_2 is put in parallel to bring the total capacitance up to a value sufficient to maintain oscillation; when C_T is very large, the switch S is opened

* See References (22), (56).
† See Section 3(*b*).
‡ See Reference (93).

and C_3 is put in series to reduce the total capacitance to a suitable value.

In the examination of condensers of small capacitance it is important to ensure that one electrode system is at earth potential, otherwise spurious frequency-changes due to the change in capacitance to earth of this electrode system may occur. This condition is established by adjusting the condenser C_1 until the points P and E are at the same potential, the latter point being connected to earth; a valve voltmeter is used to observe the potential difference between the points P and E. When large condensers are being tested, this requirement is of little consequence, since the earth capacitance is negligible with respect to the mutual capacitance between the electrodes. The capacitance of the leads to the oven is represented by the dotted-line condenser C_L. When the switch S is closed, the total capacitance C is given by the expression

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2 + C_L + C_T} \quad . \quad . \quad . \quad (1)$$

and the frequency of oscillation by

$$1/C = \omega_1^2 L \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The effect of a rise of temperature is to change the

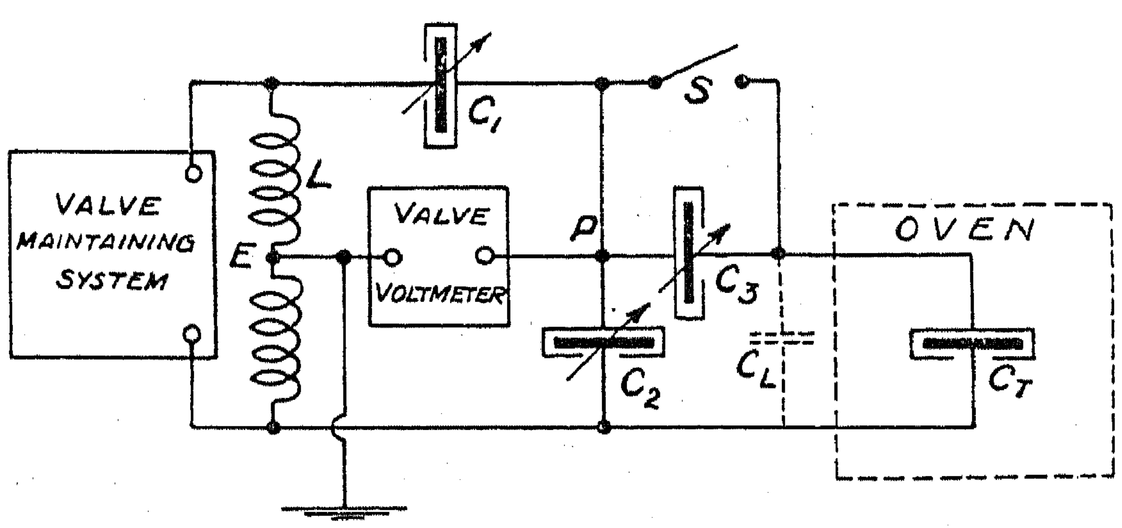

Fig. 1.—Schematic arrangement of testing apparatus.

capacitance of the test condenser by an amount ΔC_T and that of the leads by ΔC_L. Consequently, at the higher temperature, the new value of the capacitance C' is given by the expression

$$1/C' = 1/C_1 + 1/(C_2 + C_L + C_T + \Delta C_L + \Delta C_T) \quad . \quad (3)$$

and the new frequency value by

$$1/C' = \omega_2^2 L \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of the capacitance-change ΔC_T can be expressed by the equation

$$\Delta C_T = \left[1 - (\omega_2/\omega_1)^2\right]\left[1/C_1 + 1/(C_2 + C_L + C_T)\right] \left[C_2 + C_L + C_T\right]^2 - \Delta C_L \quad . \quad . \quad (5)$$

$$= U\left[1 - (\omega_2/\omega_1)^2\right] - \Delta C_L \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where U is a constant involving the known capacitance of the network.

It is seen from equation (6) that a knowledge of the change in the capacitance of the leads is necessary. Although various efforts were made to eliminate this lead effect by the use of mercury conductors in quartz tubes and special water-jacketed leads, it was found that such methods were unsatisfactory and consequently it was decided to reduce the lead effect but not to attempt to eliminate it. For this purpose, a minimum length of small-diameter copper wire was employed, ensuring that the heated portion of the leads within the oven was as short as possible. The value of ΔC_L was obtained by a separate test in which the condenser under examination was removed and the leads only subjected to a rise of temperature. In this case

$$\Delta C_L = \left[1-(\omega_2'/\omega_1')^2\right]\left[(1/C)+1/(C_2'+C_L)\right]\left[C_2'+C_L\right]^2 \quad . \quad (7)$$

where $\omega_1' = 2\pi \times$ frequency with oven cold, $\omega_2' = 2\pi \times$ frequency with oven hot, and $C_2' =$ new value of C_2 required to obtain the correct earthing condition.

Temperature-changes were produced artificially by means of an external hot-air blower which circulated warm air within an oven containing the condenser. It is necessary to adopt this method of heating since alternative means using heating elements enclosed in the walls of the oven give rise to movements of these elements with respect to the condenser under test, and such movements produce appreciable changes of earth capacitance. In the actual arrangement, no conducting material was situated within 2 ft. of the condenser under examination. The rate of temperature-rise was made as great as possible until the final desired temperature was reached; this value was then maintained constant by manual regulation of the heater current in the case of short-period tests or by thermostatic control for long-period tests; the usual rate of temperature-change was 2 deg. C. per minute and the temperature-rise was 30 deg. C.

The test procedure was as follows: each condenser was set up inside the oven and connected to the oscillatory circuit by the shortest possible length of No. 47 S.W.G. copper wire. The various network condensers were adjusted to establish the correct earthing conditions, and the frequency was measured at regular time-intervals until constancy was obtained. The temperature was now raised and maintained at a new definite value, while the frequency-drift of the oscillator was observed continuously until a constant value was again obtained. The oven was then allowed to cool and in many cases continuous observation of the cooling curve of frequency was taken, but in some cases the final value only was observed since the frequency-drifts often continued for many hours. Three tests were made on each condenser, except in those cases where thermal ageing took place, when as many as seven tests were made to ascertain whether a cyclic condition was ultimately reached. In most cases the first test gave abnormal results owing to the drying-out of the insulation, and the results obtained on this test have usually been discarded.

(3) RESULTS OF TESTS ON CONDENSERS

In presenting the results of the tests upon some selected condensers it is considered sufficient to illustrate the type of behaviour by giving two actual characteristics, one for a good cyclic case and another for a case where the performance was non-cyclic. Reference to the behaviour in individual cases is given in Tables 1 and 2.

An example of cyclic behaviour is shown in Fig. 2,

in which the temperature-variations refer to the air surrounding the condenser; this curve was obtained for Condenser T (see Table 2) and can be taken as representative of a good cyclic performance. For other cases, the time taken to reach the final value may differ appreciably from the value shown in this case, but the form of the curve is sensibly similar.

An example of non-cyclic behaviour is shown by the curve in Fig. 3, which was obtained for Condenser A (see Table 1). The behaviour of the other condensers can be expressed in a similar manner, and the curves lie intermediately between these two extremes. In some cases, the temperature coefficient of capacitance can be obtained with good accuracy. This occurs when the behaviour is cyclic and several successive tests give agreement. In other cases, however, the temperature coefficient is only a mean of a number of readings and has little practical significance. Such cases are indicated in Tables 1 and 2 by a note as to the nature of the observed performance.

Both solid-dielectric and air-dielectric condensers were tested, and, in view of the differences between these types, the results are divided into two groups.

Table 1

RESULTS OF TESTS ON SOLID-DIELECTRIC CONDENSERS

Condenser	Type	Measured capacitance	Temperature coefficient	Thermal performance	Notes on capacitance-change	Freq. of measurement
		μμF	parts in 10^6 per deg. C.			kc
A	Tubular, high voltage, dielectric unknown	300	+ 23	Non-cyclic	Capacitance value fell after each thermal cycle. Long - period changes occurred	1 000
B	High voltage, dielectric unknown	542	+ 65	Non-cyclic	Capacitance value fell after each thermal cycle, but ultimately a nearly cyclic performance was obtained	1 000
C	High voltage, dielectric unknown	557	− 110	Nearly cyclic	First heating gave low coefficient, after which nearly cyclic behaviour existed	1 000
D	" Compensated." Coefficient claimed by manufacturers to be 10 parts in 10^6 per deg. C.	1 000	+ 160	Nearly cyclic	Considerable ageing during first heating. Large time-lag between capacitance and temperature-change	745
E	High voltage, dielectric unknown	217	− 1 820	Nearly cyclic	—	1 000
F	Experimental silica tube with 2-mil copper coating deposited by metal spray	127	+ 45	Nearly cyclic	Very little ageing or time-lag between capacitance- and temperature-change	3 500
G	Tubular, " Calit " dielectric	257	+ 175	Nearly cyclic	Slight ageing at 50° C. Small lag between capacitance- and temperature-change	3 500
H	Tubular, " Calit " dielectric	238	+ 205	Nearly cyclic	Identical with G	3 500
J	Tubular, " Condensa " dielectric	219	− 200	Non-cyclic	Ageing effects considerable. Long-period time-lag between capacitance- and temperature-change	3 500
K	Tubular, " Condensa " dielectric	233	− 190	Non-cyclic	Identical with J	3 500

(a) Solid-Dielectric Condensers

The results of the tests on solid-dielectric condensers are tabulated in Table 1; all these condensers were standard commercial products with the exception of

Table 2

RESULTS OF TESTS ON AIR-DIELECTRIC CONDENSERS

Condenser	Type	Measured capacitance	Temperature coefficient	Thermal performance	Notes on capacitance-change	Freq. of measurement
		$\mu\mu$F	parts in 10^6 per deg. C.			kc
L	Fixed, monel-metal plates, brass spacing collars, paxolin base	263	+ 35	Nearly cyclic	—	3 500
M	Fixed, duralumin plates, brass spacing collars, paxolin base	257	+ 45	Nearly cyclic	—	3 500
N	Fixed, monel-metal plates, aluminium spacing collars, paxolin base	262	+ 13	Non-cyclic	Several thermal cycles did not improve non-cyclic behaviour	3 500
O	Fixed, brass plates, keramot spacing collars, mycalex end-tubes, brass clamping rods	600	− 43	Nearly cyclic	—	1 000
P	Fixed, compensated, brass plates sliding in grooves in opposite brass plate, two brass end-plates spaced by invar rods in mycalex blocks, keramot insulation	1 180	+ 20	Nearly cyclic	—	1 000
Q	Variable, aluminium plates, double paxolin stator supports	514	+ 42	Cyclic	Slight ageing during first thermal cycle	1 000
R	Variable, aluminium plates, single " Radion " stator support	510	− 65	Nearly cyclic	Considerable ageing during first thermal cycle	1 000
S	Variable, aluminium plates, single paxolin stator support	530	Negative	Severe ageing	No accurate determination possible	1 000
T	Variable, brass plates, mycalex plate insulation	640	+ 52	Cyclic	Two heat treatments required to produce cyclic state	1 000
U	Variable, series gap, brass plates, mycalex plate insulation	185	+ 60	Nearly cyclic	—	1 000
V	Variable, series gap, brass plates, mycalex plate insulation	62	+ 150 (approx.)	Non-cyclic	Coefficient increased for each successive heat treatment. Values were + 108, + 145, + 185. Heat treatment did not produce ultimate cyclic condition	1 000
W	Variable, series gap, aluminium plates, mycalex strip insulation	100	+ 100 (approx.)	Non-cyclic	Capacitance increased after each heat treatment. On cooling, capacitance-change was about half that obtained on heating	1 000
X	Variable, series gap, aluminium plates, mycalex strip insulation	40	+ 150 (approx.)	Non-cyclic	Similar to W, but capacitance was reduced after each heat treatment	1 000

Condenser F. Reviewing the results of these tests, it is seen that the capacitance coefficient may be positive or negative and its value may be very large, varying from − 1 800 to + 200 parts in 1 million per deg. C. for the condensers selected. The first five condensers (A–E) have a paper or mica dielectric and are representative of standard commercial products for radio-frequency purposes. The large differences between the

thermal performance and the temperature coefficient of capacitance of individual condensers are probably due to the complexity of the expansion coefficients of the various parts, the changes of permittivity of the dielectric with temperature, and the residual-stress redistributions which take place due to differences in the thermal capacity of the various portions. Such condensers are obviously unsuitable for use in circuits where a high degree of stability is required.

Condensers F–K have a solid dielectric with deposited electrodes in intimate contact with the insulator. The measurements made on the silica-dielectric condenser with sprayed-on electrodes (F) show that the performance agrees very closely with the temperature coefficient of permittivity of quartz (40 parts in 1 million for condensers in which deposited electrodes are used, and hence it seems clear from these results that the temperature coefficient of permittivity of Calit is not reproducible to an accuracy much better than 30 per cent.

The measurements on the Condensa condensers J and K give values of − 200 and − 190 parts in 1 million per deg. C. for the temperature coefficient of capacitance, whereas the manufacturers state that the temperature coefficient of permittivity of Condensa is − 380 parts in 1 million,* and Rohde† gives a value of − 370. The discrepancy is probably explained by the fact that the change in capacitance takes place very slowly when a temperature-change is suddenly applied, and the values given in Table 1 are derived from the observed capaci-

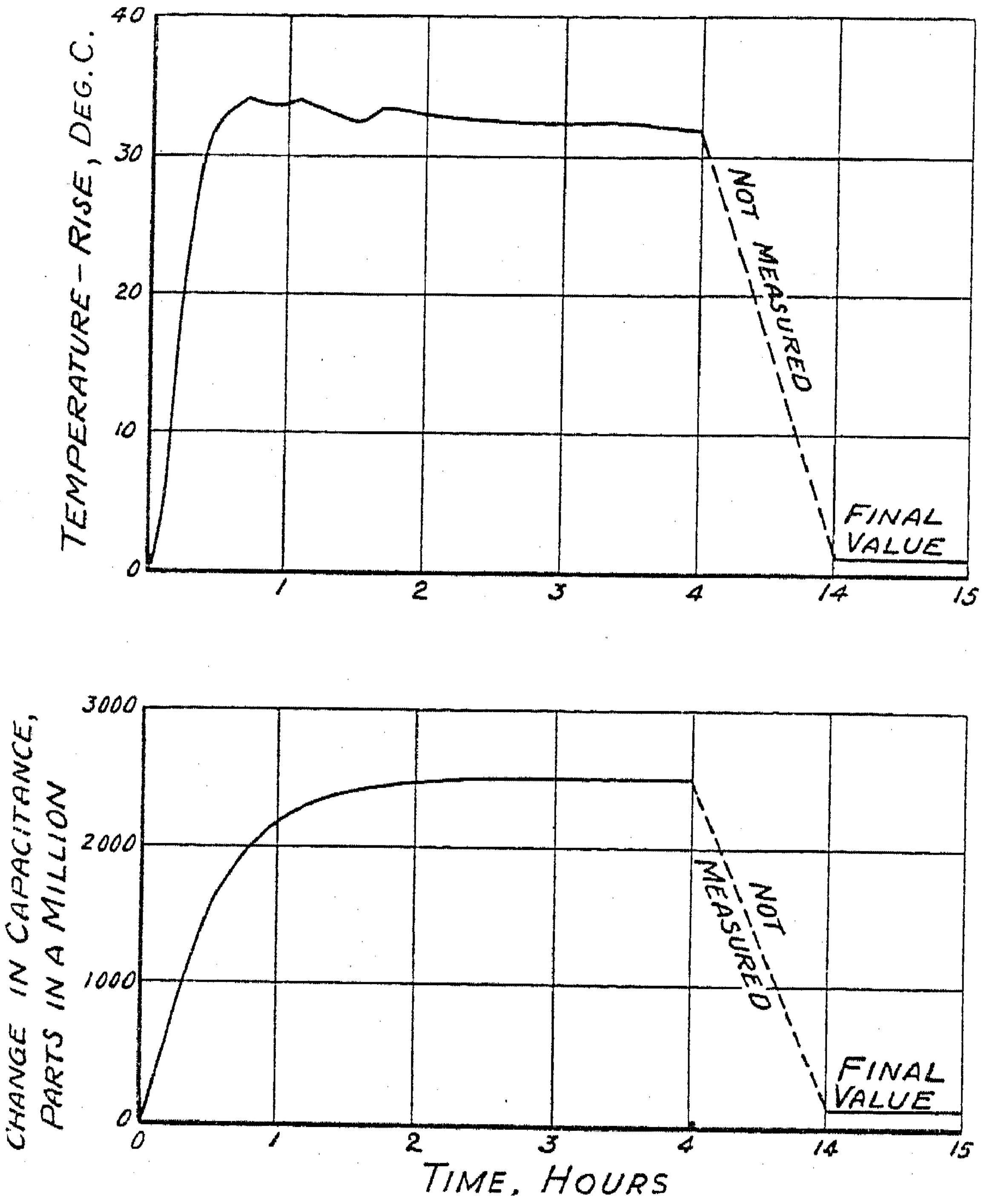

Fig. 2.—Example of cyclic behaviour, condenser T.
Atmospheric temperature = 18° C.

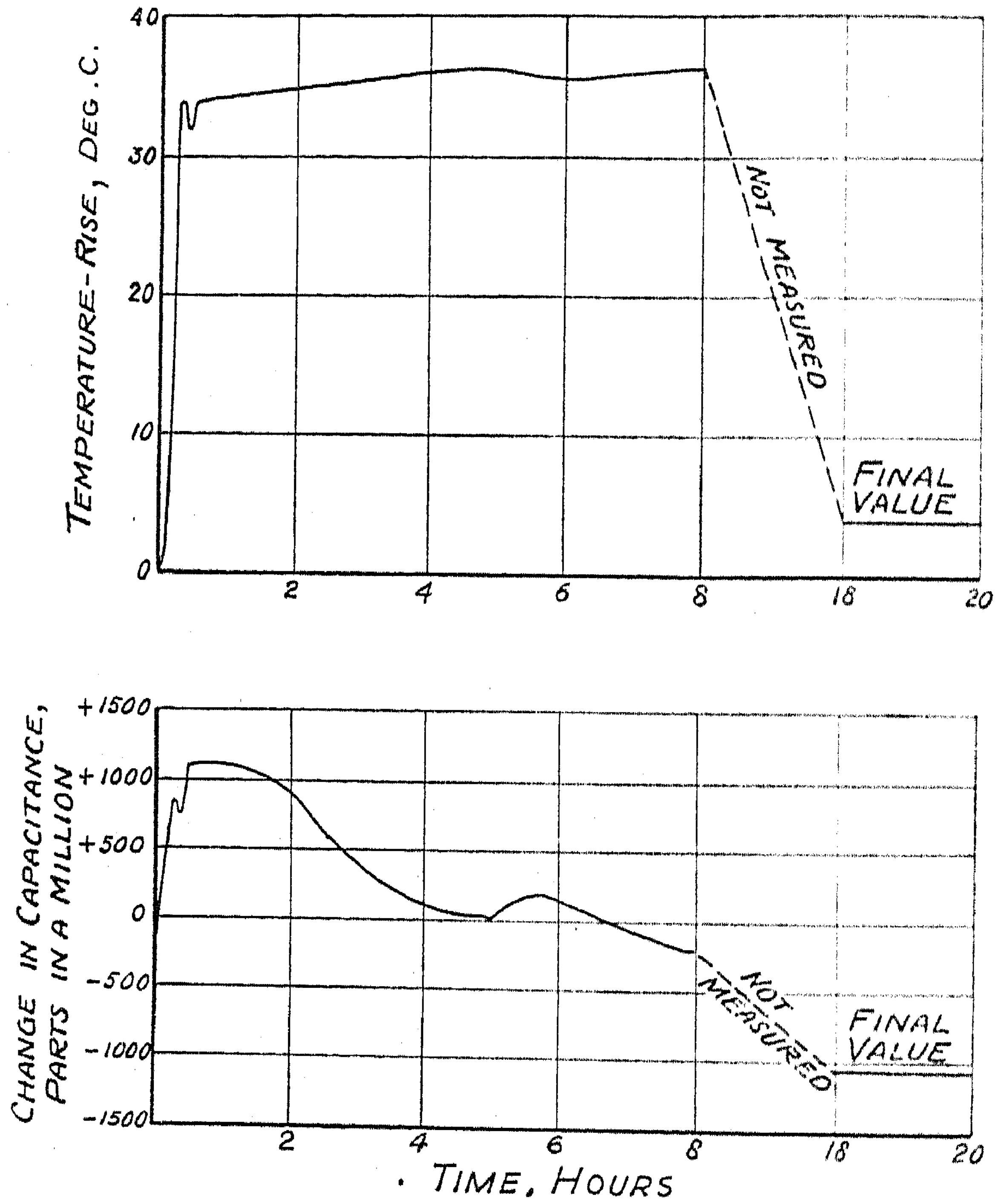

Fig. 3.—Example of non-cyclic behaviour, condenser A.
Atmospheric temperature = 18·8° C.

per deg. C.)* and that no appreciable time-lag exists between the capacitance and temperature changes. All the condensers G–K are made of ceramic materials, G and H consisting of electrodes deposited on a "Calit" dielectric and J and K of similar electrodes deposited on a "Condensa" dielectric. The Calit condensers behave in a nearly cyclic manner and have positive temperature-coefficients of capacitance of + 175 and + 205 parts in 1 million per deg. C. respectively. The temperature coefficient of permittivity of Calit is given by Handrek† as + 160 parts in 1 million per deg. C., and Rohde‡ gives values of + 220, + 180, and + 160 parts in 1 million for the temperature coefficient of capacitance of Calit condensers. It is improbable that the temperature coefficient of permittivity differs appreciably from the temperature coefficient of capacitance tance-change 100 minutes after the temperature-change occurred. If the test had been continued for a period of at least 400 minutes, a higher value of the coefficient would have been obtained, but it is doubtful whether it would have exceeded − 250 parts in 1 million. This very appreciable lag between the temperature-change and the resultant change in capacitance may be due to the poor thermal-conductivity of the material or it may be produced by some molecular effect interlinked with the mechanism producing the negative temperature coefficient of permittivity. In either case, it appears to be a serious limitation to the application of Condensa as a dielectric having a negative temperature coefficient of capacitance, for if such a condenser were used in conjunction with an inductance coil having a positive coefficient of inductance, the compensating action would

* See Reference (32). † *Ibid.*, (32). ‡ *Ibid.*, (56).

* See Reference (88). † *Ibid.*, (56).

not be effective unless the atmospheric temperature-changes were extremely gradual.

The properties of these ceramic dielectrics are discussed fully in Section (6). At the present stage in the paper it is sufficient to note that the performance of such condensers is superior to that of the more conventional types but is not nearly good enough for use in circuits where a high degree of stability is required.

(b) Air-Dielectric Condensers

The results of the tests on air-dielectric condensers are tabulated in Table 2; all these condensers were representative of good commercial or experimental practice, i.e. the results are not applicable to mass-produced products. The first group comprised a number of fixed air-condensers (L, M, N), all constructed in an identical manner but with different metals used for the circular plates and cylindrical spacing-collars. In each case, the dimensions of the corresponding parts were the same, the collars and plates were bolted together with brass rods, and the same type of paxolin base was used.

Table 3

TEMPERATURE COEFFICIENT OF CAPACITANCE, AND LINEAR EXPANSION COEFFICIENT, OF METALS USED IN CONDENSERS L, M, AND N

Condenser	Expansion coefficient (parts in 10^6 per deg. C.)		Measured temperature coefficient of capacitance (parts in 10^6 per deg. C.)
	Plates	Spacing collars	
L	13·5 (monel metal)	18·9 (brass)	+ 35
M	23·4 (duralumin)	18·9 (brass)	+ 45
N	13·5 (monel metal)	23·5 (aluminium)	+ 15

It is convenient to tabulate the expansion coefficients of the various metals used in the construction of each condenser in order to see what effect on the temperature coefficient of capacitance was produced by such variations; this tabulation is shown in Table 3.

It is observed that the temperature coefficient of capacitance increases from + 35 to + 45 parts in a million if the plates are made of duralumin instead of monel metal. The increase in temperature coefficient is, however, not as great as the increase in the expansion coefficient—from 13·5 to 23·4 parts in 1 million per deg. C. Again, the temperature coefficient is reduced from + 35 to + 15 parts in 1 million if the metal plates remain the same and the spacing collars are changed from brass to aluminium, since the greater expansion coefficient of aluminium (23·5 parts in 1 million) leads to an increase in spacing between the plates when the temperature is raised. It would not be expected that the observed temperature coefficient of capacitance would change in the precise ratio of the change in the linear expansion coefficient of the spacing collars, since stress considerations due to the brass clamping-rods would modify the expansion of these collars, but it is somewhat significant that when the differential expansion between the collars and the clamping rods was large, as in condenser N, non-cyclic behaviour occurred. This is probably due to the non-elastic changes of stress set up in the clamping rods with temperature-change.

Condenser O showed a negative temperature-coefficient, probably due to changes in the dimensions of the air-gap of sufficient magnitude to over-compensate the effect of plate distortion. Condenser P was a special experimental design in which the brass plates were secured at the ends of one side and were free to slide in grooves cut in a thick brass plate at the opposite side. These end plates were maintained a constant distance apart by means of invar spacing rods secured in mycalex insulating blocks fixed to the plate system, the design being such that the expansion of the mycalex insulator introduced no mechanical stresses and had no effect on the capacitance value. The brass condenser plates were free to expand laterally, but the effective longitudinal dimensions of the plates were maintained constant by the invar spacing rods. The capacitance should therefore have remained constant if the plates had remained flat, since the increased area, due to lateral expansion only, was counterbalanced by the increased spacing between the plates. This condenser was heat-treated at a temperature of 200° C. and the coefficient of capacitance was found to be about equal to the linear expansion coefficient of the metal plates, suggesting that plate distortion was taking place with temperature variation.

In the next group (Q, R, and S) all the condensers were mechanically similar, each having an appropriate maximum capacitance of 500 $\mu\mu$F. The differences between them consisted only in the method and material used to support the stator assembly. It is seen from Table 2 that when the stator assembly was held at both ends by insulating supports the performance was perfectly cyclic, whereas when one side of the stator only was supported this was not the case. The temperature coefficient of condenser R was negative, and in the case of condenser S the ageing effects were excessive and did not appear to diminish with the application of successive thermal cycles. This suggests that, since the weight of the stator acts as a bending moment on the beam-like insulating supports, these supports are distorted by temperature-change, and in the case of paxolin this distortion appears to be plastic, whereas for Radion it appears to be sensibly elastic. In either case, such distortion produces a reduction of the effective area of the field between the vanes and this form of distortion may produce abnormal values of the temperature coefficient. It is noteworthy that the anticipated distortion produces a reduction in area, and in both cases where single stator supports were used it is seen that a negative coefficient was actually obtained. This result suggests that if single stator supports are adopted their rigidity is of vital importance and that the properties of paxolin and Radion are unsuitable for this purpose.

The remaining condensers (T, U, V, W, and X) are representative of good modern construction, condensers T, U, and V being suitable for receivers and W and X for short-wave transmitters. An inspection of the results of the tests shows that the value of the temperature coefficient of capacitance lay between + 50 and + 150 parts in 1 million per deg. C., whereas the anticipated

coefficients were + 19 and + 23 parts in 1 million for the brass-plate and the aluminium-plate condensers respectively.

The results of the tests on air condensers reveal at once one outstanding fact of major importance. In those cases where reasonably cyclic behaviour occurred, the value of the temperature coefficient of capacitance lay between 2 and 3 times that of the linear expansion coefficient of the metal used for the plates, the values for the capacitance coefficient being + 35, + 42, + 45, + 52, and + 60 parts in 1 million per deg. C. (see Table 2); it is interesting to compare these results with published data on the stability of air condensers. Rohde* points out that the temperature coefficient of small air condensers is usually much in excess of the expansion coefficient of the metal plates. Values for this coefficient are given as + 36 and + 40 parts in 1 million per deg. C., and the thermal ageing or capacitance-change produced by one complete thermal cycle of 40 deg. C. is stated to be as great as 400 parts in 1 million in many cases. Giebe and Zickner† also find that the variation of capacitance of standard air-condensers used at the Physikalisch-Technische-Reichsanstalt is rarely less than 200 parts in 1 million and often exceeds 400 parts in 1 million.

It would be unwise to conclude from these published results that it is impossible to design air condensers of good stability—in fact, it appears that no attempt has been made hitherto to discover the causes of these large unreproducible changes. Since in all these cases the capacitance of the insulating stator supports was only a small fraction of the total capacitance, it is reasonable, in seeking for an explanation of the high values of the temperature coefficient of capacitance of air-dielectric condensers, to attribute such variations with temperature to changes of configuration. It was necessary, however, to be quite certain that no other factor had been overlooked, and, with this object in view, tests were made on a specially-built condenser which may be referred to as an "ideal" condenser, since it was made in such a manner that metal distortion was negligible and the capacitance of the solid portions of the dielectric was as small as possible. The object of this investigation was to ascertain whether the temperature coefficient of capacitance of such a condenser is predictable from the known coefficients of linear expansion of the plates and spacing pillars.

(4) EFFECTS OF TEMPERATURE-CHANGE UPON CAPACITANCE OF "IDEAL" CONDENSER

(a) Evacuation Tests

The electrodes of this specially-constructed condenser consisted of two blocks of well-annealed brass of dimensions 5 in. × 5 in. × ½ in. and 6 in. × 6 in. × ½ in. respectively. The two active surfaces were lapped flat and thoroughly cleaned and the air-gap spacing and insulation were provided by three very small adjustable screws with mica tips, thus giving accurate 3-point location. This condenser was set up as part of an oscillatory system and was enclosed in a large bell-jar equipped with a mercury pump for exhausting the air. The change in capacitance on varying the air density was observed and was found to agree exactly with the theoretical value of permittivity given by the expression

$$\kappa = \frac{1 + 2c_1 d_1}{1 - c_1 d_1} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where d_1 = air density and $c_1 = 195 \times 10^{-6}$. This result is in agreement with a previously published statement* that the permittivity of air is independent of frequency. The permittivity of air varies with temperature in the manner shown in Appendix 1, and this variation is responsible for an increase in the temperature coefficient of capacitance of a condenser of 2·15 parts in 1 million per deg. C.

(b) Heating Tests

The temperature coefficient of this same condenser was now measured, as described in Section (2), for various air-gaps between 0·005 and 0·3 in. In all cases it was found that the capacitance coefficient was about 24 parts in 1 million per deg. C., the expansion coefficient of the brass plates being 18·9 parts in 1 million. The discrepancy of 27 per cent is not large and may be due to the fact that the small air-gap screws do not reach the same temperature as the large plates owing to their very small thermal capacity and appreciable radiating surfaces. The agreement was, however, considered to be sufficiently good to establish the fact that the capacitance coefficient of such a condenser is substantially equal to the thermal-expansion coefficient of the metal plates.

(c) Expansion Tests

To find whether the performance of a condenser can be predicted from a knowledge of its dimensional changes, it was decided to conduct tests at constant temperature with special condensers in which the effects due to expansion could be imitated. In any condenser, the field can be divided into three parts, namely, the main field between the plates, the fringe field at the edges of the plates, and the stray field due to the lines of force from one electrode to earth which do not terminate on the other earthed electrode.

Two special condensers were made for this purpose. Each consisted of a square top and a square bottom plate with three screws for air-gap adjustment, the bottom plate being divided into two parts. The central square portion of this plate was identical with the top plate, but the outer hollow square or guard ring was electrically insulated by a very small air-gap. The arrangement is shown in Fig. 4, which also shows the division of the total electric field into its three components C_1, C_2, and C_3. The dimensions of the two condensers are shown in Fig. 5. It will be noticed that all the dimensions of the large condenser are about 10 per cent greater than those of the small condenser.

The capacitance of each of these condensers was measured for various air-gaps at a frequency of 347 kc with the guard-ring electrode system connected to the central square bottom plate. The guard ring was then disconnected from the main electrode, radio-frequency current at a frequency of 347 kc was supplied from an

* See Reference (56).

† *Ibid.*, (22).

* See Reference (38).

oscillator, and the currents I_1, I_2, and I_3, were measured for different air-gaps. If C = total measured capacitance,

Stray capacitance of top plate

$$= C_1 = \frac{I_1 - (I_2 + I_3)C}{I_1} \quad . \quad . \quad . \quad . \quad (9)$$

$$\text{Main-field capacitance} = C_2 = \frac{I_2}{I_1}C \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\text{Fringe capacitance} = C_3 = \frac{I_3}{I_1}C \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Care must be taken in these measurements that the main and fringe electrodes are both at the same potential; this condition can be realized by the use of low-resistance thermo-junctions for measuring the currents I_2 and I_3.

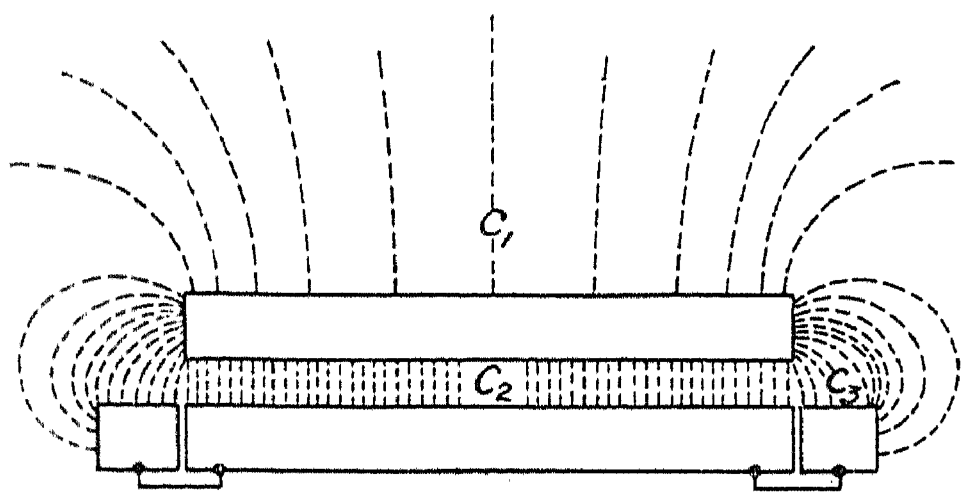

Fig. 4.—Field distribution of experimental condenser.

The values of these three components are shown in Fig. 6, in which C_1, C_2, and C_3 refer to the small condenser and C_1', C_2', and C_3' refer to the large condenser. The calculated values of the main-field capacitances C_2 and C_2' are shown by dotted lines in the curves A and B respectively. The slight differences between the observed and calculated values are probably due to the field-distorting effects of the three screws. Now, since all the dimensions of the large condenser are 10 per cent greater than those of the small condenser, the effect of an expansion of 10 per cent on the magnitude of each capacitance component can be ascertained for various air-gaps by reference to Fig. 6.

The results of this analysis are given in Table 4, in which it will be noted that the air-gap for the large condenser is in all cases 10 per cent greater than that of the small condenser. The values of the stray capacitance of the small and large top plates in the absence of the bottom plates were 10·65 and 11·77 μμF respectively, from which it is seen that a considerable portion of the field was diverted to the fringe and main fields when the bottom electrode was introduced. The stray capacitance and the fringe capacitance are not critically dependent on the air-gap, the stray capacitance increasing and the fringe capacitance decreasing as the gap is increased. The ratios C_1'/C_1, C_2'/C_2, C_3'/C_3, are, however, all sensibly the same as the increase in dimensions, and the total

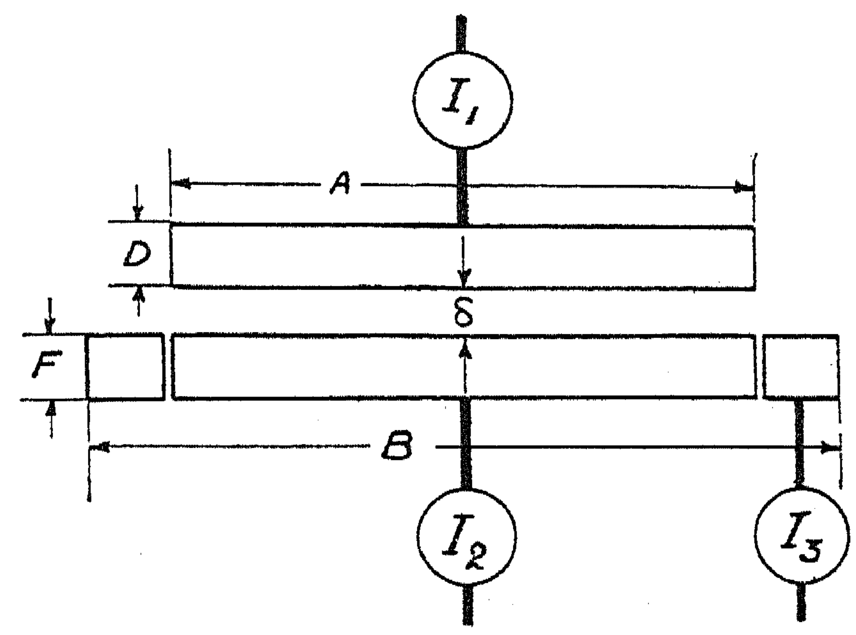

Fig. 5.—Dimensions and testing circuit of experimental condenser.

Large condenser: $A = 5{\cdot}486$ in.; $B = 6{\cdot}692$ in.; $C = 0{\cdot}553$ in.; $D = 0{\cdot}618$ in.
Small condenser: $A = 4{\cdot}994$ in.; $B = 6{\cdot}086$ in.; $C = 0{\cdot}501$ in.; $D = 0{\cdot}565$ in.

Table 4

EFFECT OF EXPANSION OF THE ELECTRODES OF A CONDENSER ON THE CAPACITANCE COMPONENTS

Small condenser				Large condenser				Small condenser	Large condenser	Capacitance ratios			
Air-gap	μμF			Air-gap	μμF			$C_1+C_2+C_3$	$C_1'+C_2'+C_3'$	Self	Main	Fringe	Total
	C_1	C_2	C_3		C_1'	C_2'	C_3'			C_1'/C_1	C_2'/C_2	C_3'/C_3	$\frac{C_1'+C_2'+C_3'}{C_1+C_2+C_3}$
mils 25	3·0	211·5	11·6	mils 27·5	3·3	235·5	12·3	226·1	251·1	1·10	1·11	1·06	1·11
50	3·2	107·0	10·0	55·0	3·5	119·2	10·6	120·2	133·3	1·10	1·11	1·06	1·08
100	3·4	52·5	7·8	110·0	3·7	59·2	8·8	63·7	71·7	1·10	1·12	1·12	1·12
150	3·8	34·0	6·5	165·0	4·2	38·8	7·7	45·3	50·7	1·10	1·14	1·18	1·12
200	4·2	24·5	5·7	220·0	4·6	27·5	7·0	34·4	39·1	1·10	1·12	1·23	1·13
250	5·0	19·5	5·2	275·0	5·5	21·5	5·4	29·7	32·4	1·10	1·10	1·04	1·09
300	5·6	16·5	5·0	330·0	6·2	18·1	5·3	27·1	29·0	1·10	1·10	1·06	1·07

capacitance-change ratio $(C_1' + C_2' + C_3')/(C + C_2 + C_3)$ is approximately equal to this same increase. The slight departures from the ratio $1 \cdot 10$ are largely accounted for by instrumental and observational errors. Previous work on the value of the capacitance of the fringe field* confirms the view that no appreciable increase in the capacitance coefficient can be accounted for by dimensional changes of this field produced by plate distortion.

It seems clear from the heating and expansion tests that the performance of these "ideal" condensers is in substantial agreement with theory, and no evidence has been obtained that any hitherto unsuspected factor exists which could account for the discrepancies observed in so many cases between the measured and calculated values of the coefficient of capacitance. The presence of a small series inductance cannot produce effects of the order required to explain these dis-

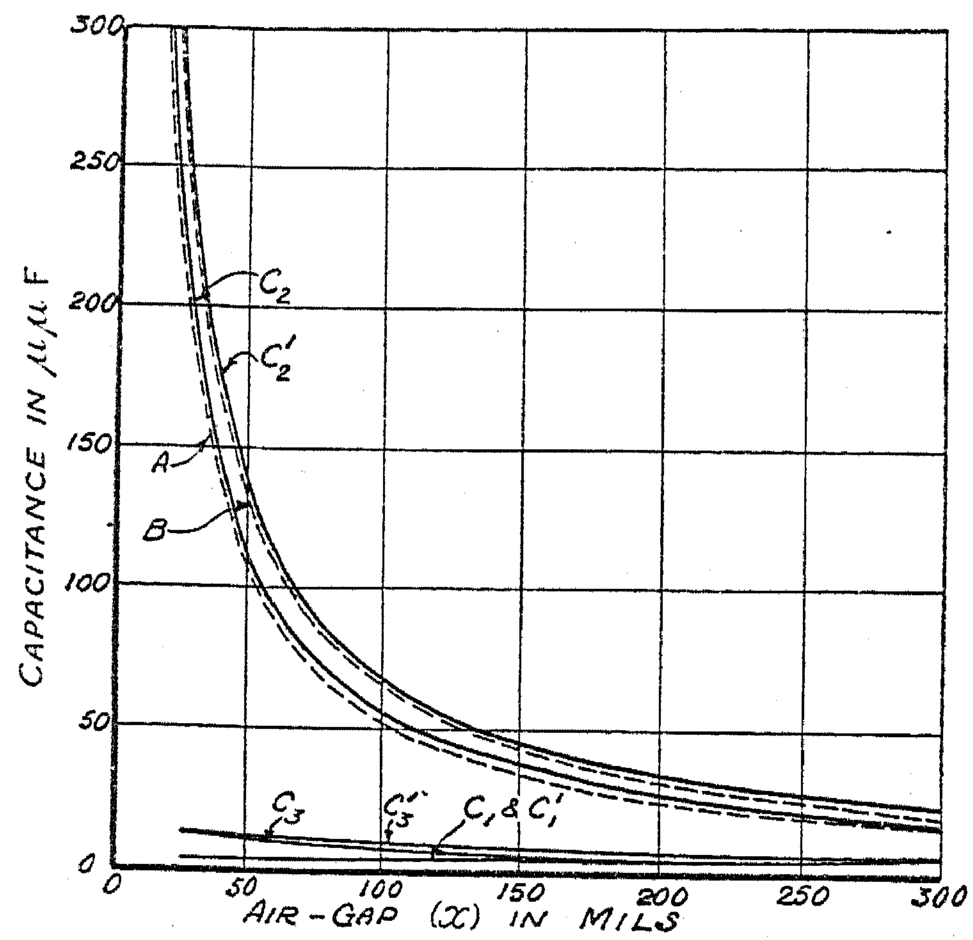

Fig. 6.—Capacitance components of experimental condenser.

crepancies, since it can readily be shown that, if a condenser of capacitance C possesses a series inductance L, then

Apparent temperature-coefficient of capacitance

$$= \Delta C/(1 - \omega^2 CL) \quad . \quad (12)$$

where ΔC is the change in the capacitance C per degree rise of temperature; and for all practical cases, for frequencies up to 10 000 kc, $\omega^2 CL$ does not exceed $0 \cdot 02$, corresponding to a 2 per cent increase in this coefficient. It therefore appears that these discrepancies must be due to mechanical deformation produced by temperature-change, and consequently a study has been made of the factors which may produce distortion as distinct from pure thermal expansion.

(5) EFFECTS OF THE VARIATION OF THE PHYSICAL PROPERTIES OF METALS WITH TEMPERATURE-CHANGE UPON THE CAPACITANCE OF CONDENSERS

(a) Residual Stress

It is very naturally assumed that considerable mechanical distortion may take place owing to the release of internal stresses on heating. Such stresses are known to exist in rolled and cast metal but can be very appreciably reduced by annealing. The observed results cannot be wholly explained, however, on the assumption that residual stresses alone distort the metal. Firstly, it is well known that such stresses are well below the elastic limit of the material, except possibly in castings which are difficult to pour owing to narrow paths in the mould. This necessarily implies that deformations of this type produced by heating will disappear on cooling, and it has been shown in Section 3(*b*) that this is rarely the case. Secondly, it is shown later in the paper that the deformation required to produce the observed capacitance-change is far greater than is likely to occur in the material. There is no doubt, however, that the release of internal stress is a factor in some cases. Measurements of the distortion produced by temperature-rise have been made by a method described in Appendix 2, and for the case of a 4 in $\times$ 2 in. plate of $\frac{1}{8}$-in. annealed brass it was found that elastic deformation of the order of 30×10^{-6} in. occurred at the centre of the plate for a temperature-rise of 30 deg. C. Such a plate is rather larger and somewhat thicker than would generally be found in ordinary small condensers. It is permissible, therefore, to say that the probable distortion produced by residual stress in a typical condenser is of the order of 10^{-6} in. per deg. C.

The above statement is made on the supposition that annealed metal is used. With cold-worked or rolled material which has not been thermally aged, the distortion may be much greater, but, since abnormally large coefficients of capacitance have been observed in the absence of such conditions, deformation must be produced by other causes.

(b) Temperature Gradient

Owing to the method of mounting condenser vanes, it is possible for temperature-changes to affect certain portions of the plates to a greater extent than others. The difference in the radiation and convection conditions on opposite faces of the plates may also give rise to temperature-gradients in the metal. Experiments were made with a brass plate of $\frac{1}{8}$ in. thickness to ascertain the possible value of such temperature-gradients. Thermocouples were soldered to the two opposite surfaces while the plate was freely supported in an air oven, heat being slowly applied from below the lower surface. Permanent differences of temperature, of about 3 deg. C., were found to exist between the surfaces when the temperature-rise was 35 deg. C. It can be shown that in a thin beam 3 in. long in which there is a temperature gradient of this value across an $\frac{1}{8}$-in. section, the curvature produced by the difference in expansion of the two surfaces is equivalent to a deflection at the centre of 15×10^{-6} in. per deg. C.

The distortion which may occur owing to temperature gradient in the metal vanes is seen to be far greater than that likely to be produced by residual stresses, and there is good reason to suspect this cause of being responsible for a considerable increase in the capacitance coefficient. It is shown in Section (7) that the rate of variation of temperature of the various parts of a condenser may differ appreciably, and consequently appre-

* See Reference (44).

ciable temperature-differences may exist depending on the rate of temperature variation of the surroundings. One would therefore expect considerable hysteresis effects to occur, and observation shows that such effects are appreciable when the rate of temperature variation is rapid.

(c) Variation of Elasticity and Moment of Inertia with Temperature

The modulus of elasticity, E, and the moment of inertia of the plate section, $\mathbf{I}$, are both dependent on temperature. The modulus of elasticity of most metals falls slightly with temperature-rise, and since the moment of the section is a function of its dimensions, expansion of the metal will vary this value. The variation of E for brass is about 0·03 per cent per deg. C. In a condenser rotor with a vertical spindle, the vanes behave as cantilevers and will be deformed by their own weight. The deflection at the end of a uniformly loaded cantilever of rectangular cross-section is given by

$$\delta = \frac{Wl^3}{8E\mathbf{I}} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where W is the total weight of the metal and l is the distance from the free end to the support. For a typical brass plate, $W = 0{\cdot}2$ lb., $l = 3$ in., E (for brass) $= 10 \times 10^6$ lb. per sq. in. at 20° C. and $9{\cdot}85 \times 10^6$ lb. per sq. in. at 70° C. The moment of inertia of the section is given by

$$\mathbf{I} = bh^3/12 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where b is the breadth parallel to the neutral axis and h is the thickness. Taking $h = \frac{1}{8}$ in., $\mathbf{I} = 1/3\,072$ at 20° C. and $\mathbf{I} = \frac{1}{3072}\left(1 + \frac{3780}{10^6}\right)$ at 70° C., and it is seen that the deflection δ is $217{\cdot}2 \times 10^{-6}$ in. at 20° C. and $214{\cdot}0 \times 10^{-6}$ in. at 70° C. Although the end deflection of the cantilever due to its own weight is appreciable, the change in this deflection is only $3{\cdot}2 \times 10^{-6}$ in. for a 50-deg. C. change in temperature. Consequently it is apparent that this source of deformation is unlikely to produce effects at all comparable with those previously considered, and in fact the effects due to variation of elasticity and moment of inertia can be neglected.

(d) Mechanical Constraint produced by Temperature Variation

In most fixed and variable air-condensers, considerable constraint is imposed upon the free expansion of the plates by the supporting rods and spacing collars. Often such rods are fixed to an insulating material which has a very different expansion coefficient from that of the metal, and, with temperature variation, bending moments are introduced. Consider the extreme case of an aluminium plate $3\frac{1}{2}$ in. long, rigidly supported at its ends. A rise in temperature of 1 deg. C. will produce a deflection at the centre of $1\,630 \times 10^{-6}$ in. if it is assumed that the deformed shape is an arc of a circle. It is apparent that this deflection will occur whatever the width and thickness of the specimen. If the thickness is considerable, the force imposed at the supports by such a deformation is large, but for thin plates it is possible that the supports may be sufficiently rigid t sustain the much smaller force required to maintai this deformation. The actual deformation for any give plate and support system could be calculated, bu usually the method of support is so indeterminate tha little would be gained by such an analysis.

In many designs of condensers, notably where tem perature compensation is attempted, the active plates ar guided or allowed to slide in grooves, and there is con siderable doubt whether the plates are free to expan and remain flat at all temperatures. To test the effec of slight constraint on the performance of such a plate experiments were made on an aluminium specimen Aluminium was selected because its modulus of elasticity is lower than that of copper, brass, or steel, its thermal-expansion coefficient is higher, and its sliding properties are inferior to those of the above metals owing to its structure. Consequently it is likely to represent the worst practical case. The dimensions of this specimen were 4·5 in. × 2 in. × 0·066 in. and its weight was 0·055 lb. It was well annealed before use and was placed on three fixed $\frac{1}{4}$-in. steel balls as shown in Fig. 7. The deformation was measured by the optical method described in Appendix 2. When the specimen was

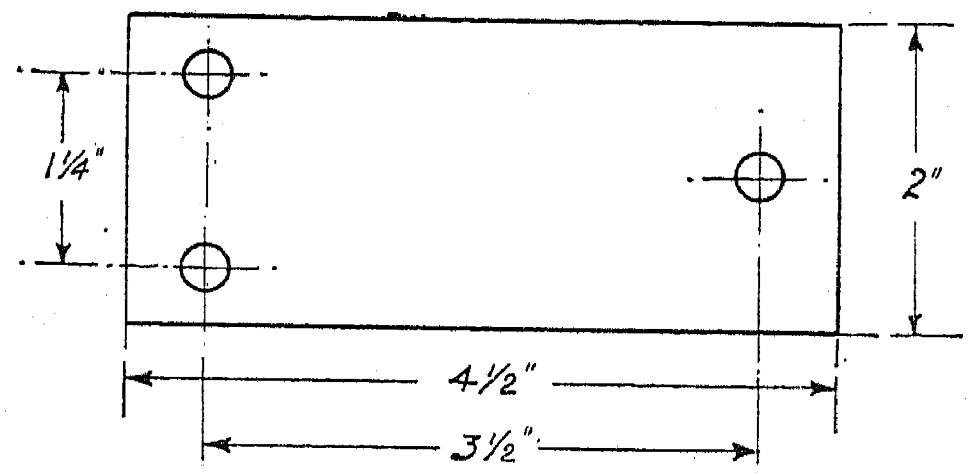

Fig. 7.—Dimensions of specimen.

heated from 15° C. to 50° C. an upward deflection of the centre was noted of the order of 500×10^{-6} in. and the behaviour was non-elastic, i.e. a long-period hysteresis effect took place on cooling. Such an effect could not be explained by the release of residual stresses since these are certainly within the elastic limit in an annealed specimen, neither could it be due to temperature-gradient effects since these would tend to deflect the specimen downwards. It was noted that the beam had a slight upward bend before heating.

The single supporting ball was now allowed to roll by being placed on a glass plane and the deflection was reduced to less than 5×10^{-6} in. for the same temperature-rise. It was clear from this experiment that the previous large deflection was due to the slight constraint imposed by the contact between the fixed steel ball and the lower aluminium face. Now the moment of inertia of this specimen was 48×10^{-6} lb. in.^{2} and the modulus of elasticity was 8×10^6 lb. per sq. in. The total load on the single ball was 0·088 lb., consisting of the sum of half the weight of the plate and the weight of one mirror (see Appendix 2). For a temperature-rise of 35 deg. C. the expansion of the beam between the supports is $2\,570 \times 10^{-6}$ in., and if the beam were completely constrained by these supports and bent into the form of a circular arc the upward deflection (assuming a slight initial upward camber) would be $58\,000 \times 10^{-6}$ in.

Regarding the beam as an eccentrically loaded strut, as shown in Fig. 8,

$$\delta = \frac{Fhl^2}{16EI} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or $\delta = 0 \cdot 000127F$ for this case, where F is the loading force on both edges of the strut. If $\delta = 500 \times 10^{-6}$,

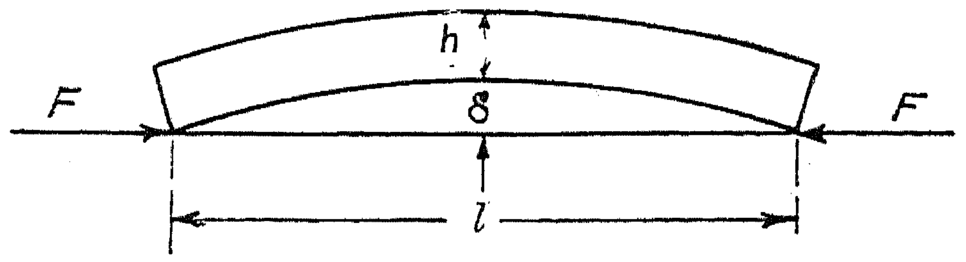

Fig. 8.—Eccentric loading of strut.

$F = 3 \cdot 9$ lb., and since the loading on each support was $0 \cdot 088$ lb. the coefficient of friction was 44.

Such a value of the coefficient of friction may appear at first sight to be very large, but it is in fact quite possible to obtain such a value for an aluminium-to-steel contact. Experiments on point contacts have shown that the coefficient of friction may be as high as 100 if the tangential force is applied sufficiently slowly. In this case the load was applied in such a manner since it was proportional to the expansion of the metal. Simple means of measuring the coefficient of friction by applying tangential loading do not give a correct value for very slow application of the load, owing to the impossibility of loading without shock. Further, all measurements of friction show a rapid increase in the coefficient as the velocity of the sliding contact is reduced, and the value at zero velocity is usually indeterminable. This result has considerable significance, for it means that the slightest constraint can give rise to appreciable eccentric forces which produce bending.

(e) Relative Magnitudes of Distorting Effects

It is now necessary to examine the probable changes in capacitance due to small deformations of the plates. For this purpose consider the type of distortion shown in Fig. 9, in which successive plates of a multi-vane condenser buckle in opposite directions. This is obviously the worst case and will set an upper limit to the possible changes in capacitance. The capacitance-change due to such buckling cannot readily be calculated

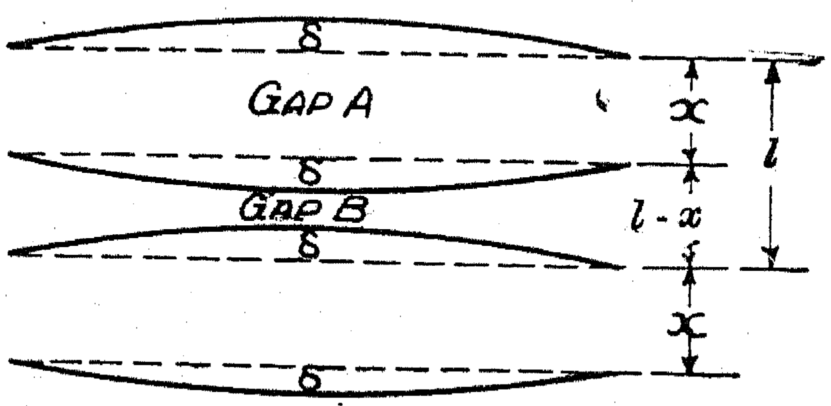

Fig. 9.—Effect of plate constraint.

exactly since the electric lines of force will no longer be straight, but a reasonable approximation can be adopted. It will be assumed that if a flat plate bends into the form of a circular arc, the change in capacitance between such a plate and a flat fixed plate is the same as would be obtained if the plate had remained flat but had moved by an amount $\delta/2$, where δ is the maximum deflection at the centre of the beam. Referring to Fig. 9, it is seen that the total fractional change in capacitance for all the vanes is the same as would be obtained if a flat plate placed between two similar plates moved by an amount δ. The change in capacitance is dependent upon the air-gap on each side of the moving plate. If the gap on one side is x and that on the other side $(l - x)$, where l is the fixed sum of the two gaps, then the fractional increase in the total capacitance is

$$1 - \frac{\dfrac{1}{x+\delta} + \dfrac{1}{l-x-\delta}}{\dfrac{1}{x} + \dfrac{1}{l-x}} \quad . \quad . \quad . \quad (16)$$

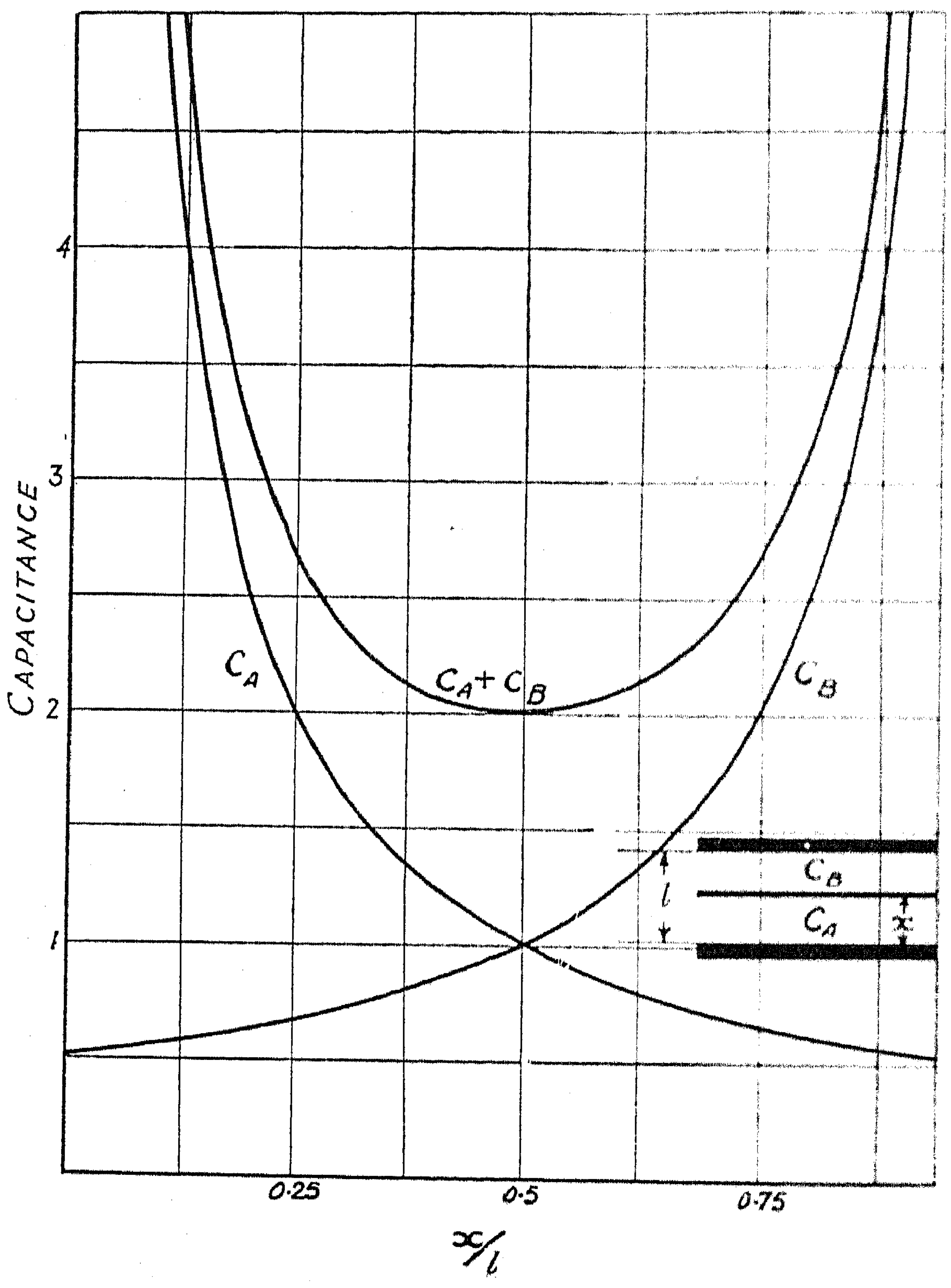

Fig. 10.—Capacitance of parallel-plate condenser.

The values of the two capacitances C_A and C_B together with their sum $(C_A + C_B)$ are plotted in Fig. 10 against the air-gap ratio x/l. It is obvious that small changes in gap will produce small changes in capacitance when the gaps on each side of the moving plate are equal (i.e. when $x/l = 0 \cdot 5$) but that the slightest departure from this value of x/l may give rise to appreciable changes in capacitance. In Fig. 11, the change in capacitance produced by small movements of the central plate are plotted for three values of x/l.

Making use of Fig. 11 together with the calculated and observed values of the distortion which may be caused by the various factors previously discussed, Table 5 has been compiled. The mean air-gap has been taken as $0 \cdot 05$ in. for this purpose. This table shows at a glance that if the air-gaps are all exactly the

same, an appreciable increase in the temperature coefficient of capacitance can only be produced by constraint effects, which could quite easily account for the observed increments, 20–50 parts in 1 million. If, however, the gaps are very slightly unequal, the effects of slight distortion are far greater and the other distorting causes may become important. A change of gap from $0 \cdot 5l$ to $0 \cdot 45l$ corresponds to 5 mils in this case. It is therefore important that the air-gaps shall be equal, to an accuracy of the order of 1 mil. This is certainly not the case in the great bulk of commercial condensers, and in fact it requires particularly fine workmanship to obtain such accuracy of gap spacing.

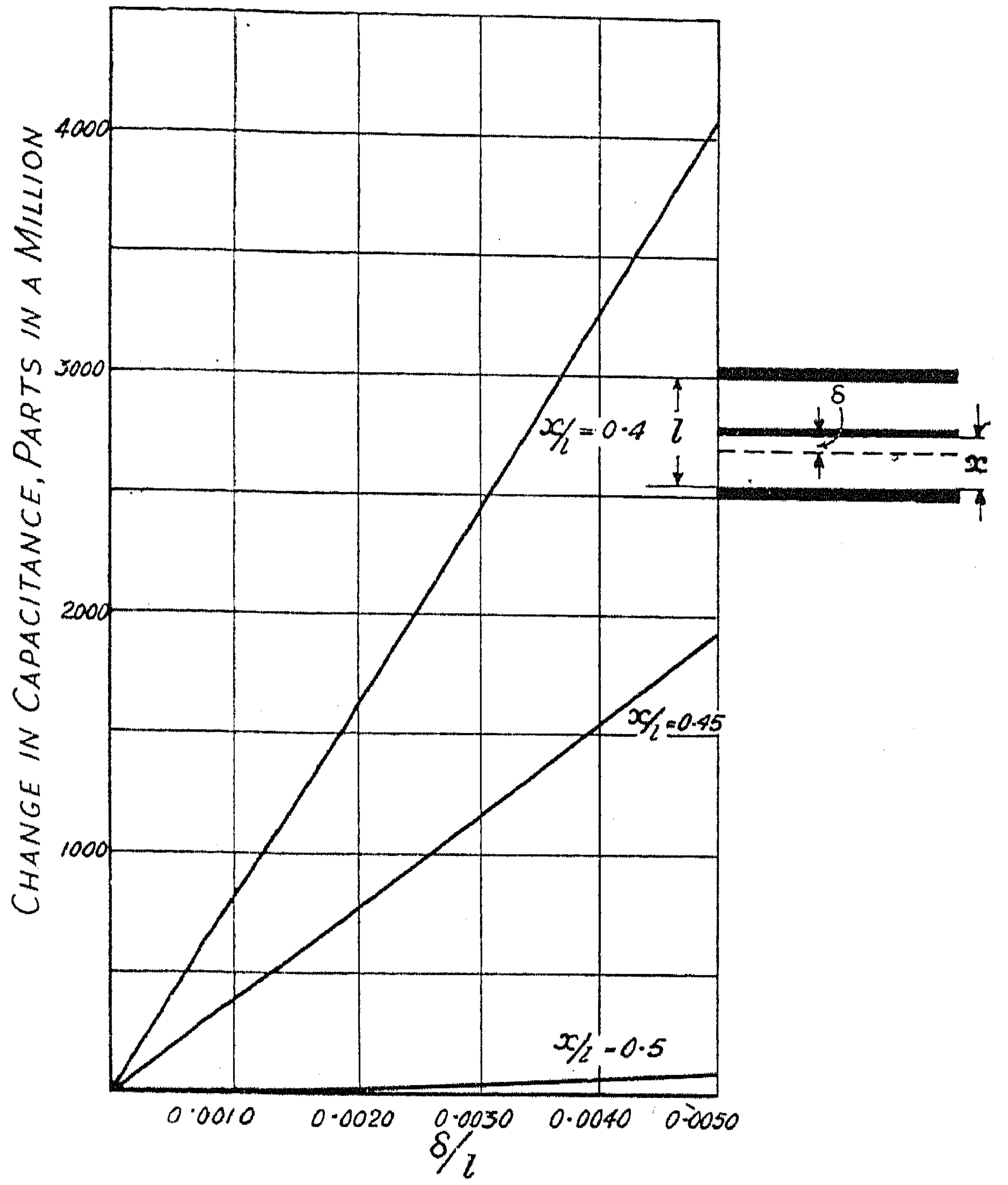

Fig. 11.—Change in capacitance due to movement of central plate.

Table 5

RELATIVE MAGNITUDE OF DISTORTING EFFECTS

Nature of distorting effect	Probable order of deformation for typical condenser vanes (millionths of an inch per deg. C.)	Increase in temperature coefficient of capacitance per deg. C. due to plate distortion		
		$x/l = 0 \cdot 5$	$x/l = 0 \cdot 45$	$x/l = 0 \cdot 4$
Residual stress (probable maximum for annealed metal)	2	0·2	8	16
Temperature gradient (probable maximum)	15	1·5	60	120
Constraint (very little)	20	2	80	160
Constraint (complete)	1 800	180	7 200	14 400

Although the figures given in Table 5 are extreme values, the relative significance of the various distorting factors would remain unchanged for smaller distortions and it can be suggested with reasonable confidence that the effects of constraint are likely to be more responsible for high values of the capacitance coefficient than any other factor, and that the second most important factor is temperature gradient in the metal portions of the condenser. It should be possible to separate these effects in particular cases by examining the manner in which the capacitance varies on heating and cooling, since temperature-gradient effects should produce a cyclic performance whereas constraint effects are unlikely to do so.

The observed coefficients of the order of twice the metal expansion-coefficient are readily explained in terms of plate distortion. In fact, it is clear that great care must be taken in construction in order to obtain a coefficient as low as twice the metal value. To eliminate these distorting effects altogether requires considerably more care in detailed design than has hitherto been given to the matter.

(6) EFFECTS OF PHYSICAL PROPERTIES OF INSULATING MATERIALS UPON CAPACITANCE OF CONDENSERS

In the case of built-up solid-dielectric condensers, the electrical stability is dependent chiefly upon the properties of the insulating material, but may also be affected by the changes in configuration produced by mechanical stresses set up by temperature variation. In solid-dielectric condensers having electrodes deposited on an insulator, the electrical stability is entirely dependent upon the properties of the dielectric material. In air condensers the capacitance of the solid-dielectric portion is usually small compared with the whole, and consequently changes in the permittivity and electrical power-loss of the solid portions are of less importance. Nevertheless, for some insulating materials an appreciable contribution to the net coefficient of capacitance may be made by the solid dielectric. It is therefore necessary to examine the changes in the electrical properties of insulating materials suitable for use in condensers designed for radio-frequency purposes, and, in view of the fact that the mechanical location of the stator assembly in the case of variable air-condensers is often entirely dependent upon the rigidity of the insulator supporting this stator, it is further necessary to study the mechanical properties of such materials.

Some selected insulating materials suitable for use at radio frequencies may be grouped as shown below. Descriptions of the properties and methods of manufacture of these materials have been published.*

* See References (1), (2), (5), (7), (9), (32), (33), (34), (35), (55), (61), (70), (72), (83).

Natural minerals	Mica	
	Quartz	
	Rock salt	
	Tourmaline	
Natural resins	Amber	
	Shellac	
Vitrified substances	Fused silica	
	Glass	
	Porcelain	
	Pyrex glass	
	Steatite (talc base; hydrated magnesium silicate)	
Rubber substances	Ebonite	
	Keramot	
Synthetic plastics	Bakelite	
	Celluloid	
	Paxolin	
	Pertinax	
	Press-spahn	
	Trolit	
	Trolitax	
	Trolitul	
Ceramic substances	Sinterkorund	Aluminium-oxide base
	Isomar	
	Calit	Magnesium-oxide base
	Calan	
	Ultra-calan	
	Frequenta	
	Frequentite	
	Ardostan	Aluminium-oxide + Magnesium-silicate base
	Sipa	
	Condensa	Titanium-dioxide base
	Kerafar	
	Tempa	
Miscellaneous substances	Fibre	
	Mycalex	
	Wood	

(a) Change of Permittivity with Temperature

The values of the permittivity and of the temperature coefficient of permittivity, together with the electric strength, of these dielectrics are tabulated in Table 6, the small figures to the right of each value referring to the authority for the measurements (see References). The permittivities of most of these substances lie between 2 and 8, with the exception of those of Condensa and Condensa C, which are 40 and 80 respectively. The change of permittivity with temperature is usually positive, the value lying between + 100 and + 600 parts in 1 million per deg. C., excluding glass, Condensa, and Tempa.

For certain natural substances such as quartz and mica, there appears to be considerable divergency of opinion regarding the value of this coefficient, the accurate determination of which necessitates the use of electrodes which must be in very close contact with the dielectric under test. It has been shown that this can only be achieved with the necessary precision by the use of mercury electrodes,* by sputtered or evaporated conducting films, or by metal sprayed on to the dielectric. It is doubtful whether these necessary experimental conditions were adopted in all the measurements giving values of the temperature coefficient of permittivity as set out in Table 6. For instance, the value for mica obtained from measurements at the National Physical Laboratory using mercury electrodes was less than + 20 parts in 1 million, whereas Handrek* gives a value of + 100 parts in 1 million. It is also stated by Handrek that pronounced ageing and non-cyclic behaviour took place in the samples tested, which he attributes to the properties of the dielectric. As a further example of such non-cyclic behaviour, Rohde† cites the case of certain mica condensers and shows that there may be considerable differences between individual samples. For different individual cases the capacitance-changes produced by temperature variation differ in both magnitude and sign. Rohde shows that ageing occurs, the behaviour becoming more cyclic as the number of thermal cycles is increased, and concludes from these results that the temperature coefficient of capacitance is rarely constant over an appreciable temperature range and the stability of mica condensers is rarely better than 200 parts in 1 million. It seems possible that these effects may be due to varying experimental conditions and not to the mica.

The value of the temperature coefficient of permittivity of vitrified substances, such as glass and porcelain, is high. Rubber substances or synthetic plastics are usually unsuitable for use as insulators in condensers where a high degree of electrical stability is desired, owing to their poor mechanical rigidity [see Section 6(*c*)].

The mechanical properties of ceramic materials are so much superior to those of most other insulating substances that their use is now becoming more general. Certain of these materials, such as Calit, Calan, Ultra-calan, and Frequenta, have temperature coefficients of permittivity of about + 100 parts in 1 million per deg. C. and a permittivity of about 6, whereas certain other materials—Condensa, Condensa C, and Kerafar—have an exceptionally high value of permittivity, a value of 90 being obtained by the use of a titanium-dioxide base. A further effect of the use of this oxide is an appreciable negative temperature-coefficient of permittivity, this being − 350 parts in 1 million for Condensa and − 720 parts in 1 million for Condensa C. The reason for this negative coefficient is at present unknown, but it is very naturally attributed to a curious molecular structure. It is claimed that intermediate values of permittivity can be obtained by varying the amount of titanium oxide in the material.‡ Recently, a new insulator known as "Tempa" has been produced having the comparatively small negative coefficient of − 20 parts in 1 million per deg. C.§

It would be anticipated that, when metal films were deposited on such ceramic dielectrics by electrolytic or other means, the observed behaviour of the condenser so formed would be sensibly similar to that of the dielectric itself. This is found to be the case; observations on Calit and Condensa tubular condensers‖ given in Table 1 show substantial agreement with the published values of the temperature coefficient of permittivity of these materials tabulated in Table 6, but it

* See References (37), (43).

* See Reference (32). † *Ibid.*, (56). ‡ *Ibid.*, (56). § *Ibid.*, (82). ‖ *Ibid.*, (32), (33).

should also be noted that the observed behaviour was not perfectly cyclic in the case of Calit and definitely non-cyclic in the case of Condensa. Published values of the ageing of ceramic materials* confirm the view that capacitance-changes of the order of 100 parts in 1 million may occur after each thermal cycle of 40 deg. C. The ageing or non-cyclic electrical performance of such substances when subjected to consecutive thermal cycles is far less than that obtained for rubber or plastic substances, but nevertheless it is often of sufficient magnitude to produce appreciable instability of capacitance.

As already pointed out, the temperature coefficient of capacitance of a variable air-condenser is affected to some extent by the permittivity coefficient of the solid insulating material. The effect is naturally greatest when the capacitance due to the solid dielectric is an appreciable fraction of the total, i.e. at the minimum-capacitance position of the rotor.

For a typical commercial variable air-condenser, the ratio of the capacitance of the solid dielectric to the total capacitance is about $\frac{1}{2}$ at the minimum-capacitance position and about 1/50 at the maximum. Consequently, if the temperature coefficient of permittivity of the dielectric material is $+ 100$ parts in 1 million per deg. C., the net temperature-coefficient of capacitance of the condenser can be increased by as much as $+ 50$ parts in 1 million at the minimum rotor setting but only by $+ 2$ parts in 1 million at the maximum rotor position. This factor is thus by no means negligible and care must be taken to reduce as far as possible the capacitance due to the solid dielectric.

(b) Electrical Losses

The selection of the most suitable insulating material for supporting the stator assembly is not dependent only on the value of its temperature coefficient of permittivity, since it is also necessary to obtain a low power factor. The values of the dielectric-loss angle or power factor of certain selected insulating materials are tabulated in Table 6, the published measurements being given for the frequency range 0·5–100 megacycles per sec. These values have been obtained by different methods of measurement† and show quite definite and serious discrepancies. For instance, the power factor of amber is given as 30×10^{-4} by one authority and 330×10^{-4} by another, Calit as 3×10^{-4} or 14×10^{-4}, Pertinax as 60×10^{-4} or $1\,000 \times 10^{-4}$, Pyrex glass as $1 \cdot 2 \times 10^{-4}$ or 27×10^{-4}, and Trolitul as $1 \cdot 5 \times 10^{-4}$ or $8 \cdot 0 \times 10^{-4}$. (See Table 6 for authorities.) These are a few of the most obvious discrepancies, but similar though smaller differences are noticed in most cases.

Although these discrepancies can be accounted for by the non-reproducibility of the materials in certain cases, this is certainly not true for natural substances or for many synthetic materials having a definite chemical composition and precise conditions of manufacture. The inevitable conclusion is that the existing methods of measurement require a thorough critical examination with the object of ascertaining more precisely the value of the power factor of many of these insulating materials.

* See Reference (56).

† *Ibid.*, (10), (41), (43), (49), (58) (66).

From the existing information, it appears that natural substances such as mica, quartz, and tourmaline, have low electrical losses, as have also most of the ceramic substances such as Calit, Calan, Ultra-calan, Condensa,* and Frequenta. Kerafar,† on the other hand, appears to have a large loss-angle, and the synthetic plastics tend to have high values, though that of Trolitul is low.

The effect of temperature-rise is to increase the power factor‡ appreciably, but this will not affect the stability of the condenser; in fact, the study of the electrical losses is only a subsidiary problem to the main investigation. It is necessary, however, to keep it in mind in designing a condenser, since a material which is suitable from the point of view of its other electrical and mechanical properties may have to be rejected on account of its high dielectric loss. For example, in the maintenance of oscillation by valves, such losses can be regarded as a decrease of the total effective equivalent shunt resistance of the circuit. In fact, if the circuit consists of an inductance of loss angle α [where $\tan \alpha = R/(\omega L)$] in parallel with a condenser of capacitance C and loss angle δ, the total equivalent shunt resistance is given by

$$\frac{1}{\omega C(\tan \alpha + \tan \delta)} \quad . \quad . \quad . \quad . \quad (17)$$

It is this shunt resistance which determines the upper limiting value of C consistent with maintenance of oscillation in a given assembly of circuit and valve. As a convenient approximation for present purposes, the condition for maintenance can be put in the form

$$\omega C\,(\tan \alpha + \tan \delta) \not> \frac{1}{|N|} \quad . \quad . \quad . \quad (18)$$

where N is an effective negative resistance produced by the valve. Now α will vary from, say, 20×10^{-4} for a really good coil at frequencies of 1 megacycle per sec. or less, to 200×10^{-4} or even more at frequencies of tens of megacycles per sec. In general, therefore, it will be a very much larger quantity than δ, which can usually be reduced to the order 10^{-4} or less. In this sense, therefore, the loss angle of the condenser will in nearly all cases be a minor factor in the total losses. This point is emphasized in Appendix 3, which shows how large δ may be at various frequencies without adding more than 10 per cent to the resistance of a typical circuit. It must be noted, however, that in some cases, particularly at very short wavelengths, when everything practicable has been done to minimize the coil loss-angle α, the quantity $(\omega C \tan \alpha)$ may be of such a magnitude relative to the available effective negative resistance that the small residual term $(\omega C \tan \delta)$ will actually become the important limiting factor in the maintenance of oscillation.

Briefly, therefore, condenser loss is, in nearly all practical cases, very small indeed compared with other sources of loss in the circuit, particularly coil resistance, and is correspondingly unimportant in relation to oscillation except in those cases where a very narrow margin of available power makes all the loss-factors of practical significance.

* See References (61), (70).

† *Ibid.*, (5), (61).

‡ *Ibid.*, (45), (48), (49).

Table 6

ELECTRICAL PROPERTIES OF SOME SELECTED INSULATING MATERIALS

(Figures in small type ndicate the references in which the authority for the values is given)

Material	Power factor (parts in 10^4 at 15–60° C.) Frequency (megacycles per sec.) 0·5	Ref.	1·0	Ref.	5·0	Ref.	10·0	Ref.	50·0	Ref.	100·0	Ref.	Permittivity κ	Ref.	Temperature coefficient of permittivity (parts in 10^6 at 15–60° C.)	Ref.	Electric strength (kV per mm)	Ref.
Amber	—		—		—		—		27·5 300	45 57	30·5 330	45 57	2·9	32, 57	—		—	
Ardostan	—		—		—		—		—		—		—		—		20	36
Bakelite	100	32, 88	160 180 380	32, 88 53 6	200 220 400	32, 88 53 91	220 270	32, 88 53	50 260 450	45 32, 88 57	13 500	45 32, 57, 88	2·8 4·7	32, 53, 57, 88 6	—		—	
Calan	3·6	36, 74, 88	2·5 3·2	71 36, 74, 88	2·8 3·6 3·7	33, 36, 74, 88 85 3, 53, 73	2·6 3·6 3·8	33, 36, 74, 88 85 3, 53, 73, 76	2·5 4·7	33, 36, 74, 88 3, 53	1·8	33	6·5 6·6	3, 53, 76, 85 33, 36, 88	+120	36, 82, 88	40	3, 36, 76, 88
Ultra-Calan	—		1·0	32, 33, 36, 71, 74, 88	1·0 2·1	32, 33, 36, 71, 74, 88 53, 73, 85	1·0 2·1 2·7	32, 33, 36, 74, 88 53, 76, 85 73	1·1 2·2	32, 33, 36, 57, 74, 88 53	1·1 2·2	33, 57 53	6·5 7·1	53, 76, 85 32, 33, 36, 57, 88	+100	36	40	36, 76, 88
Calit	4·1	32, 36, 74, 88	3·8 14	32, 36, 74, 88 3, 53	3·7 14	32, 33, 36, 74, 88 3, 53	3·4 14	32, 33, 36, 74, 88 3, 53, 76	3·2 14·2	32, 33, 36, 74, 88 53	—		6·5	3, 32, 33, 36, 53, 76, 88	+120 +160	36, 82 32	40	3, 36, 76, 88
Celluloid	—		—		490	53	480	53	430	57	450	57	3·3 3·5	53 57	—		—	
Condensa	9·7	36	8	33, 36	7·2	36, 56	6·1 6·4	36 33	5·7	36	—		40	33, 36, 70	−340 −380 −510	36, 82, 70, 88 33	17	36

Condensa C	7·1	32, 36	6·0	32, 36	4·1	36	3·2 3·8	32, 36 57	2·8 3·5	32, 36 57	—		80 90	36, 57, 32, 70	−700 −720	82 32, 36, 70	11	32, 36
Ebonite	65	36, 74, 88	64 77 110	36, 53, 74, 88 6 6	50 61	91 36, 53, 74, 88	57 107	36, 53, 74, 88 32	53 120	36, 74, 83, 88 57	150	32, 57	3·0	6, 36, 53, 57, 69	+500	89	—	
Fibre (vulcanized)	—		—		510	53	1 000	53	—		—		4·4	53	—		—	
Frequenta	—		2·5 3·2 3·8 4·2 4·7 5·1	71 33 36 3, 32 53 1	3·0 3·2	33, 36 1, 3, 53	2·8 3·4	1, 3, 33, 36, 53, 76 32	2·6 2·8	33, 36 1, 3, 53	3·0	32	2·2 5·6 5·9 6·1	32 3, 36, 76 1 33, 53	+100	36	47 150	1 3
Frequenta D	—		3·3 3·8	1 36	—		1·9 2·1	36 1, 3	1·9	36, 57	1·8	57	5·6 6·1	1, 36 57	—		27	36
Frequentite	—		10	1, 3, 90	7·5 8·0 8·8	1, 3 33 90	6·8	1, 3, 33, 76, 90	6·1	1, 3, 33, 90	—		5·9 6·4	3, 33, 76, 90 1	—		48 150	1 3
Glass (lead)	—		5·3	53	5·7 30	53 32, 91	7·2	53	10·6 11·0	53 57	14	57	6·5 6·7	32, 91, 53, 57	+2 000	89	—	
Glass (Mino)	—		4·6	53	5·0	53	5·9	53	7·4	53	—		7·5	32, 53	—		—	
Glass (Thüringer)	—		47	53	38	53	34	53	48	53, 57	65	57	7·1	53, 57	—		—	
Isomar	—		—		12·3	53	12·2	53	10·7	53	—		6·0	53	—		40	36
Kerafar R	—		1 000	36	—		750	36	—		—		90	32, 36	−600	32, 36	—	
Kerafar S	—		1 500	36	—		750	36	—		—		70	36	−500	36	10	36
Kerafar T	—		1 500	36	—		1 000	36	—		—		40	36	−300	36	—	
Keramot	—		90	6	200	91	—		—		—		4·0	6	+270 +815	89 89	—	

Table 6—*continued*

ELECTRICAL PROPERTIES OF SOME SELECTED INSULATING MATERIALS

(Figures in small type indicate the references in which the authority for the values is given)

Material	Power factor (parts in 10^4 at 15–60° C.) Frequency (megacycles per sec.) 0·5		1·0		5·0		10·0		50·0		100·0		Permittivity κ		Temperature coefficient of permittivity (parts in 10^6 at 15–60° C.)		Electric strength (kV per mm)	
Mica	1·7	32, 36, 53, 74, 88	1·7	32, 36, 53, 74, 88	1·0 1·7	71 32, 33, 36, 53, 73, 74, 85, 88	1·7	3, 32, 33, 36, 53, 73, 74, 85, 88	1·7	32, 36, 53, 57, 74, 88	1·6	32, 36, 57, 88	7·0	3, 32, 33, 36, 53, 57, 71, 85, 88	+ 20 + 60 +100	89 36, 82 32	—	
Mycalex	20	32, 36, 47, 74, 88	15 18 20	71 32, 33, 36, 74, 88 75	18 20	32, 33, 36, 53, 73, 74, 85, 88 89	18	32, 33, 36, 53, 73, 74, 85, 88	18	32, 36, 74, 88	—		6·0 8·0 8·5 8·8	47 32, 33, 71, 88 36, 75, 85 53	—		13·4 15·0	75 47
Paxolin T	—		260	6	—		—		—		—		4·2	6	—		—	
Paxolin P	—		340	6	—		—		—		—		4·3	6	—		—	
Pertinax	220	32, 36, 74, 88	280	32, 36, 53, 74, 88	350 400	32, 36, 74, 88 53	720 770	32, 36, 74, 88 53	60 900 1 000	45 57 32, 36, 53, 74, 88	20 1 000	45 32, 45, 57	5·4	32, 36, 53, 57, 88	—		—	
Porcelain	70	36, 88	55 66	36, 53, 88 6	49	36, 53, 88	63 90 110	36, 53, 88 76 3	85	36, 53, 88	—		4·7 5·6 6·0	6 36, 53, 76, 88 3	+500	32	36	76
Press-spahn	—		240	53	570	53	580	53	—		—		3·4	53	—		—	

Quartz (natural)	1·1	32, 33, 36, 74, 88	1·1 1·8	32, 33, 36, 74, 88 53	1·1 1·6	32, 33, 36, 74, 88 53	1·1 1·6	32, 33, 36, 74, 88 53	1·1 1·7	32, 33, 36, 74, 88 53	1·0	32, 33, 36, 74, 88	4·5 4·7	71 32, 33, 36, 53, 88	+20 +40	36 32	—	
Quartz (fused silica)	1·8	74, 88	1·8	33, 74, 88	1·8 2·7	33, 74, 88 53, 73, 85	1·7 2·7 5·0	33, 74, 88, 53, 73, 85 10	1·1 1·7 2·6	57 33, 74, 88 53	1·0	57	3·4 4·2	10, 75 33, 53, 57, 85, 88	—		19·8	75
Quartz (Pyrex glass)	5·0	74, 88	5·2	74, 88	5·5 6·7 8·6	33, 74, 88 73, 85 53	5·8 14·8	33, 74, 88 53, 73, 85	1·2 7·0 27·0	57 33, 74, 88 53	1·1	57	3·8 3·9	53, 85 33, 57, 88	+600	39	13·6	75
Rock salt	—		—		—		—		0·2	57	0·16	57	5·6	57	—		—	
Shellac	—		—		78	53	110	53	—		—		2·6	32 53	—		—	
Steatite	21	36, 88, 90	20 30	1, 3, 36, 53, 88, 90 71	18	1, 3, 36, 53, 88, 90	17	1, 3, 36, 53, 76, 88, 90	15	1, 3, 36, 53, 88, 90	—		5·4 6·0 6·5	75 53 3, 36, 76, 88, 90	—		150	3
Tempa	—		—		—		—		—		—		—		−20	82	—	
Tourmaline	—		—		4·7	53	3·6	53	2·9	53	—		5·0	53	—		—	
Trolit	—		270	53	280	53	290	53	—		—		5·5 6·2	32 53	—		—	
Trolitax	—		325	53	400	53	500	53	300	57	330	57	5·0	57	—		—	
Trolitul	4·1	88	2·5 3·9	71 88	3·7 4·0	88 55	1·5 4·5 8·0	57 88 45	1·5 5·4 8·0	57 88 45	—		2·2 2·4	55, 57, 88 71	—		—	
Whitewood (dry)	—		—		—		230	10	—		—		1·7	10	—		—	

Table 7

MECHANICAL PROPERTIES OF SOME SELECTED INSULATING MATERIALS

(Figures in small type indicate the references in which the authority for the values is given)

Material	Specific gravity		Tensile strength (tons per sq. in.)		Crushing strength (tons per sq. in.)		Transverse strength (tons per sq. in)		Impact strength (energy to fracture, ft. lb. per sq. in.)		Modulus of elasticity (tons per sq. in.)		Temperature coefficient of expansion (parts in 10^6 per deg. C.)		Softening temperature (°C.)	
Ardostan	2·2	36, 76	2·1	36, 76	22·2	36, 76	5·3	36, 76	0·9	36, 76	4 500	36	1·0 1·3	36, 76 70	1 300	36
Calit	2·6 2·7	36, 76, 88 3	4·8 5·1	3, 76 36, 88	60·1 61·7	3, 76 36, 76	8·9 9·5	3 36, 76, 88	1·9 2·0 2·1	88 36, 76 3	6 900	36, 88	7·8	3, 36, 76, 88	1 440	36
Calan	2·8	3, 36, 76, 88	2·8	3, 36, 76, 88	35·0	3, 36, 76, 88	6·4	3, 36, 76, 88	1·4 1·5	88 3, 36, 76	7 600	36	7·6	3, 36, 76, 88	1 390	36
Ultra-Calan	2·8	36, 76, 88	2·8	36, 76, 88	35·0	36, 76, 88	6·4 10·1	76 36, 88	1·4 1·7	76 36, 88	7 000	36	7·6 8·6	76 36, 88	1 580	36
Condensa	2·8 3·7	33 36	1·9	36	19·0	33, 36	5·7 9·5	33 36	1·0	33, 36	7 000	35	6·7 7·3	36 33	1 430	35
Condensa C	3·9	32, 36	—		—		6·6 9·5	36 32	1·5	32, 36	7 000 8 300	32 36	7·3 8·0	26 32	1 480	35
Frequenta	2·7 2·8	1, 3 76	4·8	1, 3, 76	58·5 60·5	1, 76 3	8·9	1, 3, 76	1·9 2·1	76 1, 3	6 400	1	6·5 7·0	1, 76 3	—	
Frequenta D	2·8	36	4·4	36	58·5	36	9·5	36	2·1	36	6 400	36	6·5	36	1 440	36
Frequentite	2·6 2·7	76, 90 1, 3	4·4 4·8	90 1, 3, 76	57·2 60·5	1, 76, 90 3	8·3 8·9	1, 76, 90 3	1·9 2·0 2·1	1, 76 90 3	6 400	1	7·0	1, 3, 76, 90	—	
Isomar	2·7	36	3·8	36	51·0	36	11·1	36	2·0	36	12 500	36	2·5	36	1 530	36
Kerafar S	3·6	36	3·5	36	70·0	36	9·5	36	—		7 000	36	8·0	36	—	
Mica	2·9	53	—		—		—		—		—		3·0	53	—	
Mycalex	2·7 3·4	47 9, 53, 75	3·1 3·5 3·8	75 53 47	11·2 21·6 22·0	9 53 75	8·9 10·1 11·0	9 47 75	2·8 2·9	53 75	—		8·8 9·0	53 47	450	47

Porcelain	2·4	53, 76	1·9 2·2	53 76	25·4	53, 76	4·4	53, 76	0·9	53, 76	4 400	53	3·3 3·5 5·5	76 36 53	—	
Pyrodur	2·6	36	1·9	36	47·5	36	8·2	36	1·2	36	5 700	36	4·4	36	1 645	36
Quartz (natural) . .	2·2	53	4·5	53	126·0	53	4·4	53	—		4 400	53	0·55	53	—	
Quartz (fused silica) . .	2·0	75	—		—		—		—		—		0·5	36, 75	1 400	75
Quartz (Pyrex glass) . .	2·2	75	—		—		—		—		—		3·2	75	600	75
Sinterkorund . . .	3·8	36	2·3	36	35·0	36	7·6	36	1·5	36	14 600	36	4·6	36	1 730	36
Sipa	2·2	36, 76	1·9	36, 76	25·4	36, 76	3·8	36, 76	2·0	36, 76	5 700	36	1·0	36, 76	1 400	36
Steatite	2·5 2·6 2·7 2·8	75 90 1, 3, 70 53	3·2 3·6 4·1 4·5 4·8	53 4 75 70, 90 1, 3	51·0 54·0 57·0 60·3 95·0	53 4 1, 70, 90 3 75	6·0 7·6 8·2 8·9	53 4 1, 70, 90 3	1·2 1·5 1·7 1·9 2·1	53 4 75 1, 70, 90 3	6 400	1, 53	7·0 7·6 8·0	90 53 1, 3, 70, 75	—	

(c) Mechanical Properties

However excellent a solid dielectric may be from the point of view of dielectric-loss and constancy of permittivity, it is unsuitable for supporting the stator of an air condenser if its mechanical rigidity is inadequate. This criticism applies particularly to all the synthetic plastics and rubber substances which are known to flow and distort under the application of quite small mechanical forces.* Although many substances in this class are used in small commercial condensers, tests have shown that a high degree of stability cannot be expected when such materials are employed. On the other hand, certain natural minerals, vitrified substances, and ceramic materials, possess a high degree of mechanical rigidity and are now being employed more generally in condenser construction.

Published values of the mechanical properties of a selected number of such substances are given in Table 7. It is seen that the tensile strength of these materials is about one-eighth that of mild steel (about 38 tons per sq. in.) and that the crushing strength is quite high. The transverse strength or modulus of rupture bears the same ratio to the tensile strength as for most metals, but the impact strength has a value of about 1/200th that of mild steel; in other words, these substances are very brittle. The modulus of elasticity is about half that of steel (13 500 tons per sq. in.). The temperature coefficient of expansion varies between 1 and 8 parts in 1 million per deg. C., and this small expansion is of great value in many applications of such substances. The agreement between the values given by different authorities is far better than that noticed in the case of the electrical properties, probably owing to the fact that mechanical measurements are far easier to carry out accurately.

Now although a considerable amount of data is available on the general mechanical properties of these substances, this information is not of great value in selecting the most suitable material for any particular purpose, since the stress imposed on such insulating members is, in all practical cases, very small. In many designs of condensers such insulators take the form of strips of rectangular section supporting the stator assembly and, assuming that the clamping arrangements are adequate, the electrical stability of the condenser is dependent upon the permanence of the location afforded by such strips. In other words, it is necessary to know whether the geometrical shape of such materials is affected by temperature-change.

The mechanical stability of bakelite has been examined† and it has been shown that appreciable bending and distortion of shape takes place with temperature-rise, which is attributed to the release of internal stresses. Synthetic plastics are also known to have a very poor mechanical stability,‡ and experiments on the stability of marble§ have shown that such a natural substance is liable to appreciable instability on repeated thermal treatments over quite a small temperature-range (0–40° C.). It has been found that the coefficient of expansion of marble (about $+ 6 \times 10^{-6}$ per deg. C.) varies with temperature and is actually negative at temperatures about and below 0° C.; also marble shows

* See References (14), (39). † *Ibid.*, (54). ‡ *Ibid.*, (39). § *Ibid.*, (67).

continual growth when subjected to repeated thermal cycles. Further experiments on porcelain* showed no trace of such non-cyclic behaviour, from which it was concluded that amorphous substances fused at high temperatures are likely to be more stable than natural substances, owing to the presence of cleavage planes in the latter.

Table 8

RESULTS OF TESTS ON THE MECHANICAL DEFORMATION OF CERTAIN INSULATORS DUE TO TEMPERATURE-CHANGE

Material	Deflection at centre of beam for 33 deg. C. rise in temperature (millionths of an inch)	Thermal performance	Notes on mechanical behaviour
Bakelite ..	2 800	Non-cyclic	Permanent set of $2\,000 \times 10^{-6}$ in.
Bakelite ..	3 900	Non-cyclic	Permanent set of $2\,100 \times 10^{-6}$ in.
Calit ..	10	Cyclic	—
Condensa ..	19	Cyclic	—
Ebonite ..	18 500	Nearly cyclic	—
Keramot ..	4 200	Nearly cyclic	Permanent set of 320×10^{-6} in.
Keramot ..	5 200	Nearly cyclic	Permanent set of 300×10^{-6} in.
Marble ..	300	Nearly cyclic	—
Mycalex ..	156	Nearly cyclic	—
*Quartz (fused silica)	100	Non-cyclic	No return on cooling
Quartz (pyrex glass)	45	Nearly cyclic	—
Steatite ..	90	Nearly cyclic	—

* Only one specimen was examined, and the non-cyclic behaviour observed may not be a general characteristic of this material. The result is nevertheless important and shows that fused silica cannot safely be assumed to be cyclic in behaviour under temperature-change.

It was considered advisable, therefore, to carry out tests on the mechanical stability of a few typical insulating materials when subjected to temperature-change. For this purpose, a standard size of 4 in. × 2 in. × 0·2 in. was adopted and the specimens were examined by means of the optical method described in Appendix 2. The results of these tests are tabulated in Table 8. It is seen that the distortion of shape produced by temperature variation was large for bakelite, ebonite, and keramot, and was very small for the two ceramic materials which were tested; mycalex, steatite, and quartz had intermediate values. In several of these cases a permanent bending moment was applied by loading the centre of the beam, and no appreciable change of deformation was noted, suggesting that the effects were due to the release of quite appreciable stresses.

* See Reference (67).

It is not easy to derive the probable distortion for a beam of different section, since it would be unwise to assume inverse proportionality between the distortion and the moment of inertia of the cross-section, but it is likely that the effects would be greater if the section were reduced. Taking a typical stator-supporting system consisting of two beams of insulating material, it appears probable that the movement of the stator in a direction parallel to the rotor axis may be about 20×10^{-6} in. for a 33 deg. C. rise in temperature in the case of ceramic materials such as Calit or Condensa and of the order of $5\,000 \times 10^{-6}$ in. for materials such as bakelite. Assuming that the distortion is proportional to temperature, and referring to Fig. 11, it is clear that this distorting effect will produce a negligible capacitance-change in the case of Calit or Condensa but may be responsible for an increase of several hundred parts in 1 million in the temperature coefficient of capacitance in the case of bakelite, particularly if the original gap spacings are unequal. It appears, therefore, that a reasonably high degree of mechanical stability with temperature-change is required and that natural mineral, ceramic, and vitrified substances, are much superior in this respect to rubber substances or synthetic plastics.

Although such ceramic materials can be moulded to quite accurate and small shapes,* it is impossible to machine such materials by ordinary methods. Many of these substances can only be cut by a diamond saw, and grinding methods are necessary to obtain accurate dimensions. Nevertheless, the texture is such that high-grade parts can be produced if care is taken to use the most suitable methods of machining.

(7) EFFECTS OF THERMAL PROPERTIES OF MATERIALS UPON CAPACITANCE OF CONDENSERS

An examination of the curves relating capacitance-change with time, when the temperature of the surrounding air was changed rapidly over a large range, shows that this change lags behind the temperature variation to a varying extent. In some cases this time-lag was merely a few minutes, whereas in others it amounted to several hours. This phenomenon is due to the different thermal properties of the various portions of the condenser, produced by the large differences in surface conditions, mass, and specific heat, of the various parts. If the thermal constants of the different members are appreciably different, the capacitance-change which takes place after a temperature-rise has been made and maintained for a long period may be very different from the maximum transient changes which exist during the

* See References (35), (55).

stabilizing of the temperature of each part. In many cases where a low temperature-coefficient of capacitance is obtained if the temperature variation is slow, the maximum change of capacitance may be far greater when rapid temperature-changes occur, and in certain applications of condensers it is necessary to limit such transient capacitance-changes. This factor is of special significance in the design of temperature-compensated condensers, for, although it is possible to design a satisfactory compensating arrangement for slow temperature-variations, it is exceedingly difficult to obtain continuous compensation during a rapid change of temperature of the surroundings.

This aspect of condenser design has hitherto been entirely neglected, but a perusal of actual results obtained on typical condensers shows that it is of primary importance in the production of satisfactory means of stabilization. The study of the thermal properties has an additional value in the fact that a thermal analysis of a condenser affords a ready means of delineating the various types of distortion which occur with temperature variation. If the thermal time-lags of the various portions can be calculated, it should be possible from the form of any capacitance-change observation to isolate the various contributory factors.

For thermal analysis, an air-dielectric condenser may be regarded as made up of the following separate portions:—

Stator assembly (or one electrode system, in fixed condensers);
Rotor assembly (or the other electrode system, in fixed condensers);
Frame;
Insulation.

In all cases, the stator and rotor are distinct thermal entities and so is the insulator, but the frame is often in good metallic contact with the rotor and can be regarded as part of it. In certain cases, where a massive frame is used, the contact to the rotor is such that a better approximation is obtained by considering the frame separately.

Although in the tests described in Section 3(*b*) the change in the temperature of the surroundings was not immediate, it was an approximation to this condition, and for the purposes of analysis it will be assumed at this stage that an immediate change of temperature of the surroundings took place. Postulating this condition, the problem resolves itself into the determination of the rate of change of temperature in the various portions of the condenser assembly.

(a) Methods of Heat-Transfer and Determination of Theoretical Heating Characteristics

The rate of temperature-rise at the centre of a body when the surroundings are suddenly raised in temperature is dependent upon the rate at which heat enters the surface and passes inwards from that surface to the centre. The three modes of heat transfer—conduction, radiation, and convection—must be considered separately and then combined.

The conduction of heat through bodies of various shapes, initially at a uniform temperature θ_0, when the temperature of the surfaces is altered to, and maintained at, a value θ_1, is most conveniently expressed by Fig. 12, reproduced from a paper by A. Schack.* In this figure, θ = temperature at centre of body, t = time in seconds, k = thermal conductivity of material, s = specific heat of material, $k/(s\rho)$ = diffusivity of material, and ρ = density of material. It must be remembered that this theoretical curve only applies to practical cases if the surface layer is immediately changed in temperature. Usually this bounding layer has a coefficient of heat transfer between the surface and the surroundings. Now $k/(s\rho)$ is $0\cdot 34$ for brass and $0\cdot 83$ for aluminium, and taking a thick rotor or stator plate in which $d = 0\cdot 2$ cm it is readily seen that the time taken to heat the entire plate is about $0\cdot 044$ sec. for brass and $0\cdot 018$ sec. for aluminium. In the case of large thermal insulators, however, the

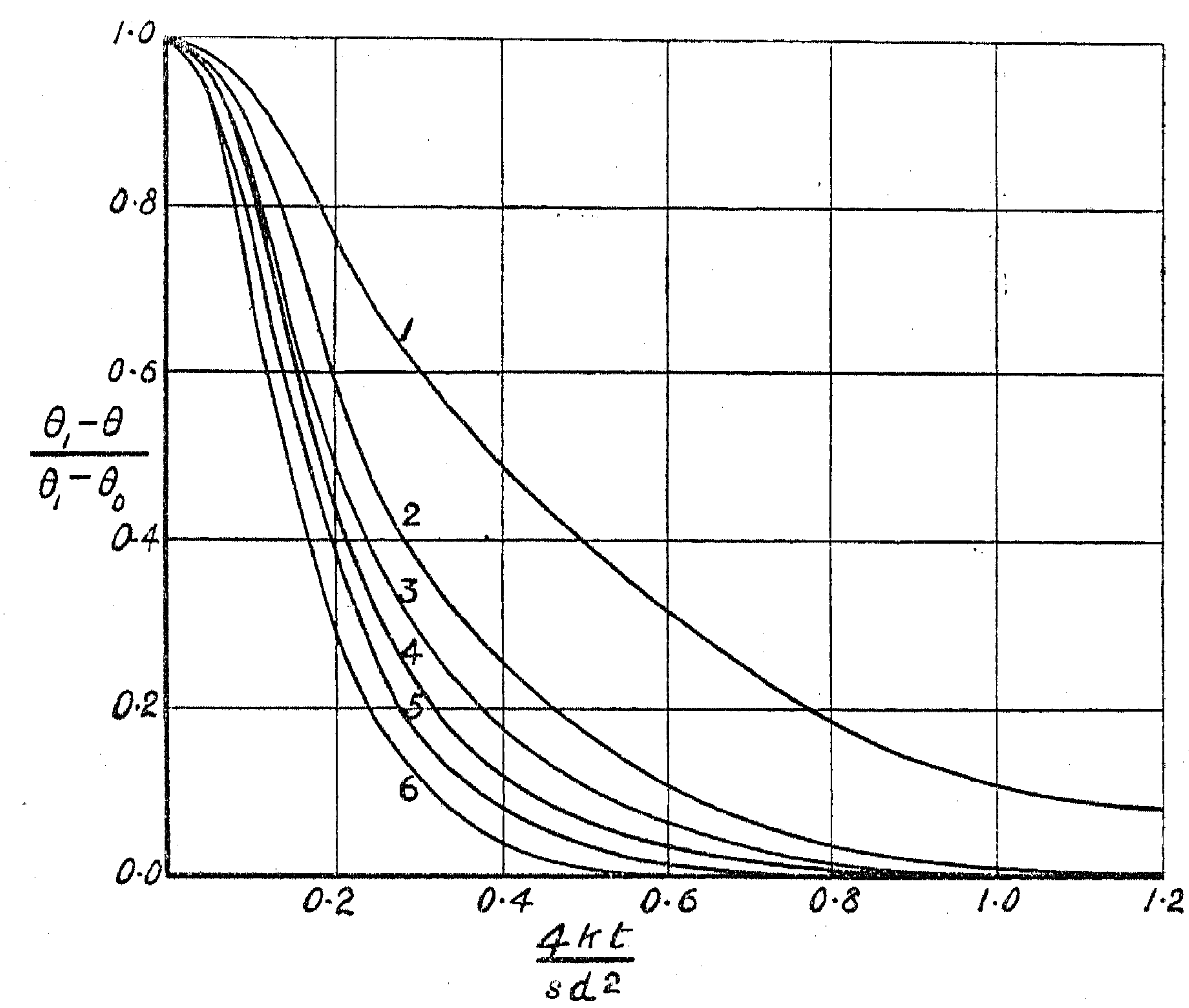

Fig. 12.—Temperature at centre of bodies of various shapes.

1. Slab, d = thickness.
2. Square bar, d = side.
3. Long cylinder, d = diameter.
4. Cube, d = side.
5. Cylinder, length = diameter, d = diameter.
6. Sphere, d = diameter.

effects of thermal conductance have an appreciable effect upon the rate of flow of heat.

Assuming perfect conductivity, the rate of temperature-change due to radiation is given by the expression

$$\frac{ms}{2eA_R\theta_1^3}\Big[(\text{arc tan }\theta/\theta_1 + \text{arc coth }\theta/\theta_1) - (\text{arc tan }\theta_2/\theta_1 + \text{arc coth }\theta_2/\theta_1)\Big] \quad . \quad (19)$$

where

m = mass, in grammes;
e = emissivity of surface, in calories per cm² per sec. per deg. K;
= emissivity coefficient (100 for perfect black-body radiator) × Stefan's constant ($1\cdot 37 \times 10^{-12}$ calorie per cm² per sec. per deg. K.)†

* See Reference (60).
† °K. signifies the absolute scale of temperature, on which 0° C. = 273° K.

A_R = effective area submitted to radiation;
θ_1 = final temperature of body and surroundings (°K.);
θ = temperature at time t (°K.);
and
θ_2 = initial temperature of body (°K.).

The derivation of expression (19) is given in Appendix 4. The equation deals with the case where the body is cooling, i.e. $\theta_1 < \theta_2$. The law for heating is obviously identical.

The value of the function

$$(\text{arc tan } \theta/\theta_1 + \text{arc coth } \theta/\theta_1) - (\text{arc tan } \theta_2/\theta_1 + \text{arc coth } \theta_2/\theta_1) \quad . \quad (20)$$

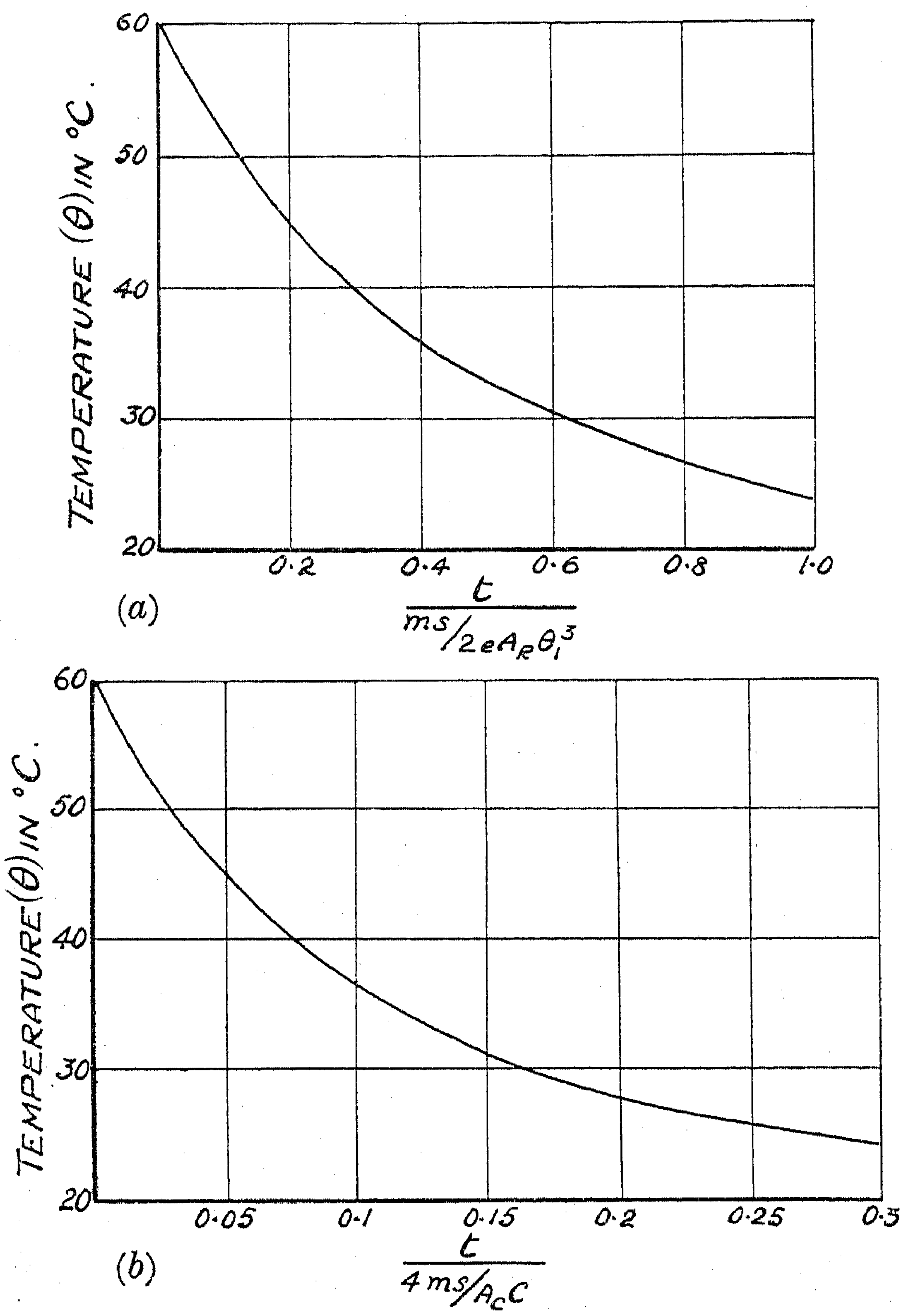

Fig. 13

Upper curve: Radiation law. $\theta_1 = 293°$ K., $\theta_2 = 333°$ K.
Lower curve: Convection law. $\theta_1 = 293°$ K., $\theta_2 = 333°$ K.

for the values of $\theta_1 = 20°$ C. and $\theta_2 = 60°$ C. is plotted in Fig. 13(*a*). These values cover the range of oven temperature used in the tests on condensers. From this characteristic, it is possible to determine the temperature-change which occurs in any given time for a cooling body whose constants are known, due to radiation only.

The rate of temperature-change due to convection is given by the expression

$$\frac{4ms}{A_cC}\left[\frac{1}{(\theta-\theta_1)^{\frac{1}{4}}} - \frac{1}{(\theta_2-\theta_1)^{\frac{1}{4}}}\right] \quad . \quad . \quad (21)$$

where
A_c = effective area submitted to convection currents
and
C = convection coefficient, in calories per cm² per sec. per deg. K.

The derivation of this expression is given in Appendix 5. The value of the function

$$\frac{1}{(\theta-\theta_1)^{\frac{1}{4}}} - \frac{1}{(\theta_2-\theta_1)^{\frac{1}{4}}} \quad . \quad . \quad . \quad (22)$$

for the values $\theta_1 = 20°$ C. and $\theta_2 = 60°$ C. is plotted in Fig. 13(*b*). From this characteristic it is possible to determine the temperature-change which occurs in any given time, owing to convection only, in a cooling body whose constants are known.

The combination of these two heat-losses is best done graphically. Referring to Fig. 14, let the convection and radiation characteristics be plotted. Then, taking any convenient small unit of time OA, the fall in temperature due to convection is BC and due to radiation is BD. The total fall is therefore BE = BC + BD. Now in the next equal unit of time AF, the fall in temperature will be GH + JK = LM, where NG = OJ = AF. By continuing this construction for each succeeding interval, the final characteristic can be determined. Any degree of accuracy can be obtained by making the time-intervals sufficiently small.

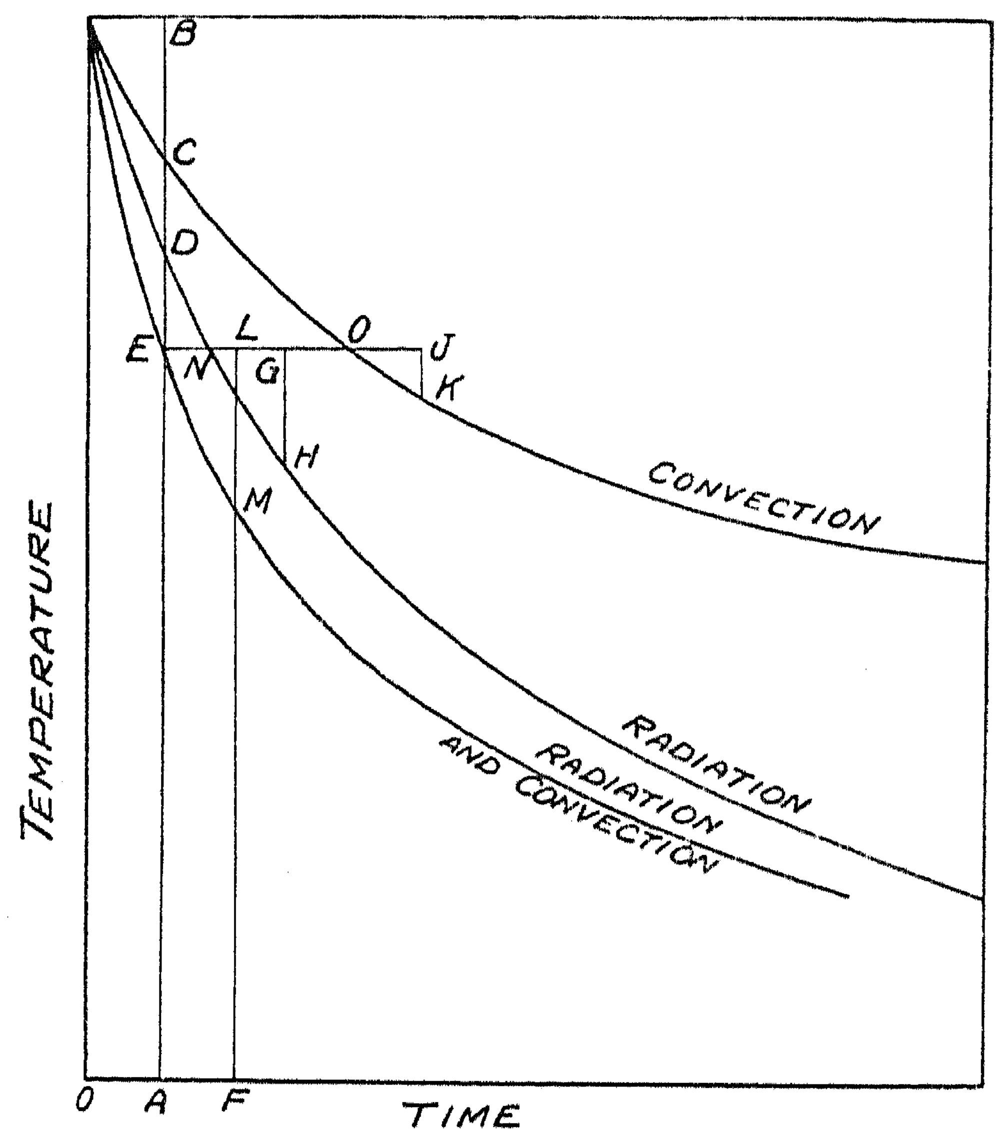

Fig. 14.—Method of deducing cooling curve from radiation and convection characteristics.

By inverting any of these curves, the corresponding heating characteristic is obtained, but in all cases this deduced characteristic is applicable only to the case when the external temperature is suddenly changed. In the actual tests this was not the case, and if the rate of the oven temperature-change is known, it is possible to deduce the temperature-change of the body by means of a graphical construction. Referring to Fig. 15, let the calculated temperature-rise of the body and the measured temperature-rise of the surroundings be as shown. Then in unit time OA, the temperature of the

body would have risen by an amount AB, if the surroundings had immediately changed from O to C.

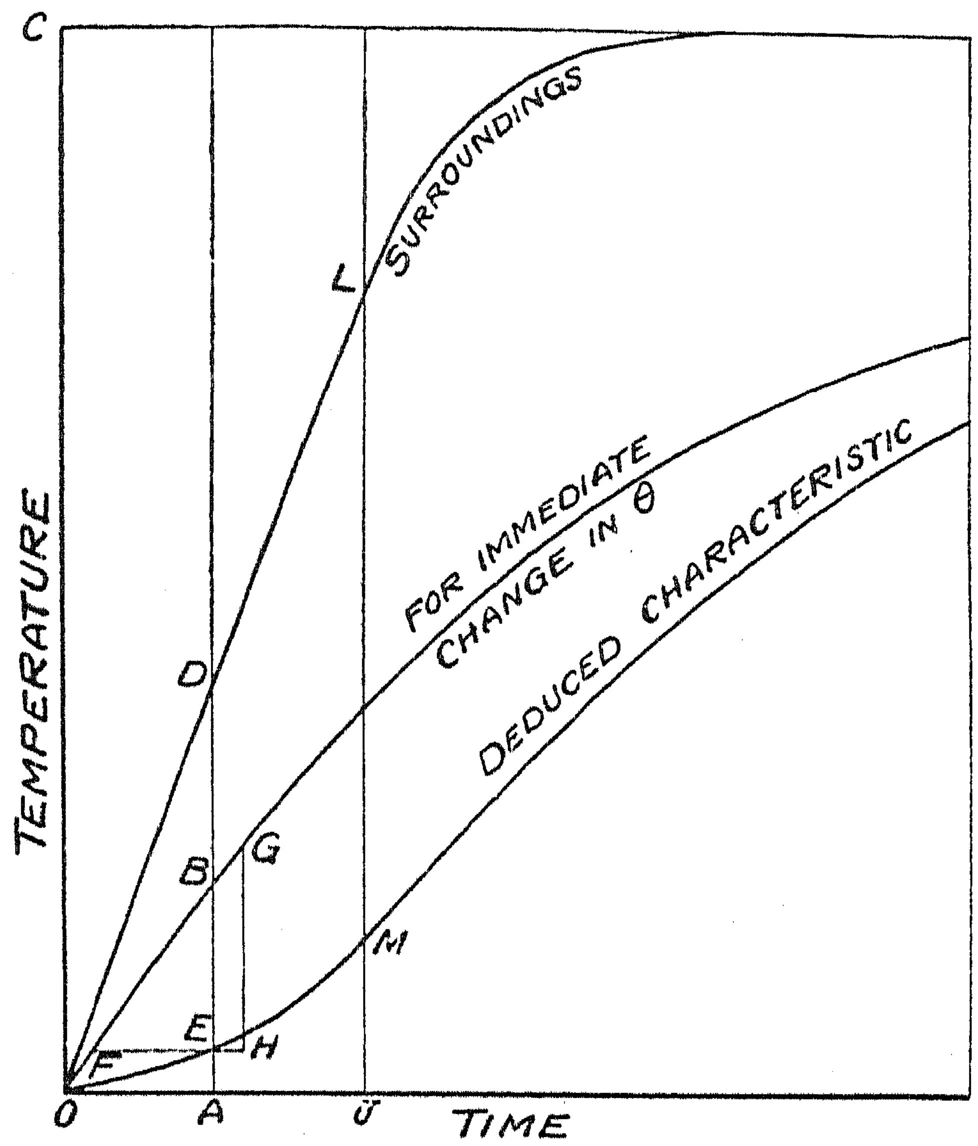

Fig. 15.—Method of deducing heating curve for defined temperature-change of surroundings.

The mean temperature of the surroundings during this period is, however, AD/2, and so the actual rise in temperature will be given approximately by

$$AE = AB \times \frac{AD}{2OC}$$

By continuing this construction for succeeding intervals of time, the final theoretical characteristic can be obtained, the accuracy depending only on the number of intervals chosen. For the purpose of illustrating the method of construction, a large time-interval has been selected, but in all actual characteristics deduced by this method the time-interval was much smaller.

Before this analytical method can be applied to an actual case it is necessary to determine the value of the convection coefficient C, the remaining constants involved being known for metals and some other substances.*

(b) Determination of Thermal Constants of Metals and Insulators

For the purpose of determining the value of the convection coefficient C for still air, a cylindrical metal can was used in which the diameter was equal to the length. This can was polished and filled with water, the total water equivalent of the combination being 456 grammes. A cooling curve was taken, and since the emissivity coefficient for polished tin is 5 per cent, the heat lost by radiation was small. It was found from this test that a convection coefficient of $0 \cdot 54 \times 10^{-4}$ calorie per cm^2 per sec. per deg. K. gave exact agreement between the deduced and observed cooling curves. Further experiments with metal cans of varying shapes and sizes and different surfaces gave good agreement between the rates of cooling as observed and as predicted from the known thermal constants.

To test the validity of the constants and the applicability of the method to cases involving the cooling of metal

Table 9

Constants of Materials used in Condensers T, U, V, W, and X, for the Temperature Range 0–100° C.

Material	Density (ρ)	Coefficient of expansion (α)	Specific heat (s)	Thermal conductivity (k)	Diffusivity [$k/(s\rho)$]	Emissivity coefficient	
	grammes/cm^3	parts in 10^6					per cent
Keramot ..	1·7	+ 40 to + 70 (variable)	0·23	0·0057	0·00147	Smooth	70 (approx.)
						Rough	90 (approx.)
Mycalex ..	2·45	+ 6	0·22	0·0014	0·00258		95 (approx.)
Brass	8·10	+ 18·9	0·074	0·204	0·339	Polished	5
						Matt	21
						Lacquered	65 (approx.)
Aluminium ..	2·71	+ 25·5	0·214	0·480	0·826	Polished	4
						Rough	7

Now during the next time-interval AJ, the temperature would have risen from F to G, where FH = OA = AJ, if the surroundings had been at the final temperature C; but the mean temperature of the air is (AD + JL)/2, and consequently the actual temperature at the end of the second time-interval will be

$$JM = AE + GH \left(\frac{AD + JL}{2OC}\right)$$

bodies, tests were made on various masses, the temperature at the centre of each being measured by an embedded thermocouple. It was found that the rate of heat-transfer from the surface to the centre was so rapid that the mass could be considered a perfect thermal conductor for all practical purposes. If this is so for a body in which the ratio mass/area is large, it will be

* See References (16), (18) (42), (53), (62), (68), (69), (78), (86), (87).

more nearly true for the metal portions of a condenser in which this ratio is usually very small.

Turning now to the insulator, experiments were carried out in a similar manner to determine the thermal properties of the materials used in condensers T, U, V, W, and X, namely keramot and mycalex.* In both these cases, various shapes were taken and a thermo-junction was embedded in the material. By plunging it into hot water and observing the rate of change of temperature at the centre, the diffusivity of the material was determined. Cooling curves in air were also obtained, giving approximate values for the emissivity coefficient. It was found from the tests that, for small sections of insulating material, the rate of change of temperature due to conduction was much more rapid than that due to radiation and convection. Consequently, to a fair degree of accuracy, it is possible to regard the body as a perfect thermal conductor having specified surface conditions. For all the cases involved in the tests, this treatment was found to be valid.

Lastly, the specific heat of the materials was measured, from which the thermal conductivity was readily calculated. The values for these materials are given in Table 9, and for the sake of completeness the known constants of brass and aluminium are added, together with approximate values of the density and expansion-coefficient of the insulating materials.

(c) Application of Analytical Methods to Resolution of Factors producing Capacitance-change

In applying the foregoing thermal method of analysis to the observed capacitance-changes for different cases, it must be realized that an actual condenser is a complex thermal system and that precise calculation is impossible. Nevertheless, considerable agreement has been obtained between the observed and calculated performance.

Taking Condenser U (see Table 2) as a typical example, the dimensions, mass, and thermal characteristics of each part were obtained. These were as follows:—

Stator (one plate assembly):

Mass	= 158 grammes.
Total air surface	= 200 cm^2.
Specific heat	= 0·074 (brass).
Emissivity	= 65 per cent (lacquered surface).

Rotor (including shaft):

Mass	= 317 grammes.
Total air surface	= 375 cm^2.

Specific heat and emissivity same as for stator.

Insulation (one plate):

Mass	= 158 grammes.
Total air surface	= 157 cm^2.
Specific heat	= 0·22 (mycalex)
Emissivity	= 95 per cent.

From these data, the calculated heating curve for each part was obtained. This is shown in Fig. 16. It is observed that the rate of change of capacitance closely followed the calculated temperature-rise of the mycalex insulator, and consequently it is difficult to escape from the inference that the observed temperature-coefficient of capacitance in this case was due to the deformation of the insulating material. The rate of temperature-rise of the metal portions was much greater than that of the insulator.

* See References (30), (51).

In a further case (Condenser V) a similar method of analysis was adopted, and the observed and calculated temperature variations are shown in Fig. 17.

In this case, the performance was non-cyclic, due to the fact that keramot possesses poor rigidity and when distorted by a rise in temperature only partially returns to its original state.

The insulating material may not always be responsible for long-period time-lags, condenser X being an example of this type. In this case (see Fig. 18) the initial change in capacitance appeared to be due to the expansion of the metal rotor and stator, and the mycalex insulation; in fact the observed characteristic was a mean between these two calculated curves. Here, however, a longer-period change took place, owing to the slow temperature-rise of the heavy frame casting, the expansion of which produced a shift of relative stator position, causing a reduction of capacitance.

Condenser P had very thin plates, and the insulation consisting of four keramot washers played an unimportant part in the mechanical system. After the insulating material had been thoroughly dried, the capacitance-changes with temperature-variations were nearly cyclic and the coefficient was low. In this case the thermal properties of the two fixed-plate assemblies only need be considered, since expansion of the short mycalex bars did not affect the relative position of the plates. The constants for one plate assembly were as follows:—

Mass	= 310 grammes.
Total air surface	= 900 cm^2.
Specific heat	= 0·074 (brass).
Emissivity	= 5 per cent (polished).

The calculated performance is shown in Fig. 19, from which it is seen that the thermal-lag in this case is very small and agrees closely with the calculated behaviour of the metal plates.

It has already been pointed out that large transient capacitance-changes may occur when the temperature of the surroundings is altered rapidly, and that it is desirable to reduce such effects. This can be done by so designing the condenser that the rates of loss and gain of heat of each portion are similar. If this condition is satisfied, no temperature-differences will exist between the various parts of the assembly, irrespective of the type of thermal cycle applied to the surrounding air. Equations (19) and (21) show that this requirement can be satisfied if the values of the quantity (mass × specific heat/area) and the emissivity coefficient of each part are both similar. The emissivity coefficient can be altered within wide limits by polishing, roughening, or painting the surfaces, but it is more difficult to obtain equality between the values of (mass × specific heat/area) for the insulating and metallic positions.

In the cases of solid-dielectric condensers, where the interior is inaccessible, the effective specific heat of the condenser considered as a thermal body is unknown. If a value is selected to give agreement between the observed and calculated heating characteristics, it is found that the forms of the characteristics agree, and conse-

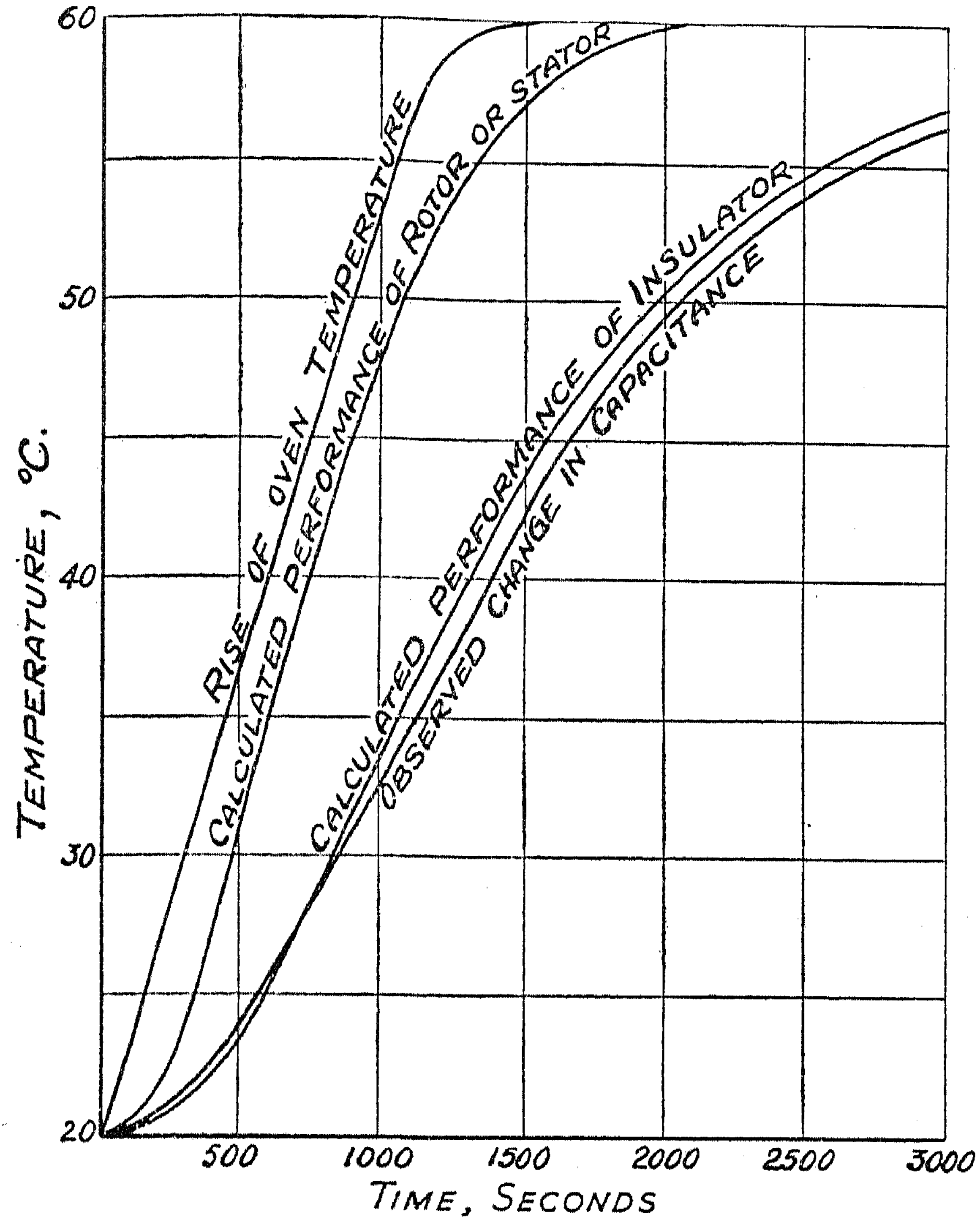

Fig. 16.—Thermal performance of condenser U.

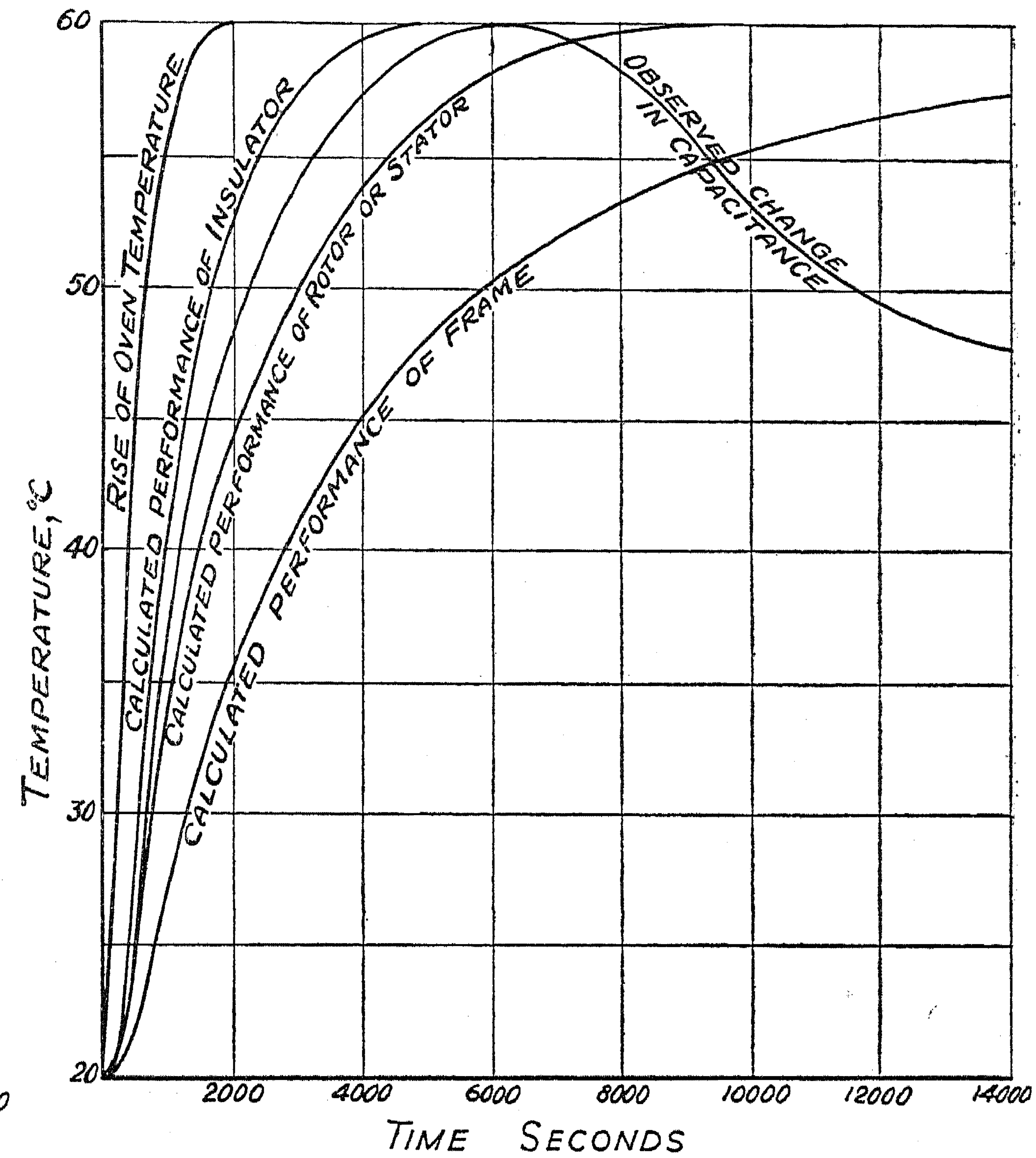

Fig. 18.—Thermal performance of condenser X.

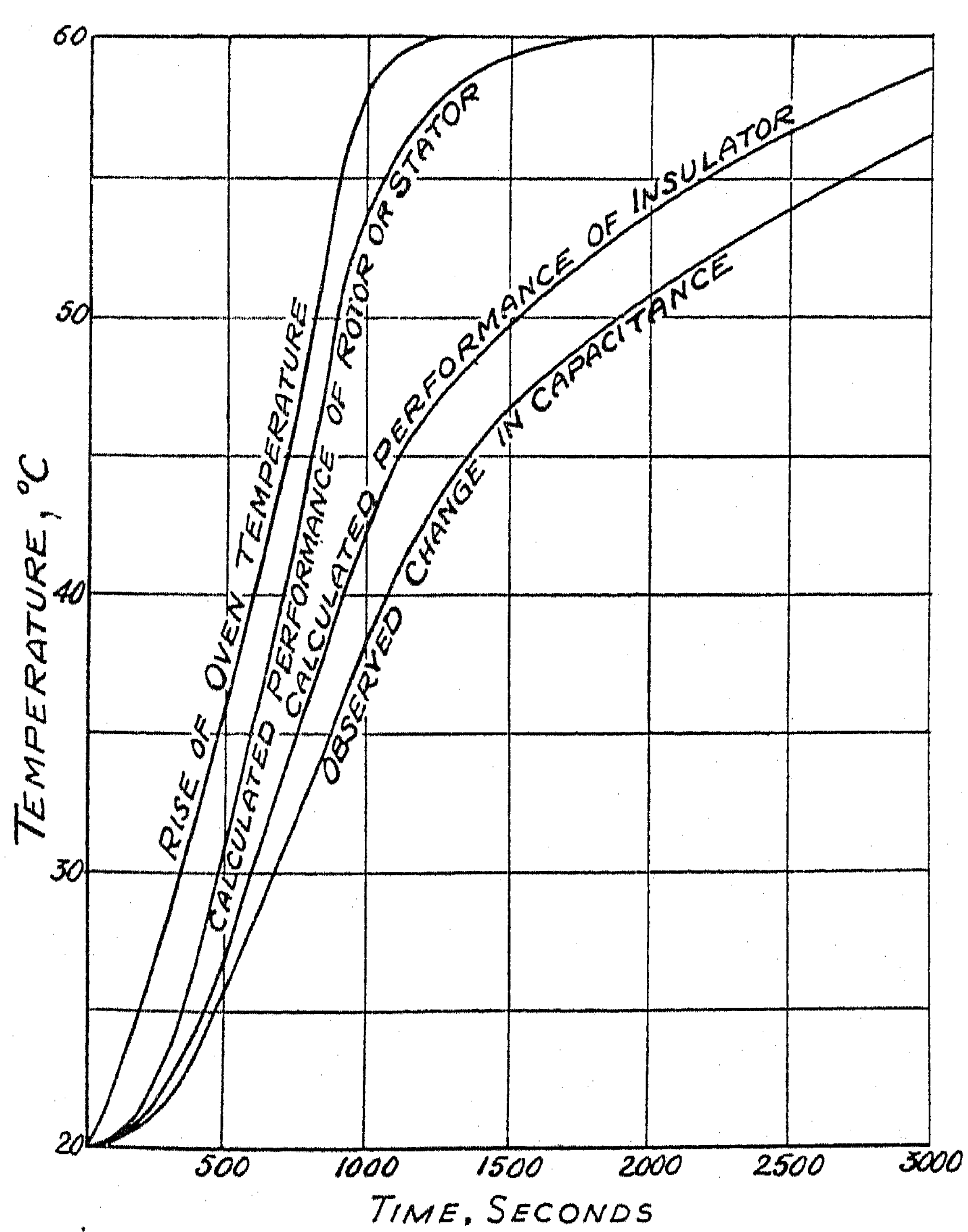

Fig. 17.—Thermal performance of condenser V.

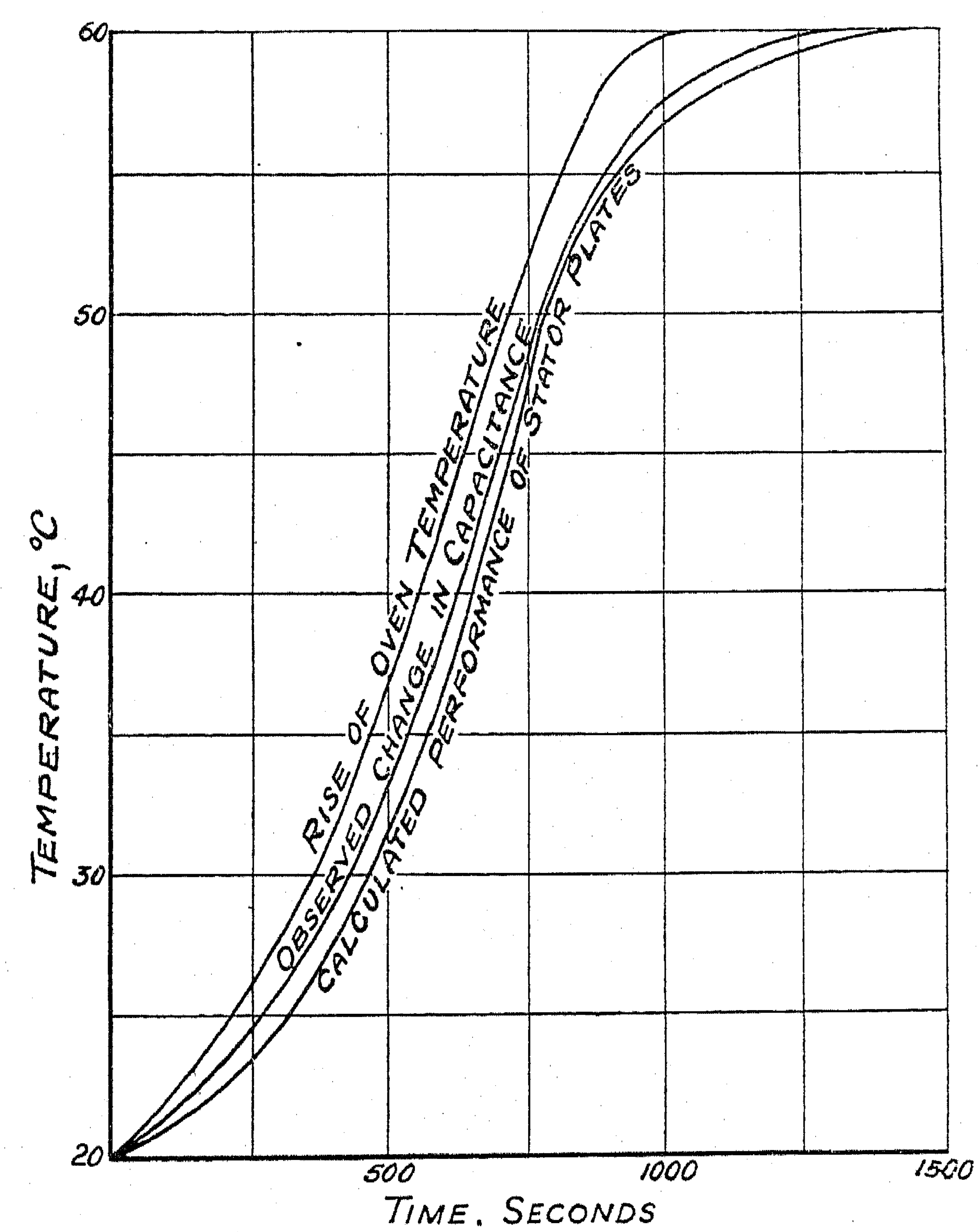

Fig. 19.—Thermal performance of condenser P.

quently it is permissible to look upon the condenser as a perfectly conducting mass having a definite specific heat which is obtained experimentally. The time-lags of such condensers are far greater than those of air condensers, and the effective specific heat varies considerably. Owing to the large differences in design and the absence of knowledge regarding the inner construction of most of the solid-dielectric condensers which were tested, no attempt has been made to correlate the experimental specific-heat value with the specific heat of the materials used in the construction.

The agreement between the calculated and experimental observations of the rate of capacitance-change with temperature variation is such that this method of analysis can be adopted for the resolution of many problems connected with condenser design. The method not only enables the various causes of distortion to be separated but also indicates the factors which produce non-cyclic behaviour in specific cases.

(8) DESIGN OF AIR-DIELECTRIC CONDENSERS

Reviewing the results of the investigation described in the preceding sections, it is clear that a number of different conditions must be satisfied if a high degree of electrical stability is to be obtained. These are set out below in order of relative importance:—

Metal portions of condenser (stator, rotor, and frame)	(*a*) Absence of mechanical constraint in the assembly of each part and in the method of support.
	(*b*) Uniform temperature over whole of each portion.
	(*c*) Absence of residual stress.
Insulating portions of condenser	(*d*) Small value of the temperature coefficient of permittivity, and reduction of capacitance of solid dielectric to a minimum value.
	(*e*) High degree of mechanical stability.
	(*f*) Accuracy of location of the insulator supporting the stator assembly.
	(*g*) Small dielectric loss and high electric strength (for transmitting condensers).
General thermal requirements applying to all component parts	(*h*) Value of (mass × specific heat/area) to be same for each part
	(*j*) Value of emissivity coefficient to be identical for each part.

These conditions are often unfulfilled in present methods of construction. For instance, it is a common practice to assemble the rotor and stator of a condenser by means of collars and clamping rods, often made of metals different from that of the vanes, with the result that appreciable constraint effects can occur, owing to differential expansions. Again, even if the same metal is used for all the parts of a metal assembly, appreciable bending forces may still be produced if the thermal capacities of all parts are not similar. Stability of mechanical shape is not likely to be obtained if the vanes of a condenser are clamped between spacing collars.

It seems clear that the best type of stator or rotor is one in which the vanes are integral with the spacing members, i.e. the plate system is machined from a solid metal block.* Such a method of construction is highly satisfactory when cast or forged metal is used, particularly if it is annealed several times during machining to release internal stresses. The machining of such a block is best done by means of a multiple cutter; a further advantage of this method of machining is that it introduces very little surface stress.

Various other methods of achieving this object have been suggested. One such method of stator construction involves the welding of the plates into grooves cut in a solid supporting member,† and a further well-known method is to die-cast these parts.‡ A far simpler arrangement applicable to the rotor assembly embodies a locking device which rigidly fixes and welds each separate vane to a spindle of the same metal.§ All such arrangements have for their object the production of an assembly in which constraint and residual-stress effects are small, but in all of them it is important that the mass of the spacing pillars or webs should be so adjusted that the thermal capacity of these parts is substantially the same as that of the vanes. The question of the best thickness for the plates is one of difficulty; thick vanes naturally have a far greater mechanical rigidity, but temperature-gradient effects may be serious. Unfortunately the thermal system is so complex that a detailed analytical treatment is impossible, and improvement in design will have to develop as a result of trial.

Requirements (*d*), (*e*), and (*g*), suggest the use of ceramic materials for insulating purposes, but for such substances accuracy of location of the stator assembly is difficult. Even for these materials the deformation and ageing with temperature-change is by no means negligible and the use of strips of insulating material is liable to produce instability. Unfortunately it is often necessary to use a considerable length of insulating material to give the necessary electrical strength in the case of high-voltage condensers, but for receiving purposes there appears to be no justification for the use of long strips. A further difficulty of using such materials arises from the fact that precise location of the stator system is not easy to attain, owing to the fact that accurate machining is impossible. Fixing to a ceramic insulator by means of screws in moulded holes may cause considerable uncertainty of location. Joints in which a ground tubular insulator is forced between two cylindrical tapered metal sleeves have been successful in the case of a mycalex insulator.|| It is difficult to satisfy both the thermal and electrical requirements at the same time, since the thermal properties of insulating materials are so different from those of metals, but improvement in this direction is not impossible.

Considering now the assembly of the stator and rotor, the usual method of stator support is quite unsuitable in respect of thermal stability. In most cheap commercial receiver condensers the stability depends upon the mechanical properties of an insulator of plastic characteristics, and such a design cannot be expected to have good thermal stability. In high-class precision

* British Patent 26709/1911.
† See Reference (63).
‡ British Patents 22931/1914, 5371/1915, 212199 252132, 252941.
§ British Patents 248855, 301586.
|| See Reference (47).

condensers the stator is sometimes located in the manner shown in Fig. 20. The stator blocks are clamped between cylindrical quartz tubes held in position between a top and bottom plate by several rods. The bottom plate supports the rotor on a ball centre, the shaft of this rotor being free to slide vertically in the top plate. This is a very unsuitable design from the point of view of stability. A very small difference in expansion between the stator block and the clamp rods gives rise to a large bending moment in the bottom plate, and the whole rotor will move in accordance with the deformation of this plate acting as a beam. In one well-known make of condenser the clamp rods are made of steel, the stator of aluminium, and the bottom plate of brass. The differential expansion alone gives rise to a vertical movement of the stator of 0·2 mil per deg. C., which for a 0·1-in. gap gives a temperature coefficient of + 42 parts in 1 million if the gaps are exactly equal. If the air-gaps are unequal by as little as 1 mil the coefficient is increased to about 100 parts in 1 million. Even if care is taken in selecting the materials, the large differences in the thermal capacities

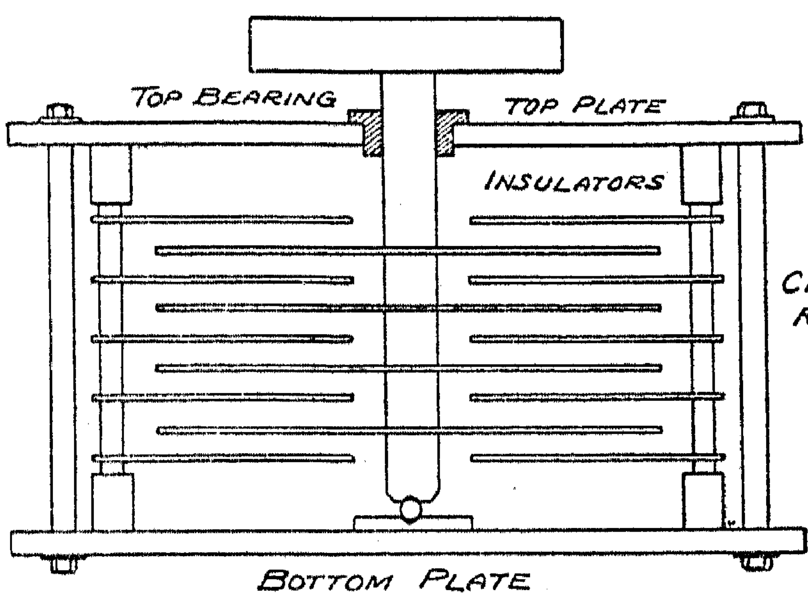

Fig. 20.—Typical assembly of condenser.

of the parts produce appreciable hysteresis effects, and the stress variations in the stator columns may produce mechanical constraint.

Various alternative means of constructing condensers have been suggested from time to time. The concentric-plate condenser* is attractive from the point of view of stability, owing to the fact that its capacitance is not critically dependent upon the location of the rotor axis, but unfortunately the capacitance for a given overall volume, and the ratio of the maximum to minimum capacitance, is much less than for a parallel-plate condenser. The first objection can be overcome to some extent by the use of multiple concentric semi-cylinders, and the latter by the use of telescopic tubes.† Hemi-spherical,‡ helical,§ and conical|| electrode systems have also been suggested and may be of value in future development work, but in all these devices the stability conditions already enumerated must still be satisfied. A type of concentric die-cast stator and rotor assembly in which the section of the interleaving vanes is triangular¶ offers distinct possibilities, since such a system is somewhat easier to construct than a parallel-vane system, but high accuracy of location of the various parts is required.

* British Patents 14834/1914, 212199, 212381, 249803, 251049, 268089, 86816, 325802.
† British Patents 8172/1902, 1855/1903, 218281, 228006 239751, 263948.
‡ British Patent 154386.
§ British Patents 191618, 271920, 299197 ,304583.
|| British Patent 268211.
¶ British Patents 26709/1911, 252132

A critical examination of such special methods of assembly has not yet been made, since the investigation is still in progress, but there appears to be no reason why such methods of construction should produce an appreciable improvement in electrical stability. If the requirements already stipulated are satisfied, a parallel-plate system should be quite satisfactory.

(9) METHODS OF COMPENSATION FOR TEMPERATURE-CHANGE

Means of providing automatic compensation for temperature variation have been developed both for solid- and for air-dielectric condensers. In all existing methods the compensating devices are appropriate to slow temperature variations, and are often unsatisfactory when the temperature variations are rapid.

(a) Condensers with Compensated Dielectric Material

In some of the simpler arrangements for fixed solid-dielectric condensers, use is made of compensatory electrical properties of the insulating material, but in the case of condensers built up of sheets of metal foil spaced by a solid dielectric the results are unsatisfactory. For instance, Condenser D (see Table 1) had a temperature coefficient of capacitance of + 160 parts in 1 million per deg. C., whereas the manufacturers claimed that its coefficient did not exceed + 10 parts in 1 million. Another published result of tests on " non-temperature-coefficient " paper condensers shows that for a resin-wax dielectric the temperature coefficient of capacitance was − 700 parts in 1 million per deg. C.*

If ceramic materials are used, a far better degree of compensation can be obtained. Rohde† shows how two condensers made of Calit and Condensa respectively may be connected in parallel to give zero resultant temperature coefficient, i.e.

$$\frac{\Delta C}{C} = \frac{\dfrac{\Delta C_1 C_2}{C_1} + \dfrac{\Delta C_2 C_1}{C_2}}{C_1 + C_2} \quad . \quad . \quad . \quad (23)$$

where C_1 and C_2 are the capacitances of the two condensers and ΔC_1 and ΔC_2 are the changes in capacitance for unit temperature-change. For compensation

$$\frac{\Delta C_1 C_2}{C_1} = - \frac{\Delta C_2 C_1}{C_2} \quad . \quad . \quad . \quad . \quad (24)$$

Such an arrangement has been used to compensate the positive temperature-coefficient of a coil by making

$$\frac{\Delta L}{L} = - \frac{\Delta C}{C} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where ΔL and ΔC are the changes in inductance and capacitance respectively for unit change of temperature, and $C = C_1 + C_2$.

This method is applicable only to cases where a stable oscillator is needed for a single frequency. Also, compensation obtained by the use of two condensers, each of which has a large temperature coefficient of permittivity, requires very high accuracy of adjustment; this is difficult to achieve in ceramic materials. For repro-

* See Reference (21).
† *Ibid.*, (56).

ducibility to a given design, the value of the permittivity and the temperature coefficient of permittivity of both dielectrics must be constant to an accuracy of at least 1 per cent, and it seems most unlikely that this can be achieved unless the chemical composition can be reproduced to a high degree of precision. Also, the time-lag and " ageing " of Calit is very small, whereas the lag of Condensa is large, from which it would appear that compensation is only effective for very slow rates of temperature-change, and this property of Condensa renders such a compensating system unsuitable for many purposes. Notwithstanding these difficulties, the properties of these materials are of great interest and their use may be justified in certain circumstances.

(b) Other Methods of Compensation

Other available methods of compensating air-dielectric condensers for temperature variation are as follows:—

(i) Use of invar plates and frame, to eliminate expansion effects.

(ii) Use of an invar frame with brass plates sliding in grooves to maintain the ratio area/air-gap constant.

(iii) Use of bimetallic expansion device, to vary the air-gap so as to compensate for the increase in the effective area of the plates.*

The first method has been advocated and appears very attractive. No measurements of the thermo-mechanical stability of invar have as yet been made, but it is well known from general experience that its secular stability is poor. Its electrical resistance is high, and to obtain a low series resistance it is necessary to deposit copper or other low-resistance metal on the surfaces. Such a type of construction is expensive and its merits are at present unknown. It is likely that considerable constraint effects would exist unless the stator and rotor were both machined from the solid. Although this procedure is not impossible, the cost would be high—about £5 for material for a small receiving condenser. Die-casting is a possible alternative.

The second method has been used, and tests on such a condenser (P, Table 2) show that compensation was not obtained—the coefficient of capacitance was + 20 parts in 1 million per deg. C. This was probably due to the constraint imposed by the sliding action of the plates in the grooves cut in the frame. This constraint effect is likely to be appreciable in any such design, and it seems inexpedient to pursue this principle of compensation.

The third method involves no great departure from normal design practice and is easy to apply to a condenser which satisfies the requirements enumerated in the previous section. It seems to possess the advantage of comparative simplicity with a high degree of stability at reasonable cost. The method has been used in two trial models.

(c) Experimental Compensated Variable Condenser No. 1

In the first of these experimental condensers, the object was to ascertain what degree of compensation could be obtained for slow temperature-changes, associated with a high degree of secular stability, regardless of size and cost. The assembly is in consequence somewhat massive and expensive, but the results show that a very high degree of capacitance-stabilization can be obtained.

The condenser is illustrated in Fig. 21. In order to reduce residual stress and constraint effects, the stator and rotor vanes were machined from a brass forging which was annealed several times during the machining process. The rotor (1) consisted of 10 semicircular vanes and a shaft, while the stator (2) consisted of 11 vanes which were approximately rectangular, together with a heavy brass web. The rotor shaft was supported by two plain bearings (3) which were bolted to two rigid supports (4), and end location was provided by a shoulder on the shaft and a spring as shown. The other bearing had no end location and the shaft was free to slide axially. The stator was supported at three points by means of ground spherical feet resting on a cone, a **V**-groove, and a plane, as shown at (6). This type of support gives precise location and eliminates stresses due to differential expansion between the stator and the base-plate. The three stator supports were insulated from the base-plate by small keramot washers which gave good mechanical location, though at some slight sacrifice of power factor.

The brass base-plate was machined as shown in the plan view, and the central portion supporting the stator was connected to the main frame by 4 thin brass beams (9). This construction permits the central mass to move slightly with respect to the main frame with only one degree of freedom, and eliminates the difficulties of providing sliding rails or guides for a stator carriage. The rigidity in the vertical plane is very great. The central portion was connected to the main frame by means of a steel rod (10) provided with the necessary clamping arrangements and adjustments whereby the effective length of connection to the main frame might be varied. Owing to the differential expansion between this brass frame and the steel rod, the stator carriage moves relative to the rotor supports with change of temperature.

By setting the rotor so that the air-gap δ_2 on one side of each vane was twice as large as the gap δ_1 on the other side, the increase in capacitance produced by the metal expansion could be compensated by a decrease in capacitance due to the increase in the small gap δ_1. Variation of the length of the steel rod provided a continuous adjustment of this compensating action. The brass base-plate was mounted on a heavy brass casting (8) by means of insulated bolts. A drum scale (12) and window was provided for setting the rotor position, the friction driving-shaft being taken through an oil-packed air-tight gland (13). The whole condenser was enclosed in an air-tight metal case to prevent changes of air density.

The action of the compensating mechanism is as follows:—

Let δ_1 = small air-gap spacing (see Fig. 21).
δ_2 = large air-gap spacing (see Fig. 21).
t = thickness of rotor or stator vane.
A = area of one face of rotor vane.

* See Reference (81) and British Patents 335164 and 335526.

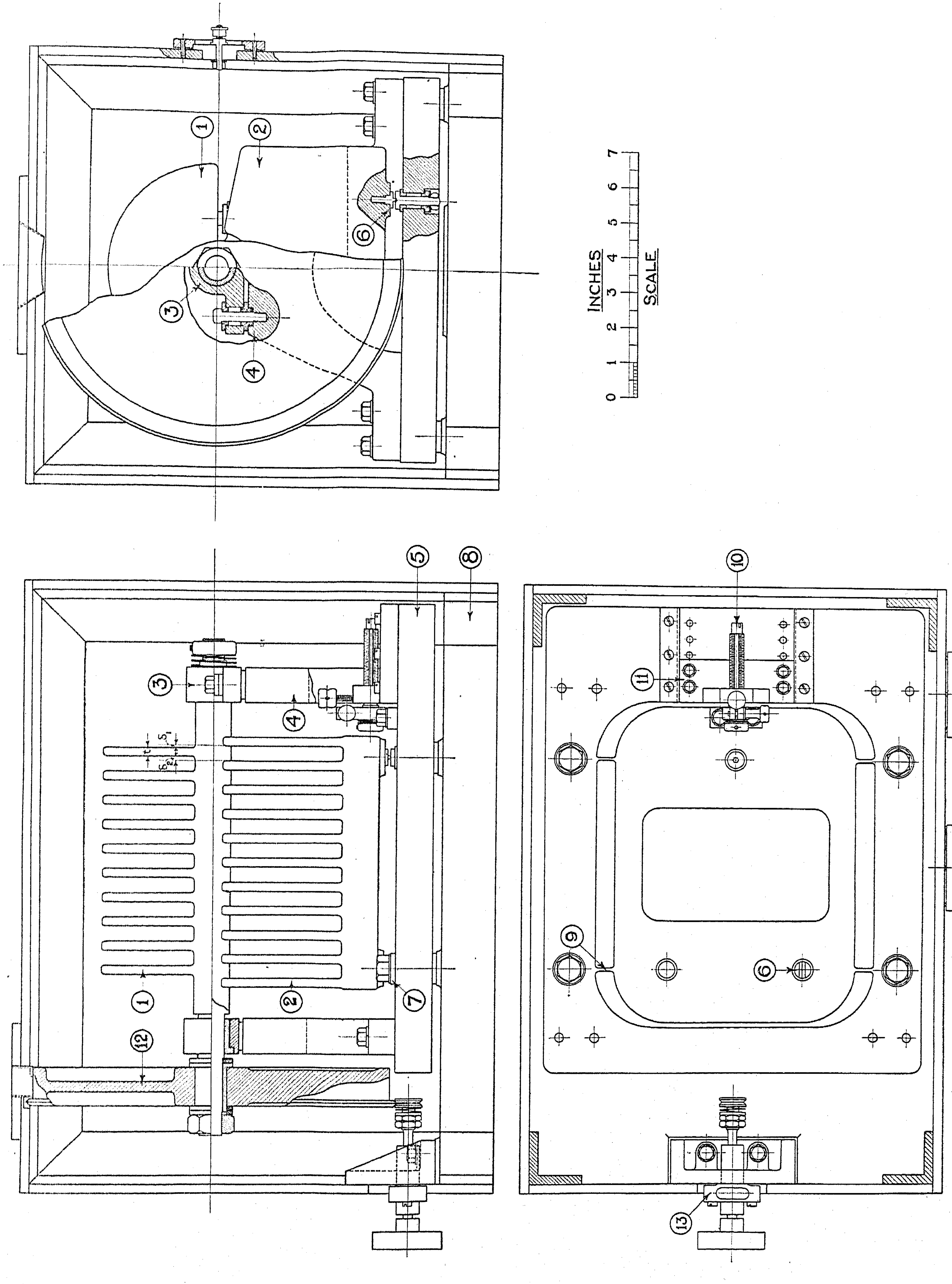

Fig. 21.—General assembly of compensated variable condenser No. 1.

α = coefficient of linear expansion of vane.
θ = temperature-change.
$\beta\theta$ = lateral movement of stator.

Then, owing to the linear expansion of the metal, the *increase* in the capacitance of the condenser formed by the two gaps δ_1 and δ_2 in parallel is

$$\frac{\alpha A}{4\pi}\left[\frac{1}{\delta_1}+\frac{1}{\delta_2}\right]\theta \quad . \quad . \quad . \quad . \quad (26)$$

The *reduction* in capacitance due to the motion $\beta\theta$ is

$$\frac{A}{4\pi}\left[\left\{\frac{1}{\delta_1}+\frac{1}{\delta_2}\right\}-\left\{\frac{1}{\delta_1+\beta\theta}+\frac{1}{\delta_2-\beta\theta}\right\}\right] \quad . \quad (27)$$

and since $\beta\theta << \delta_1$, or δ_2, this *reduction* in capacitance may be written as

$$\frac{\beta A}{4\pi}\left[\frac{1}{\delta_1^2}-\frac{1}{\delta_2^2}\right] \quad . \quad . \quad . \quad . \quad (28)$$

Taking the values of α and γ as $18\cdot9 \times 10^{-6}$ and $11\cdot6 \times 10^{-6}$ for brass and steel respectively, when $\delta_1 = \frac{1}{16}$ in., it is seen that l should be $0\cdot393$ in. Provision was made for adjustment of this length, since the small stresses imposed on the rod by the bending of the beams (9) produced an extension which could be compensated by a slightly increased operating length of the steel rod.

The thermal properties of the three relevant masses —stator, rotor, and frame—together with the data required to calculate the thermal properties of each part, are given in Table 10.

From these constants it can be shown that the heating curves for the three masses are as shown in Fig. 22 when the temperature of the surroundings is suddenly changed from 20° to 60° C. It will be seen that the stator and rotor are very nearly identical in thermal performance, but that the frame could be improved by reducing the ratio of its mass to its area. Since the temperature of the compensating steel rod is sensibly the same as that of the frame, it would be expected that a lag would take place between the temperature-changes of the stator and rotor and those of the compensating mechanism.

Table 10

THERMAL PROPERTIES OF STATOR, ROTOR, AND FRAME, OF TEMPERATURE-COMPENSATED VARIABLE AIR-CONDENSER

	Stator	Rotor	Frame
Mass (m), grammes	14 850	5 670	19 100
Total air surface (A), cm^2	3 650	2 100	2 150
Radiation constant $\frac{ms}{2eA\theta_1^3}$	$0\cdot44 \times 10^4$	$0\cdot29 \times 10^4$	$0\cdot96 \times 10^4$
Convection constant $\frac{4ms}{AC}$	$2\cdot23 \times 10^4$	$1\cdot48 \times 10^4$	$4\cdot87 \times 10^4$

Specific heat (s) = $0\cdot074$ (brass).
Emissivity (e) = 98 per cent (lampblack).*
Air temperature θ_1 = 293° K. = 20° C.

* The stator and rotor vanes were painted with lampblack to increase the emissivity coefficient.

Now for compensation

$$\beta\left(\frac{1}{\delta_1^2}-\frac{1}{\delta_2^2}\right)=\alpha\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}\right) \quad . \quad . \quad . \quad (29)$$

or

$$\beta=\left[\frac{\delta_1\delta_2}{\delta_2-\delta_1}\right]\alpha \quad . \quad . \quad . \quad . \quad (30)$$

In the present design $\delta_2 = 2\delta_1$ and consequently $\beta = 2\delta_1\alpha$.

The motion $\beta\theta$ is produced by the differential expansion between the brass base-plate and the steel rod (10). Let the effective length of this steel compensating rod be l and its expansion-coefficient be γ, which must be less than α. Then for a temperature-rise θ, the movement of the stator with respect to the rotor will be

$$l(\alpha-\gamma)=\beta \quad . \quad . \quad . \quad . \quad . \quad (31)$$

and hence

$$l(\alpha-\gamma)=2\delta_1\alpha \quad . \quad . \quad . \quad . \quad (32)$$

or

$$l=2\delta_1\frac{\alpha}{\alpha-\gamma} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The condenser was set up with the standard temperature-compensated coil,* as a series-fed Hartley oscillatory circuit. The leads were carefully arranged to be as short and rigid as possible, and the battery supply leads were taken to a remote-control point through an earthed metal tube. The frequency range of the circuit was found to be 3 000–6 000 kc, and the condenser was set to give a frequency of 4 000 kc.

The circuit was now enclosed by an oven in which the temperature could be automatically maintained at any value by a photocell-operated thermostat. The temperature was raised 30 deg. C. and the frequency-change was noted, from which result the correct adjustment of the compensating screw was determined, since it was known that the temperature coefficient of inductance of the coil was less than 1 part in 1 million per deg. C.

* See Reference (93).

The condenser having been set to this new condition, the oscillator was maintained at a temperature of 50° C. for several days and then three thermal cycles over the range 20–50° C. were applied. During this period slight ageing effects took place which became progressively less after each thermal cycle.

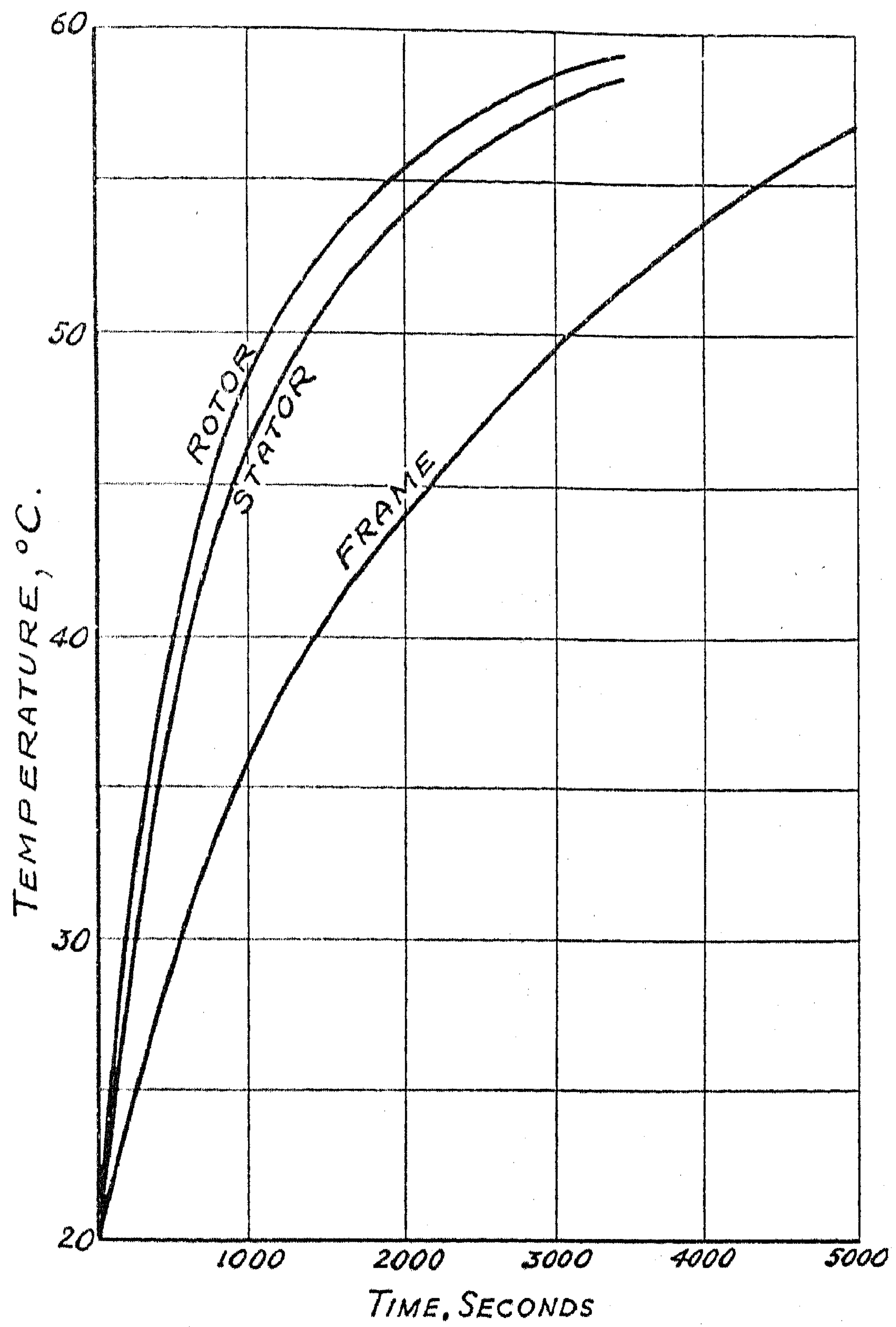

Fig. 22.—Thermal performance of compensated variable condenser No. 1.

Finally, a long-period test was undertaken to ascertain the frequency stability. The temperature was raised from 26° C. to 50° C. in steps of 1 deg. C., the temperature at each step being maintained for 2 hours. The highest temperature of 50° C. was maintained for 17 hours and then the circuit was allowed to cool naturally. The frequency variations are shown in Fig. 23, the dots giving the actual observations; the accuracy of measurement was about 5 parts in 1 million. It will be seen from Fig. 23 that there is no sign of a definite temperature coefficient and that the frequency remained constant to within ± 30 parts in 1 million during a period of more than 100 hours, during which time temperature variations of over 30 deg. C. took place. An increase of 10 per cent in the anode potential gave an increase in frequency of 9 parts in 1 million, and an increase of 10 per cent in the filament current gave a decrease in frequency of 16 parts in 1 million.

The results show that a high degree of frequency stabilization can be obtained by attention to details in the manner shown, but it is realized that the high stability of this condenser has been obtained by a partial sacrifice of some of the electrical properties. Quartz mounting has been attempted but has been found to be quite unsuitable, owing to the fact that precise location of the stator cannot be obtained. It is desirable to attempt to improve the electrical properties while still maintaining the present stable mechanical construction.

(d) Experimental Compensated Variable Condenser No. 2

The success obtained with the condenser described above suggested the production of a more practicable form of the same system, and consequently the essential

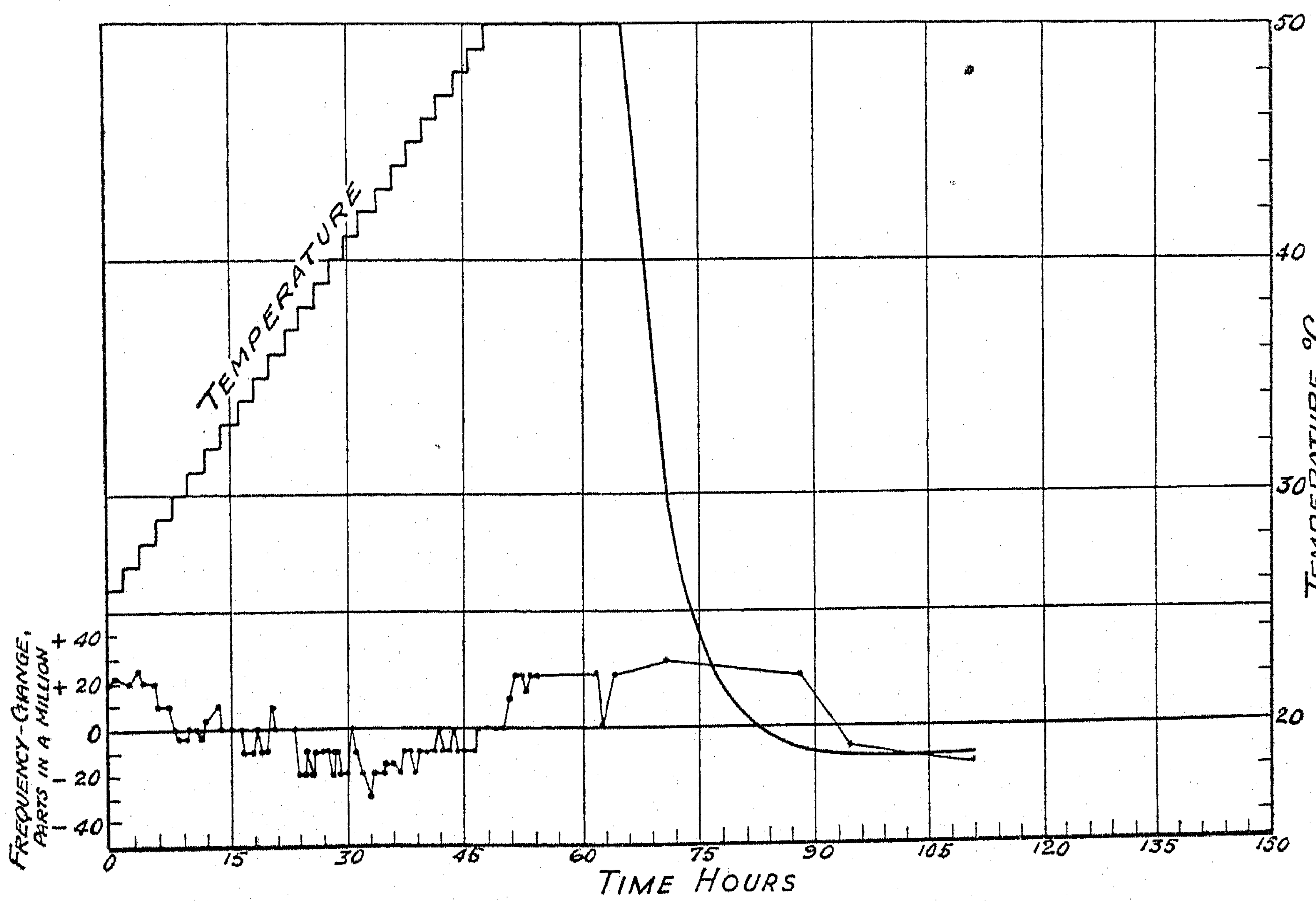

Fig. 23.—Effect of temperature on frequency of compensated circuit at 4 000 kc.

principles were incorporated in a second design illustrated in Fig. 24. In this case conventional practice was adhered to as far as possible, but certain important differences were necessary to satisfy the requirements already specified.

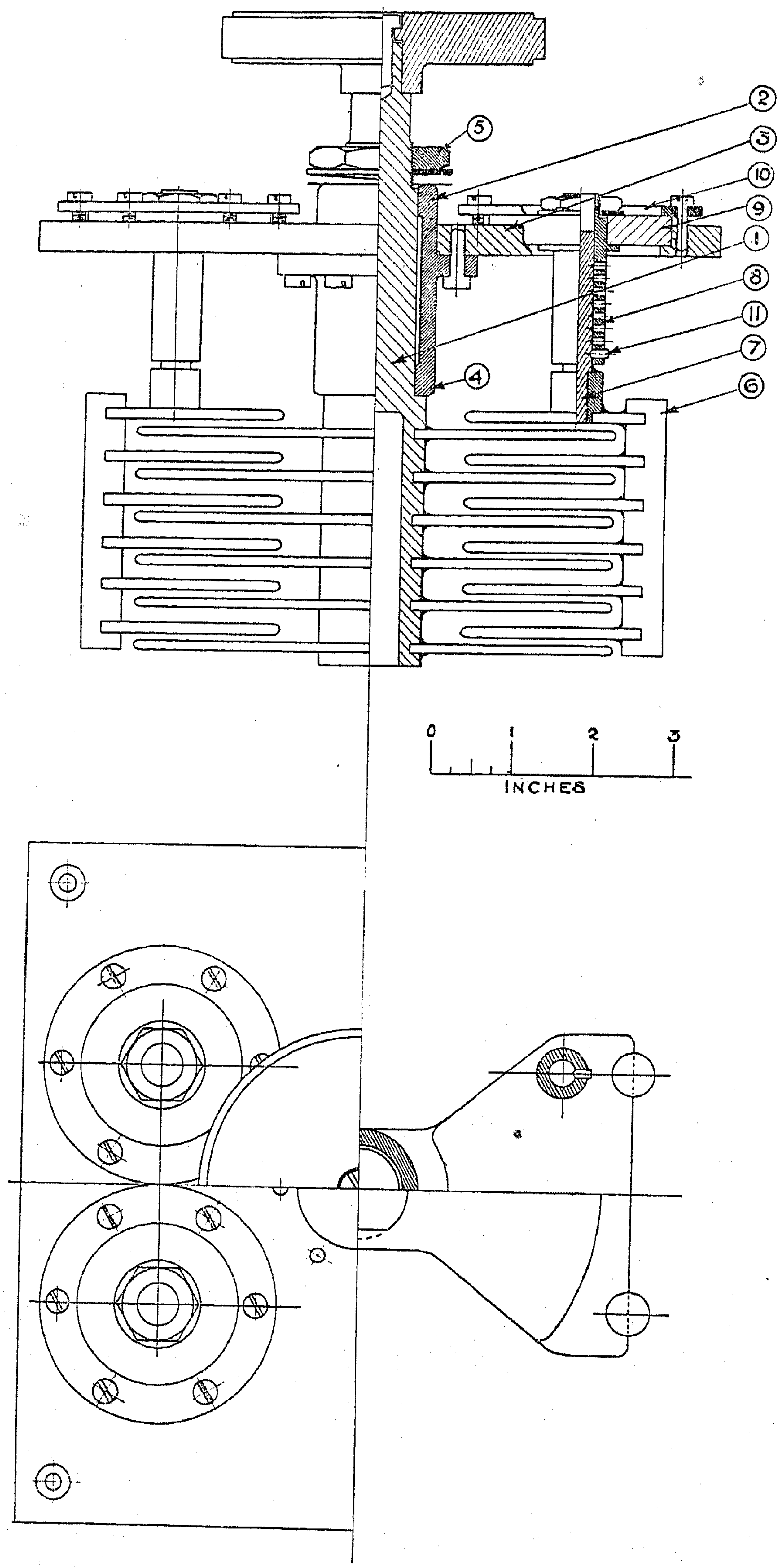

Fig. 24.—General assembly of compensated variable condenser No. 2.

All the metal parts of this condenser were machined from brass castings, and exactly the same composition was obtained in each part by pouring these castings from the same mix. Each part was well annealed between the preliminary and final machining operations. The rotor plates were let into grooves cut in the spindle (1) and soldered at the points shown. This spindle was located by the massive bearing (2) screwed to the top plate (3) and was registered by the face (4) and spring locking nut (5). The stator plates were similarly soldered into grooves cut in the spacing rods (6) and were located by means of four invar rods (7) clamped by the brass sleeves (8). These sleeves were in turn clamped to four marble insulators (9) located in the top plate and fixed by the rings (10). By means of the movable grub-screws (11) the length of the compensating rods (7) could be adjusted to give the required compensating action, the air-gaps on one side of the rotor plates being twice those of the other side as in the case of the previous condenser. This type of construction reduces constraint

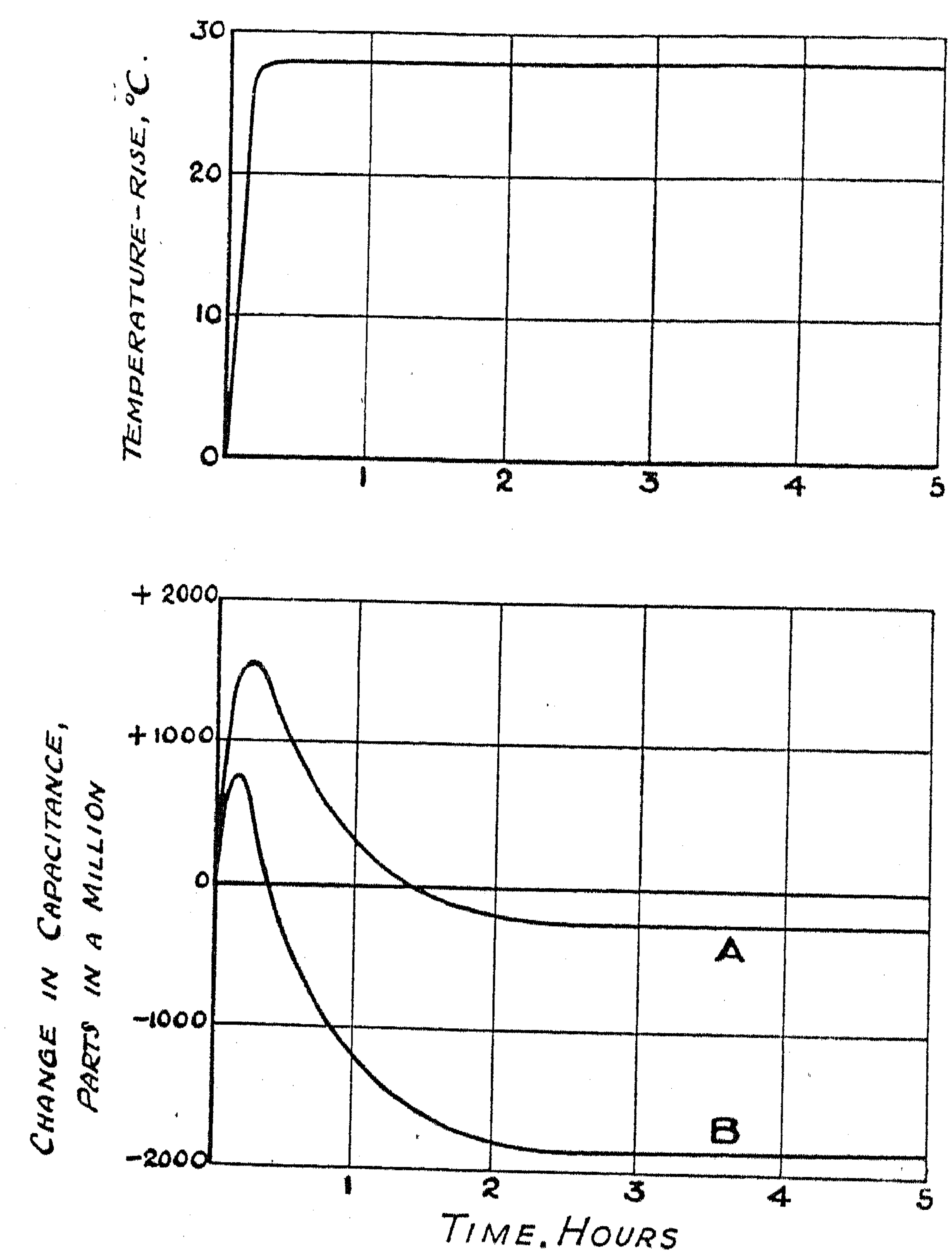

Fig. 25.—Performance of compensated variable condenser No. 2.

Atmospheric temperature = 24° C.

and residual stresses to a minimum and gives good stator location.

The performance of this condenser is shown in Fig. 25 for two settings of the compensating rods (7). In the first case, the length of each invar rod was 0·95 in., and curve B depicts the change of capacitance which occurred when a rapid temperature-change was made. It is seen that an increase of capacitance took place initially, and then a slow reduction occurred due to the compensating action. The final capacitance value was remarkably constant, and the net temperature-coefficient of capacitance was − 65 parts in 1 million per deg. C. In the second case, the length of the invar rod was 0·26 in. and the performance is shown by curve A: in this case, a similar behaviour was noted and the net temperature coefficient was − 8·6 parts in 1 million per deg. C. By the use of equation (33), it can be shown that these observed temperature-coefficients of capacitance agree

quite well with the calculated values for the two lengths of compensating rod; the theoretical values are $-77\cdot4$ and $-7\cdot3$ parts in 1 million per deg. C. Exact compensation was not possible in this experimental condenser, since a sufficiently short length of invar rod was not provided for in the design, but it is obvious that such a condition could be obtained by a slight lengthening of the brass sleeves (8).

When this condenser was subjected to variations of temperature at a constant rate of 2 deg. C. per hour, the capacitance variation did not exceed ± 40 parts in 1 million for a total temperature-change of 9 deg. C., showing that the initial increase of capacitance observed in the previous cases was due to the rapid rate of temperature-change. The performance of this condenser was remarkably cyclic; a large number of tests were made and almost perfect agreement was obtained, a result never before achieved. The secular stability is likely to be high and the performance is calculable and reproducible, but the thermal requirements have not been completely satisfied.

It appears, therefore, from the results obtained on these condensers that compensation for slow temperature-change can be obtained, together with a high degree of secular stability. The problem of providing compensation for any rate of temperature-change involves further study and will be considered in the near future.

(10) ACKNOWLEDGMENTS

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APPENDIX 1

Change in Permittivity of Air due to Changes of Temperature and Pressure

$$\kappa = \frac{1 + 2c_1d_1{}^*}{1 - c_1d_1} = 1 + 3c_1d_1 + 3c_1^2d_1^2 + \ldots$$
$$= 1 + 3c_1d_1(1 + c_1d_1) + \ldots$$

where d_1 = air density $= p/(1 + 0\cdot003665\tau)$,
$c_1 = 195 \times 10^{-6}$,
p = pressure (atmospheres),
and τ = temperature (°C.).

If p is constant, a temperature-change of 1 deg. C. corresponds to a density change of $3\,665 \times 10^{-6}$. The change in κ is therefore

$3 \times 195 \times 3\,665 \times 10^{-12} = 2\cdot15$ parts in 10^6 per deg. C.

APPENDIX 2

Method of Measuring Deformation of Specimens when Subjected to Changes of Temperature

Most designs of condenser are constructed of metal and insulating material in sheet form, and to study the thermo-mechanical properties of such sheets it is necessary to measure the deformations other than normal thermal expansion produced by temperature variations. It is convenient to use a standard rectangular form for all specimens and to treat each specimen as a beam. Such specimens are not initially flat and consequently it is necessary to measure either the actual deformation at the centre when the ends of the beam are definitely located or the change in curvature of the beam. It is essential to measure the deformation of the specimen without imposing any external bending moment, and it is also important to ensure that no appreciable temperature-gradient exists at any time during the test. The first condition can best be satisfied by resting the plate on three steel balls which are free to roll. This gives precise location and complete freedom to distort in any direction. Although it is not difficult to measure small deformations at constant temperature, it has been found exceedingly difficult to obtain accurate results when the recording apparatus is subjected to temperature variation, owing to the difficulties of eliminating distortion from the measuring apparatus.

An attempt was made to measure the distortion at the centre of such a beam by an electrical method which consisted essentially of fixing a light condenser plate to the beam centre and measuring the change in capacitance between this plate and a fixed vane by a beat-frequency system. Unfortunately, difficulties of locating the fixed plate arose and ambiguities occurred due to the unknown change of permittivity with temperature of the insulating material supporting this plate. The method was abandoned and various photocell systems were tried. The idea adopted in these cases was to allow the moving centre of the beam to vary a gap through which light was transmitted and to compare the illumination with that given by an adjustable gap, photocells being used as the measuring instruments. These arrangements were found to be unsatisfactory, owing to the fact that the stability of photocells is insufficient for measurements of this accuracy. In another arrangement, use was made of a rocking mirror of which one end was located on the centre of the specimen and the other on a fixed support, observation of the angular shift of this mirror being made by a collimated beam and telescope. The sensitivity was found to be satisfactory, but difficulties arose from the fact that this method measures the relative movement between the top face of the specimen and a fixed point. Although the location of this point was made satisfactory after many attempts, it was found that the indentation produced by the weight of the beam on the surfaces of the balls altered with temperature and the observed deflection at the centre did not give a true measure of the actual change of beam curvature. This indentation amounted to many millionths of an inch, and since it was necessary to obtain an accuracy of the order of 10^{-6} in., the method had to be abandoned.

After the failure of all these methods it was realized that it was difficult to measure accurately the deflection of the centre of the beam on account of the vertical movement of the specimen produced by ball-indentation and expansion of the supports. In accordance with this view a further method has been developed and has proved to be entirely satisfactory, although a little

* See Reference (92).

sensitivity has had to be sacrificed. In this method the change in curvature of the beam, produced by heat, is measured, and the deformation at the centre is determined on the assumption that the distorted form is an arc of a circle. For the very small deflections under consideration, the change produced by assuming any other reasonable distorted shape is negligible and the dimensions of the recording apparatus have been so chosen that the deformation calculated on this assumption is quite accurate.

The apparatus is shown schematically in Fig. 26. The sheet specimen is placed on two steel balls at one end which are located in countersunk holes cut in a " Rayax " brick mounted on three adjustable feet. (" Rayax " is a special refractory material of high stability and low coefficient of expansion which gives good vertical location.) The support at the other end consists of a similar steel ball which is placed on a glass plate. In this manner the constraint imposed by the supports on the expansion of the plate is reduced to a negligible amount. The dimensions of the standard rectangular test specimen and the ball supports are shown in Fig. 7. Two light glass mirrors stand on the top face of the specimen, the location being defined by three ground feet 1 mm square. The inner surfaces of these blocks are made optically flat and act as mirrors, and temperature variation produces no measurable distortion of the blocks. Observation of any angular change between these surfaces is made by means of a collimated light beam and telescope with micrometer eyepiece. The great advantage of this system is that any movement of the mirrors which does not alter the angle in the vertical plane between the mirror surfaces is unobserved, and consequently ball-indentation, support-expansion, and specimen-expansion effects, are all eliminated. To reduce temperature-gradient effects as far as possible, very thin tinfoil to prevent the oil from coming into contact with the material. The foil is so thin that it has no effect on the distortion of the plate.

APPENDIX 3

Effect of Dielectric Loss upon Effective Resistance of a Radio-frequency Circuit

The effective resistance of a typical oscillation circuit at frequencies of the order of 100 megacycles per sec. is between 5 and 10 ohms, and this value remains reasonably constant over the range 1–100 megacycles per sec. Taking a value of 5 ohms as a typical circuit resistance, the permissible values of the condenser loss-angle δ for various frequencies and values of capacitance to produce a 10 per cent increase in circuit resistance are shown in Table 11.

Table 11

VALUES OF PERMISSIBLE LOSS-ANGLE TO GIVE 10 PER CENT INCREASE IN CIRCUIT RESISTANCE

Capacitance	$10^4 \tan\delta$ (δ = arc tan $R\omega C$) Frequency, megacycles per sec.		
	1	10	100
$\mu\mu$F 10	0·314	3·14	31·4
100	3·14	31·4	314
1 000	31·4	314	3 140

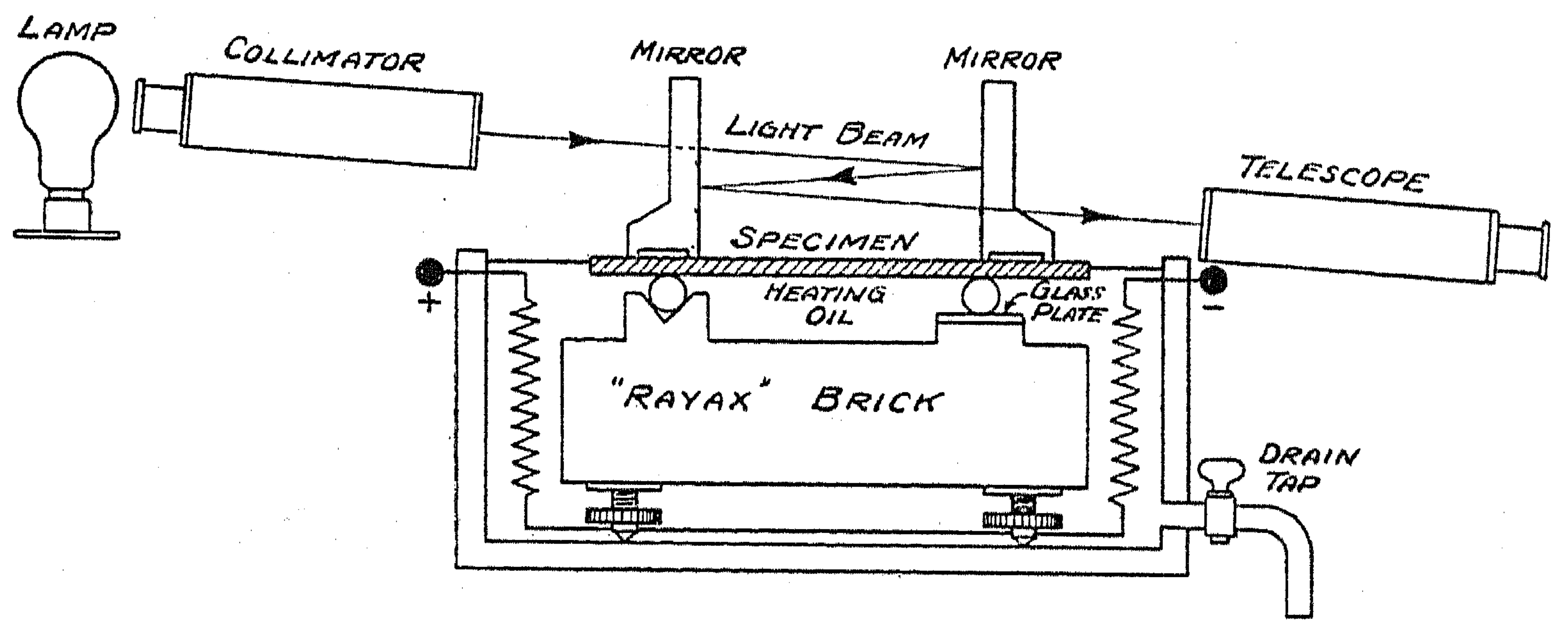

Fig. 26.—Arrangement of apparatus for measuring deformation of sheet specimens.

the specimen is heated by a surrounding oil bath, the level of the oil being just below the top surface. Transformer oil is used and is heated by two mats immersed in the liquid. Rapid cooling can be obtained by draining off the liquid so that the level falls well below the test specimen.

A movement of 3×10^{-6} in. at the centre of the beam produces a shift in the image corresponding to one division on the micrometer eyepiece. The minimum observable shift is about 10^{-6} in. The specimen is heated from 15° C. to 60° C. in about 10 minutes and is cooled in about the same time. When insulating materials are being tested, the specimen is wrapped in

It is seen from Table 11 that the dielectric loss must be small for small condensers at medium frequencies, but that for values of the capacitance of about 100 $\mu\mu$F at frequencies of 10 megacycles per sec. and above, a value of $\tan\delta = 20 \times 10^{-4}$ is low enough for most purposes.

APPENDIX 4

Rate of Change of Temperature of Thermally Conducting Body Cooling by Radiation only

Consider the cooling curve of a hot body at temperature θ_2 cooling to its surroundings at θ_1. Assume perfect thermal conductivity within the body and no

loss of heat by convection. Then, by Stefan's law, the heat passing through the surface in time dt is

$$eA(\theta^4 - \theta_1^4)dt$$

where e = emissivity, A = area, and θ = absolute temperature at time t.

This must equal the heat lost by the body, namely

$$ms\left(-\frac{d\theta}{dt}\right) dt$$

where m = mass, and s = specific heat.

Therefore $$\frac{eA}{ms}dt = -\frac{1}{\theta^4 - \theta_1^4}d\theta$$

or $$adt = -\frac{-d\theta}{\theta^4 - b^4}$$

where $a = eA/(ms)$, and $b = \theta_1$.

Now $$\frac{-d\theta}{\theta^4 - b^4} = \frac{1}{2b^4}\left[\frac{1}{1 + \theta^2/b^2} + \frac{1}{1 - \theta^2/b^2}\right]d\theta$$

Therefore, integrating both sides, and remembering that $\theta/b > 1$, we get

$$2ab^4t = b \operatorname{arc\,tan}\frac{\theta}{b} + b \operatorname{arc\,coth}\frac{\theta}{b} + K$$

and, since $\theta = \theta_2$ when $t = 0$,

$$t = \frac{ms}{2eA\theta_1^3}\left[\left(\operatorname{arc\,tan}\frac{\theta}{\theta_1} + \operatorname{arc\,coth}\frac{\theta}{\theta_1}\right) - \left(\operatorname{arc\,tan}\frac{\theta_2}{\theta_1} + \operatorname{arc\,coth}\frac{\theta_2}{\theta_1}\right)\right]$$

APPENDIX 5

Rate of Change of Temperature of Thermally Conducting Body Cooling by Convection only

Consider the cooling curve of a hot body at temperature θ_2 cooling to its surroundings at θ_1. Assume perfect thermal conductivity within the body and no loss of heat by radiation. Then the heat passing through the surface in time dt is

$$CA(\theta - \theta_1)^{5/4}dt$$

where C = convection coefficient, A = area, and θ = absolute temperature at time t.

This must equal the heat lost by the body, namely

$$ms\left(-\frac{d\theta}{dt}\right) dt$$

Therefore $$CA(\theta - \theta_1)^{5/4}dt = -msd\theta$$

or $$\frac{AC}{ms}dt = \frac{-d\theta}{(\theta - \theta_1)^{5/4}} = -\frac{d(\theta - \theta_1)}{(\theta - \theta_1)^{5/4}}$$

Therefore, integrating both sides, we get

$$\frac{AC}{ms}t = 4(\theta - \theta_1)^{-\frac{1}{4}} + K$$

and, since $\theta = \theta_2$ when $t = 0$,

$$t = \frac{4ms}{AC}\left[\frac{1}{(\theta - \theta_1)^{\frac{1}{4}}} - \frac{1}{(\theta_2 - \theta_1)^{\frac{1}{4}}}\right]$$

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[The discussion on this paper will be found on page 336.]

DISCUSSION BEFORE THE WIRELESS SECTION, 6TH MAY, 1936, ON TWO PAPERS BY MR. THOMAS*

Mr. F. M. Colebrook: I should like to emphasize the fact that certain aspects of modern radio engineering involve an extremely high degree of precision. For ordinary engineering practice an accuracy of 1 in 1 000 is quite good enough in nearly all cases, but in certain cases of radio communication—for example, in working at 30 megacycles per sec. with a tolerance of ± 100 cycles —the precision involved is of the order of parts in a million. It is for this reason that, in his investigation of the causes of frequency-variation, the author has had to carry out measurements to a quite unusual degree of refinement and precision and has had to improvise and invent special methods capable of the required degree of precision.

Table 8 of the second paper gives the results of some work on the mechanical properties of certain materials used in the construction of condensers. It is surprising to note that a specimen of quartz, in the form of fused silica, is described as non-cyclic in shape variation, with respect to thermal changes, with no return to the original shape on cooling. This is a very important matter, as fused silica is now being used as a former for standard inductances, and is likely to be used increasingly for that purpose in the future. It must be pointed out that the measurements refer to a single specimen, the only suitable specimen available at the time the measurements were made. The results cannot, therefore, be assumed to be representative, but they serve to show that fused silica cannot safely be assumed to be cyclic in thermal behaviour, and should be tested in this respect before being used in the construction of standards.

Mr. A. J. Gill: In the first paper the author refers to the possible application of his standard inductances to transmitters. As far as my experience goes, if one wants stability in a transmitter it is necessary to employ very low power in the originating oscillator, and if possible to use some method of frequency doubling. It is almost impossible to stabilize at a high-power stage and yet get good constancy of frequency in the emitted wave.

Regarding the question of power-supply variations, I should like to know whether the author's results were obtained with a dynatron circuit, and, if so, what power was being used. Experience seems to indicate that one must employ very little power; and if this is done, it is not very difficult to put the condenser and the coil in a thermally-controlled compartment.

There is another aspect of this question, namely, that if one attempts to use an *LC* drive for a transmitter one has to face the problem of adjusting the condenser to the required accuracy. To adjust a variable condenser to an accuracy of 2 or 3 parts in 1 million is almost an impossible mechanical feat. This is one of the difficulties of utilizing the high degree of stability mentioned by the author, even if one can get it.

With regard to the non-cyclic behaviour of these materials, as far as metalwork is concerned non-cyclic behaviour is a sure indication that the material has some initial stress in it; and it would have been rather interesting if some of the author's specimens had been cut out of the solid material.

The coil which the author showed as the final result of his investigation is very interesting, but it is subject to the disadvantage that it has only one turn per layer, which would make it very bulky for a given inductance. It rather reminds me of a design which was developed by one manufacturer, in which a copper spiral coil was cut from a flat sheet of material and then riveted to a mycalex sheet so that it could not move except with the mycalex. If this method could be applied to the author's coil, one might be able to arrange it in two layers, one of which would move in relation to the other and produce the compensation.

Mr. N. Lea: The author emphasizes the importance of avoiding stresses in the structure either of an inductance or of a condenser, and I would suggest that in order to obtain the stabilities which we require we must avoid approaching the elastic limit much more definitely than would an engineer in other branches of his work. Roughly speaking, I suspect that a stress must be not more than about one-tenth of that inferred from the normally understood elastic limit. This point is of great importance in a design such as that of the condenser described in the second paper, where it is necessary to consider questions of internal stresses even in such details as the attachment of the pillar to the insulator used at the extremity of it. Obviously, if one squeezes a brass washer on to a piece of mycalex, for example, and subjects the combination to a change of temperature, the brass washer will creep radially (with doubtful symmetry) inwards and outwards; and it may be that over quite an ordinary change of temperature the skidding stresses and shear stresses are very serious indeed.

Some years ago I was bothered with a design produced by somebody else, which a third party had tried to push to a precision far beyond that originally intended by the designer. Trouble resulted, and I found that this was largely due to the very serious internal stresses. When these were removed by a change in materials and a proper regard for some of the principles outlined by the author, I found very close adherence to a temperature coefficient of 30×10^{-4} to 35×10^{-4} per deg. C. for an *LC* circuit consisting of copper for the inductance and brass for the whole of the condenser parts.

Has the author evolved any ratio as between the temperature coefficient which one would expect from the linear primary expansion of the material only and the actual one which is obtained as a result of the secondary effects which he mentions? On many occasions, it appears, one may encounter a temperature coefficient of about double the linear value.

In most of his experiments the author seems to have employed designs having no screens closely associated with them. For commercial purposes we must include screens in the design, even though these complicate the thermal analysis. My experience is that the screen often constitutes a major thermal difficulty.

I should have liked to hear more about the methods of design which are possible in reducing frequency

* "The Stability of Inductance Coils for Radio Frequencies" (see vol. 77, p. 702); and "The Electrical Stability of Condensers" (see page 297).

perturbations during temperature-changes. The use of invar for compensating purposes is helpful because it means that the designer can almost forget the rate of change in temperature of the compensating rod in comparison with the device which that rod is attempting to compensate.

Dr. L. Hartshorn: The author has tackled the question of inductance stability from the point of view of the frequency stability of an oscillator circuit. At the National Physical Laboratory we also have to tackle the question from the point of view of inductance standards, and from this aspect the order of accuracy dealt with by the author is less significant, for we cannot, in general, measure an inductance to 20 parts in a million. Further, the author is aiming at producing an oscillatory circuit which will maintain its frequency constant over the range 15°–50° C.; that is to say, he is considering tropical temperatures. At Teddington we are not considering tropical temperatures, and we never let any of our standards exceed a temperature of 30° C. We are only concerned with producing a standard which will be stable over a very small range of temperature, e.g. 10°–25° C., and I think this is also the field covered by manufacturers of standard coils.

The temperature coefficients which the author obtained with his representative coils are very much higher than the coefficient of expansion of the metal; and it might be thought curious that bodies like the N.P.L. should not have noticed this before, as they have been making measurements to an accuracy of a few parts in 100 000 for many years. According to the records available, the temperature coefficients of inductances measured over the small range of temperature I have referred to are usually about 23 parts in a million; that is, for uncompensated standards wound of copper wire. Most of them are wound with many layers of wire on marble bobbins, but some have spaced windings on keramot formers. The coefficient of expansion of the metal is 17 in a million, and we have always regarded the measured coefficient of 23 in a million as being equal to this coefficient of expansion, plus an extra 6 parts in a million due to internal stresses. This rule has so far been found good enough for most purposes. It is necessary to remember, however, that the conditions in these standards are quite different from those in most short-wave coils. The former are all wound with thin wire, usually stranded. Frequently the coil is immersed in oil, and after the winding process it is nearly always raised to a temperature of about 110° C. and kept at this temperature for a considerable time, with the idea of securing whatever measure of annealing is practicable, bearing in mind that silk-covered wire is used. There is no doubt that such annealing does get rid of a very considerable amount of strain in the wire; I have known annealing produce a change of inductance of 1·5 parts in 1 000. The temperature coefficient of 23×10^{-4} per deg. C. has been found applicable to coils of large diameter with fairly heavy wire and to coils of small diameter with fine wire.

Turning to the second paper, we get a similar difference in outlook on the subject of condensers. When we approach a variable condenser in the ordinary way, what we understand by stability is the accuracy with which we can reproduce a given capacitance by setting the condenser to a given reading. Such an interpretation introduces problems which are rather different in some ways from those mentioned by the author. The chief point is that the bearing becomes of fundamental importance; the author has been able to leave the bearing out of consideration because he is going to set his frequency to a standard wavelength. In the more general problem we cannot assume that an external reference standard of this kind will be available, and it therefore frequently happens that the factors studied by the author are less important than others such as the bearing, ease of setting and reading, the scale law, and irregularities of scale. When designing a condenser a compromise usually has to be made, and the factors that are allowed to dominate any particular design will depend on the use for which the instrument is intended. In this connection it should be noticed that the design of Fig. 20, though deservedly criticized by the author from his point of view, gives the lowest dielectric losses for a given insulating material and a given mechanical rigidity, and is therefore very satisfactory for many purposes. The faults mentioned by the author should of course be corrected, provided the economics of the problem will permit this to be done.

There is only one other point I would mention, and that is the difficulty the author had in finding any power-factor value which could be regarded as representative of named materials. He was inclined to think that the trouble lay in the methods of test, but I feel that it is more likely to be due to differences of manufacture in the products mentioned. I certainly have come across samples of Pyrex glass with power factors as low as the best he has mentioned, and I have also come across samples with power factors worse than the worst he has mentioned; and similarly with regard to the ceramic materials. I believe there are similar variations among the natural dielectrics. We have also found that samples of the newer ceramic materials give different values when measured in different conditions, e.g. with electrodes of different sizes. There is little doubt that these materials are not always homogeneous.

Mr. D. A. Bell (*communicated*): There are one or two points in connection with the method of measurement described in the first paper on which I am not quite clear. Since there are lead inductances L' of considerable size which are not contained within the constant-temperature enclosure of the reference tuned circuit, it becomes necessary to measure accurately the frequency both with the test inductance in circuit and with it short-circuited for every observation of its inductance value. But if the use of the reference circuit no longer guarantees a known frequency with the test inductance short-circuited, what is the advantage of using a reference circuit, complete with inductance, rather than a mere reference condenser? The disadvantage of including the inductance is that it reduces the sensitivity by a factor of the order of 2, since the 50-μH branch of the circuit containing the variable inductance is shunted by a constant 68 μH.

Next, with regard to maintaining the centre-point earth on the inductance, it seems to me that the reduction in earth-capacitance effects resulting from this precaution

can at most amount to a ratio of 2 : 1. For example, we may consider as a simplified case a coil with an earthed metal sheet at an equal distance from each end, the effective capacitance to earth from each end being C_0. Then with a centre-point earth we have the arrangement of Fig. A, where the effective capacitance across the whole circuit consists of the two elements in series, i.e. it is $\frac{1}{2}C_0$. Taking the other extreme (one end earthed), only one capacitance is active, so that the effective capacitance thrown across the circuit is C_0. Since in the author's experiments there were no heating circuits around the test oven, it could perhaps have been unscreened and constructed of insulating materials, thus minimizing the capacitances to earth from the test inductance; was this actually the case?

Since the publication of Mr. Thomas's paper on inductances, it has been suggested by Groszkowski* that the temperature coefficient of inductance is affected by temperature change of resistance owing to the fact that the current distribution in the conductor at radio frequencies is a function of specific resistance. As this would tend to account for the difference between the value of 23 in 10^6 usually found by Dr. Hartshorn at telephonic frequencies and the various larger values

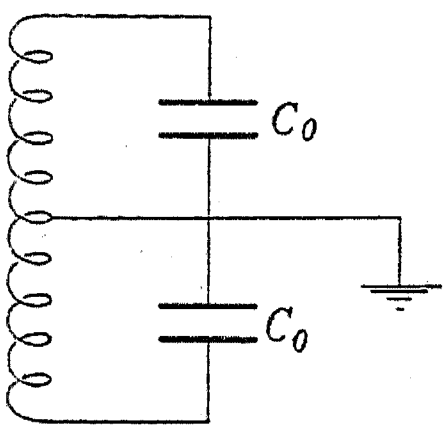

Fig. A

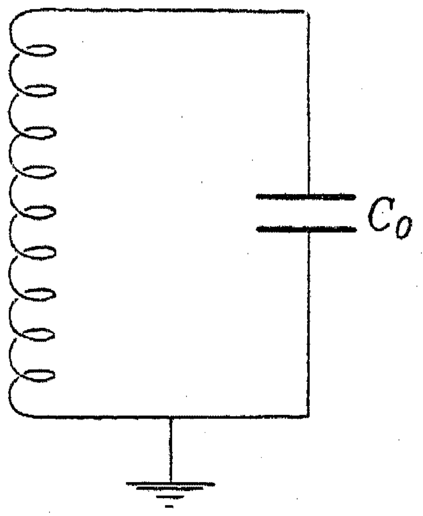

Fig. B

measured by Mr. Thomas at radio frequencies, I should like to know whether he has calculated the possible magnitude of such an effect for any of the coils he has measured.

With regard to the second paper, I should like to ask why the "ideal" condenser was designed as shown in Fig. 4, instead of following the usual guard-ring principle where the single plate covers the whole of the divided plate, so that the field over the the protected central portion is uniform. Had this principle been followed, it would have been possible to study directly the change of fringe capacitance as the gap was changed, a variation which can only be deduced from Table 4 by comparing the measured values for C_2 and C_2' with those calculated for a uniform field; the difference is from about 5 per cent to 13 per cent, being greatest, of course, with wide gaps.

Mr. Gill suggests that a stable oscillator must almost inevitably be of low power; but since the temperature-rise of the tuned circuit can be kept as low for a high-power oscillator as for a low-power one by making the components large enough to have the same ratio of cooling surface to energy dissipation, there appears to be no justification for this assertion on thermal grounds. To minimize change of frequency with supply-voltage variation, however, the efficiency of the oscillator may have to be sacrificed: stability demands amongst other things a lightly-loaded tuned circuit having a steep phase-angle/frequency characteristic at resonance, and a tolerably sinusoidal generated voltage, both of which are opposed to high efficiency. This lack of efficiency may make it impracticable to generate at very high power; but it should be feasible to construct a stable oscillator with a d.c. input of 100 watts and radio-frequency output of 30 watts.

* *Wireless Engineer*, 1935, vol. 12, p. 650.

Mr. W. H. F. Griffiths (*communicated*): The title of the paper on inductances should have indicated that the paper dealt only with coils for short-wave transmitters, for the author's selection of coils for test and his own ultimate design were limited to a few microhenrys. Moreover, the drastic treatment during test was justified only if the coils subjected to it were intended for use ultimately under similar conditions where the temperature-changes were to depend upon the heat generated by coil losses rather than upon changes of ambient temperature.

As the title does not indicate the narrow limits which are obviously intended, I should like to mention that no reference is made to the performance of my design of thermal-compensated inductance coil. Such coils, however, might not behave at their best under the extremely drastic test conditions which the author imposed—conditions which would never be even approached in practice with coils properly used as standards, the purpose for which they are intended.

I should also like to point out that the author was evidently under a misapprehension regarding the design of the two coils M and N of Table 1 on page 708 (vol. 77). These coils were of a relatively simple and inexpensive mass-produced type in which no attempt is made to obtain the correct ratio of axial to radial expansion or to obtain good location of the conductor. Both coils would, in my opinion, have behaved much better under normal laboratory conditions.

I am very interested in the second paper, because for many years I have been working on the problem of the electrical stability of condensers and have succeeded in producing a stable type of mica condenser in which the temperature coefficient of capacitance between 15° C. and 30° C. does not exceed $\pm$ 20 parts in 10^6 per degree C. for all capacitances, and is the same whether tested at audio or radio frequencies.

With regard to air condensers, both variable and fixed, I am able to obtain regularly temperature coefficients of capacitance under 10 parts in 10^6 per degree C., and in special cases under 5 parts in 10^6. Careful workmanship, which has, moreover, to be entrusted to selected craftsmen of long experience in work of this type, is essential in the execution of the designs.

The lowest "natural" temperature coefficient which I have experienced without any intentional compensation is + 13 parts in 10^6 per degree C. It is obtained on a quartz-insulated fixed air condenser of 1 000 $\mu\mu$F capacitance consisting of a fairly massive structure with plates, washers, and other parts, of similar material. I agree with the author, however, that with insufficient experience in condenser design it is quite easy to obtain temperature coefficients of a very high order. I recently

tested a well-known precision condenser of the type mentioned by the author on page 325 and found its temperature coefficient to be + 130 parts in 10^6 per degree C., thus confirming his estimate of the possible augmentation of coefficient due to air-gap inequality.

Mr. B. Williams (*communicated*): A knowledge of the effects of temperature on the inductance of coils and the capacitance of condensers is of paramount importance in the design of transmitters and receivers for use in military aircraft, which may be subject to a difference in temperature between ground and air of about 70 deg. C. A combined temperature coefficient of inductance and capacitance of the order of 30 parts in 10^6 will give rise to a change of 10 kc in a working frequency of 5 megacycles per sec. This frequency-change puts serious limits on the number of available channels and necessitates retuning of the aircraft and ground receivers. Owing to the small permissible size and weight it is not possible to use compensated coils and condensers of the type described in the paper, and for the same reason the use of a very small-power master-oscillator drive and numerous amplifying stages is not possible. The need for flexibility of working frequency precludes the use of crystal control.

For transmitters, various types of compensated coils operating chiefly on the bimetallic principle have been tried, but in no case has a satisfactory mechanical design combined with cyclic properties and low or zero temperature coefficient been obtained. In this connection commercial temperature-compensated coils have been examined, but their behaviour has in general been found inferior to that of ordinary coils as regards both cyclic properties and temperature coefficient. The usual type of coil used in small-power short-wave aircraft transmitters is that referred to as Coil B in Table 1 of the first paper. This coil, which is of the variable-inductance type, is cyclic and has a temperature coefficient of about 20 parts in 10^6. The author's figures confirm measurements made previously at the Royal Aircraft Establishment. Other coils used in aircraft transmitters show temperature coefficients of the order of 30–40 parts in 10^6.

The effects of temperature on condensers have been found to be much more serious. Small fixed mica condensers of the type used in broadcast receivers have a temperature coefficient of the order of 2 parts in 10^3 and are usually non-cyclic. Variable air condensers have differing temperature coefficients according to their size and construction. The thickness of dielectric, the centrality of the moving-plate system with respect to the stator, the insulating material, and the method of securing the stator to the framework of the condenser, all have widely differing effects. A laboratory standard condenser usually has a temperature coefficient of between + 35 and + 45 parts in 10^6. A condenser such as is used in the tuning circuits of a receiver has a positive coefficient of about 100 parts in 10^6. Small variable air-dielectric trimmers, as used in trimming intermediate-frequency transformers, have a coefficient of about 250 parts in 10^6. These coefficients are all positive and the condensers are usually cyclic. Trimming condensers of the mica compression type are usually non-cyclic and possess a high temperature coefficient. On small variable air condensers it is not possible to employ any method of temperature compensation. It is of prime importance, however, that every condenser should be cyclic, and the usual practice is to examine a number for temperature coefficient and cyclic performance. In super-heterodyne receivers the sensitivity of the receiver depends on the stability of frequency of the first frequency-changers and in a minor degree on the stability of frequency of the intermediate-frequency transformers. In this respect it is fortunate that the temperature coefficient of iron-cored inductances, particularly of the closed-core type, is usually highly negative and partially compensates for the positive coefficient of the variable condenser. Since the new ceramic condensers (which, because of their small size and the precision with which they can be made, are an attractive proposal for aircraft receivers) have also a negative temperature coefficient of a high order, it is possible, by a combination of iron-cored inductance, ceramic fixed condenser, and small air-dielectric trimmer, to adjust the temperature coefficient of the whole circuit to such proportions as to render communication workable on a receiver which is preset on the ground and is operated at a height of 30 000 ft. It can be shown that, in a circuit consisting of an inductance L having a temperature coefficient α, and two condensers C_1 and C_2 having temperature coefficients β and γ, the temperature coefficient of the LC value of the circuit is

$$\delta = \frac{C_1(\alpha + \beta) + C_2(\alpha + \gamma)}{C_1 + C_2}$$

For zero temperature-coefficient

$$\frac{C_1}{C_2} = \frac{-(\alpha + \gamma)}{\alpha + \beta}$$

The behaviour of electrolytic condensers used in smoothing circuits has also been investigated, and the results obtained preclude the use of them for transmitters and receivers employed in high-altitude aircraft. Down to − 30° C. the condensers show a linear decrease in capacitance, the temperature coefficient being about + 7 to + 8 parts in 1 000. The capacitance decreases very rapidly below − 30° C., and tends to vanish at − 50° C.

Mr. H. A. Thomas (*in reply*): Much of the discussion does not call for detailed reply, and I am indebted to Messrs. Colebrook and Williams for the additional information which they have contributed. Both Mr. Colebrook and Mr. Lea refer to the thermal tests on materials. It is not claimed that these tests were exhaustive; they were carried out with the sole object of ascertaining the probable order of magnitude of the deformations which can arise in actual condensers, and Table 5 clearly shows the probable increase in temperature coefficient of capacitance due to various conditions. This Table shows that a very considerable increase in coefficient may be expected with the slightest constraining forces if the centring of the plate system is imperfect.

Mr. Lea points out that I have not considered the design of coils and condensers involving screens. At the present stage of development, it was considered advisable

to concentrate on the major problem of an unscreened coil, since the immediate application of the work does not demand the production of very small receiver coils where screening is essential. For medium-power transmitters the coil need not be screened, and this application of the work has temporarily obscured the problem of compensating a small screened coil. The problem will, however, be considered at a later date.

Both Dr. Hartshorn and Mr. Gill refer to the difficulty of adjusting a condenser to the accuracy suggested in the paper. The application of frequency-stabilized valve oscillators which I have been considering throughout this work does not involve a high degree of discrimination on the part of the condenser. Once the LC circuit has been adjusted to some suitable operating value, it is necessary that it shall remain as constant as possible, but it is unnecessary for the condenser to be capable of precise re-setting. There is consequently a great difference between the stability of a condenser at any particular setting and the calibration accuracy with which the instrument can be reset to any particular value. Since frequency variation in a radio transmitter gives rise to change of received audio-frequency telephone signal, which may be so great as to make the signal vanish, the practical application of this work only involves constancy at a particular setting of the condenser. With further reference to Mr. Gill's remarks, it is not suggested that a simple variable air condenser can be set to an accuracy of a few parts in a million. In practice, one would of course use a small vernier condenser for the fixed setting.

Mr. Bell's communication calls for detailed reply. With reference to his first point, it should be made clear that the object of the reference circuit was to enable coils of very different inductance values to be measured at about the same frequency. A reference condenser would have been simpler, and is used in all measurements on one coil only. In the tests under consideration, however, it was inconvenient to alter the frequency of the measuring circuit over a large range owing to the limitations of the existing multi-vibrator equipment. A further difficulty of a single reference condenser would have been the necessity of providing a centre tap on each coil, and this was impracticable.

It is pointed out on page 704 (vol. 77) that the object of adjusting the potential of the centre point of the coil to that of the surrounding earth without the use of a direct earth connection was to measure the performance of the coil as it would be used in most applications with this centre tap connected to earth. It was not suggested that the capacitance of the coil to earth would thereby be eliminated, but only that the condition of measurement was such as to give a true measure of performance. Actually, every precaution was taken to reduce such capacitance effects, the oven being built entirely of non-conducting materials and the coil kept as far as possible from earthed conductors.

With regard to Mr. Bell's suggestion that the temperature coefficient of inductance is affected by temperature change of resistance of the conductor, I would point out that exact computation of the change of inductance produced by variation of the specific resistance of the metal is difficult except in simple cases. For a straight rod the calculation is readily made by the use of the following formula:—*

$$\frac{\Delta L}{L} = -\frac{\left(1 - \dfrac{4}{x}\cdot\dfrac{Z}{Y}\right)}{\left(4\log\dfrac{2l}{\rho}\right) - 3}$$

where ΔL = change of inductance at frequency f;
L = d.c. value of inductance;
l = length of conductor (cm);
ρ = radius of conductor (cm);
$x = 2\rho\sqrt{(\pi p\mu/\sigma)}$;
σ = specific resistance of material,
= 1 722 e.m.u. at 20° C. for copper;
μ = permeability of material;
f = frequency $= \omega/(2\pi)$;
Z/Y = a function of x (see Table 22).

The temperature coefficient of resistance of copper is 43×10^{-4} per degree C.

For coils wound in the manner described in the paper, it can be shown that the change of inductance will not exceed about twice the value obtained by assuming the wire to be straight. The maximum change of inductance from the d.c. value is produced at infinite frequency, in which case the current distribution is unaffected by the resistance of the material. This maximum change is given by

$$\frac{\Delta L}{L} = -\frac{1}{\left(4\log\dfrac{2l}{\rho}\right) - 3}$$

For intermediate frequencies, it is necessary to determine the variation of the function $\left(1 - \dfrac{4}{x}\cdot\dfrac{Z}{Y}\right)$ with change of resistivity of the conductor. The maximum change of this function is produced at rather low frequencies for wires of about 0·4 cm diameter. For frequencies of the order used in the tests described in the paper, and for dimensions comparable with those used, it will be found that the variation in inductance for a 1 degree C. rise of temperature is of the order of 1 to 4 parts in a million. The higher value represents the worst case (coil H), where the test frequency was about 1 megacycle per sec. At the normal working frequency of this coil (10 megacycles per sec.), the coefficient is very small. The effect of change of resistance with temperature for the other coils is very small.

The object in view in the tests on the "ideal" condenser was to ascertain what expansion factors were of significance in the case of small fixed and variable air condensers, in which the stator plate usually overlaps the rotor plate. The model made for this purpose simulates the actual conditions and, by isolating the overlapping portion, the field distribution can be studied.

I agree with Mr. Bell's closing remarks regarding the temperature compensation of medium-power circuits. Mr. Gill assumes that no improvement can be made in existing coils for transmitters, but already the results of the work described in the paper have been used for

* E. B. ROSA and F. W. GROVER: "Formulae and Tables for the Calculation of Mutual and Self Inductance," *Bulletin of the Bureau of Standards*, 1912, vol. 8, p. 5.

the purpose of constructing a coil to carry 50 amperes which has a temperature coefficient of only a few parts in a million. In an actual frequency-drift run on a 1-kW transmitter, it was found that the drift was only one-tenth that produced with an ordinary helical coil.

The test results given in the paper were obtained with a Hartley circuit of small power (about 3 watts), but some of the coils have been developed for large power and have been tested under conditions similar to those which exist in an actual transmitting installation.

With reference to Mr. Griffiths's communication, I note that the expectations of the manufacturers in respect of coils M and N are considerably less than I had assumed from a perusal of their published information. It is quite clearly stated in the Introduction* to the paper that the present discussion "is restricted to the form of coil which is in current use in practically all short-wave applications, namely, an open-type helical coil of a few turns wound on a former or a coil in which the turns are sufficiently rigid to be self-supporting." I am pleased to learn that Mr. Griffiths finds that exceptional workmanship is required to produce condensers having low temperature-coefficients of capacitance, and I thank him for the interesting figures which he gives for the performance of some precision condensers.

* Vol. 77, p 703.

DISCUSSION ON "THE DROITWICH BROADCASTING STATION"*

Mr. S. R. Kantebet (India) (*communicated*): Dealing first with the question of site selection, the importance of this, so far as the ultimate success of a modern high-power broadcasting station is concerned, cannot be over-stressed. From the paper, site selection appears to have been done by (*a*) boring to ascertain the nature of soil strata under the station, and (*b*) actual field-strength measurement. I should like to know why it was found necessary to bore to a depth of 300 ft., as from the radiation point of view it would appear needless to go so deep. Radio-frequency currents do not penetrate more than a mere fraction of this depth into the ground; and in fact, as the main earth wires are sunk only 9 in. in the soil, the amount of current beyond this depth will be negligible. As for foundation strength, the nature of the soil beyond a depth of 10–15 ft. would not be of much importance, since the mast load—including stays and the concrete base block—would not exceed 1·25 tons per sq. ft., which is not much more than the loading due to an average 6-story building. Has this deep boring been of help either from the radiation or from the mechanical-strength point of view?

Turning to the subject of the earth system, a system of 72 copper wires of No. 16 S.W.G. radiating from the aerial base and laid in furrows 9 in. deep is doubtless a simple and economic structure. May I ask whether the earth system is not too near ground level to be unaffected by daily variations in temperature and soil humidity. It appears to be a compromise between a well-insulated counterpoise and a deep-laid earth net, well beyond the reach of temperature and moisture variations. I should be glad to know whether any earth-resistance measurements have been made over a period long enough to cover both daily and seasonal variations in soil constants. These figures should prove instructive. We in India have constant difficulty in finding suitable earth connections, both for transmission and for direction-finding work.

As regards the aerial-transformer house, the aerial coupling gear is in duplicate, and the spare set can be brought into circuit without delay by means of isolator switches. From Fig. 4, the transformer house appears about 600–700 ft. from the transmitter building. Is the operation of change-over switches carried out by some system of remote control, or has the operator to run to the transformer house and effect the change?

The use of aerial systems for directional effects on medium waves is fast coming into use, and it seems practicable to design for any polar curve—from a perfect circle, through a cardioid, to an elongated "D," and finally to a flat beam. As an interesting instance may be quoted the polar curve of the 50-kW station WOR at Cartaret, New Jersey. The station is 17 miles south-west of New York, on the line New York—Philadelphia. These two are thickly-populated modern industrial centres with high noise-level. At right angles to the above line are smaller towns of importance and, still farther away, quiet rural areas. The aerial system gives a field equivalent to a 120-kW radiation in the direction New York–Philadelphia and to one of only 5 kW in the direction at right angles. Have any special devices been fitted to maintain the requisite phase relationship between the aerial and the reflector at Droitwich? Some further details of the reflector would be appreciated.

It would be interesting to know whether the open-wire feeder, being exposed to the intense radiation field of the aerial, produces radio-frequency feed-back; and in particular whether it affects the polar curve of the aerial itself. Does the polar curve given in Fig. 36 take into account the effect of the long overhead feeder?

It is a big step forward to be able to predict the

* Paper by Sir Noel Ashbridge, Mr. H. Bishop and Mr. B. N. MacLarty (see vol. 77, p. 437, and vol. 78, p. 432).

characteristics of projected aerials of great size from tests on equivalent models. It is not clear from the paper what actual tests were carried out on the models; were they transmission or reception tests (I notice that experiments were carried out at Tatsfield receiving station)? What were the modulation band widths employed on the $\frac{1}{7}$ and $\frac{1}{10}$ scale experiments? Can the test-results obtained on the model be assumed *in toto* for the real aerial, or has any correction factor to be applied? Information on all these points would greatly assist in the design of aerials for high-power stations.

Dealing with the subject of filament heating, the filament currents of cooled-anode valves drawing several hundred amperes can be switched on and off only very slowly, lest the sudden rush of current, or its drop, should produce local mechanical stresses on the filament. In fact, this operation should take at least 1–2 minutes. Since at Droitwich each CAT 14 valve has its own supply generator, connected direct on to the filament without

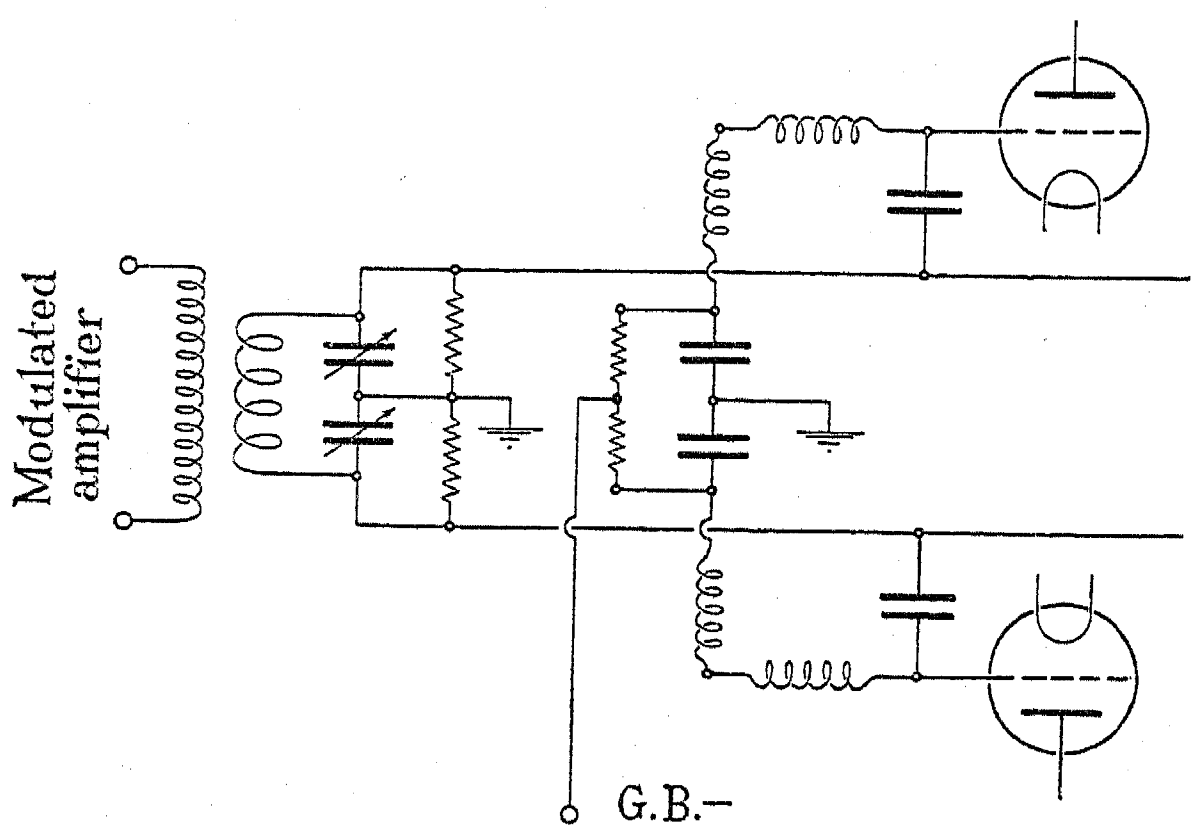

Fig. A

any intermediate switchgear, has any special delay mechanism been fitted to prevent sudden increase or decrease of current on starting or stopping of the motor-generator?

Regarding the filament of the modulator valve, it is not understood why a simple high-insulation transformer was not preferred to the cumbrous insulated motor-generator set.

In the main power amplifier, with four CAT 14 valves working in parallel push-pull, what provision is there to show that the high-frequency load is shared equally by all the four working valves? In Fig. 19, the way of leading grid negative bias to the grid terminals is not clear. Should not the connection be as shown in Fig. A? In the authors' diagram the bias lead appears to be short-circuiting the bias resistance.

Sir Noel Ashbridge, Mr. H. Bishop, and **Mr. B. N. MacLarty** (*in reply*): Mr. Kantebet does not appear to have appreciated the reasons for making a trial bore on the site to a depth of 300 ft. This work had no relation to investigations regarding the conductivity of the soil in the neighbourhood of the station, but was intended to ensure that the possibility of subsidence did not exist. As mentioned in the paper, serious subsidences have occurred in the vicinity of Droitwich owing to the existence of extensive salt deposits. Mr. Kantebet's observations regarding the depth of radio-frequency current penetration into the soil, and the loading on the foundations of the buildings and mast, are therefore irrelevant.

In reply to the question as to whether the type of earth system used at this station is affected by variations in temperature and soil humidity, it can be stated that no variations have been observed due to excessive dryness. This is probably due to the fact that the soil under the aerial is covered with thick grass, which of course tends to conserve the moisture. The wide variations in resistance of this type of earth system observed by Mr. Kantebet may be due to the different conditions which may exist in India. It may be of interest to mention that the only weather condition which causes variation of earth resistance is prolonged frost, when the ground beneath the aerial may become frozen to the depth of a few inches. This condition is of infrequent occurrence in this country and can be compensated for by slight adjustment of the aerial tuning circuits.

When criticizing the adoption of manual switchgear for bringing into use the spare aerial tuning equipment Mr. Kantebet has lost sight of the fact that the reason for breakdown must be investigated, and this in any case requires the presence of an engineer in the aerial-transformer house. Further, it is necessary in any case to inspect the aerial tuning equipment after a breakdown to ensure that a flashover has not occurred and caused a fire. If this point be taken into consideration it will be appreciated that the use of automatic switchgear would offer little advantage.

We agree that directional aerials are now being used extensively for the purpose of adjusting the polar diagram of the aerials of medium-wave broadcasting transmitters. In the directional aerial system cited in the paper a latticed network has been incorporated in the aerial coupling circuits in order to give the requisite phase-difference of $\pi/2$ between the aerial and reflector. The mutual impedance between the balanced transmission lines and the radiating wires is negligible; the polar curve (Fig. 36) is a measured curve and includes any small effect which the presence of the transmission line might cause.

The tests carried out upon the model aerial system at Tatsfield consisted of measurements of driving-point impedance and of the received field round the aerials when driven. The tests were carried out with unmodulated signals, since the band width of the aerials was readily calculated from the impedance characteristics. The principal correction necessary in order to predict the performance of the real aerial, as compared with that of the model, is the difference in conductivity of the earth at frequencies employed with the model and the real aerial respectively. This presents little difficulty at the short distances involved in the measurements.

With regard to the question as to whether delay mechanism has been fitted to prevent sudden increase or decrease of current in the filaments of large valves at Droitwich, suitable interlocks have been provided which prevent the starting of the motor-generators unless the field regulator is set at the minimum-voltage position.

Further, the field regulators are motor-operated, the motor speed and gearing being designed to produce a relatively slow movement in the regulator. It is therefore impossible to increase the filament current at a rate which may be dangerous.

With regard to the danger of sudden interruption of current, this point is covered by the fact that the valve filament is permanently connected to the armature of the generator, without intermediate switchgear. The rate of decrement of current is therefore governed by the rate of collapse of the generator field. It should be remembered that, owing to the thermal characteristics of the filament, rapid application of voltage is far more dangerous than rapid interruption.

With regard to the lighting of the filaments of the modulated amplifiers, it appears that Mr. Kantebet has fallen into a common error on this point. Although at first sight it appears that a highly insulated transformer would be a more simple and economical method of lighting these filaments, when the matter is fully investigated on a practical design basis it is found that there is little to be said for either system; and, taking into account the fact that the motor-generator eliminates all danger of ripple on the carrier, it may be said to possess the advantage. It is well known that to reduce the carrier hum due to a.c. lighting even to a reasonable limit, it is necessary to use a number of modulators heated from a polyphase system with Scott-connected transformers.

It is necessary also to provide induction regulators capable of regulation to zero voltage, and capable of extremely fine regulation around the working voltage. The transformer feeding the heating current to the valves must be of very low capacity, which involves special design and precludes the possibility of using oil for cooling or insulation. The foregoing indicates bulky and expensive plant, for which spares must be provided.

The motor-generators which were installed at Droitwich have proved to be entirely satisfactory, and the porcelain couplings have not shown sign of failure during the first 18 months of working. Difficulty is not experienced in obtaining equalization of anode d.c. input and high-frequency output between the four CAT 14 valves. This is due to the accuracy with which the valves are manufactured, their inherent characteristics, and the fact that they are operated as Class C amplifiers.

We note that Mr. Kantebet questions the accuracy of Fig. 19. The method of connection shown is correct. The resistance shunting the grid bias is the grid-bias generator loading resistance, which is used to swamp low-frequency pulses of grid current that would otherwise flow through the grid-bias generator armature and produce variation in grid-bias voltage in sympathy with the modulation.

DISCUSSION ON

"SOME INVESTIGATIONS ON THE AXIAL SPIN OF A MAGNET AND ON THE LAWS OF ELECTROMAGNETIC INDUCTION" *

Mr. W. Fordham Cooper (*communicated*): It is undoubtedly of great value for such a careful search to have been made to determine whether any evidence can be found to justify the assumption of the rotation of a magnetic field about its axis. Such a search for error is analogous to Sir Oliver Lodge's "ether drift" experiments and to the Michelson–Morley experiment, all of which are important because no effect was found. Had the present authors detected a rotation of the lines of force, it would have been necessary to modify or supplement the equations of a magnetic field. The implication that such experiments can distinguish between the two statements of the law of induced e.m.f., i.e. the rate of change in total flux linkage of a closed circuit and the rate of flux cutting of an element, is not, however, justifiable. It is clear from the account of the experiments that the e.m.f. is easily calculable on the latter assumption from the speed of rotation. The following shows how this is directly equivalent to the rate of change of linkage.

ABC (see Fig. A) is a conductor ring with two conducting arms connecting the centre to the circumference, OA being fixed and OA′ rotating. If the ring is placed in a uniform axial field, then, as A′ rotates, the flux through OABA′O will increase and that through OACA′O decrease at exactly the same rate as the flux is cut by OA′. An e.m.f. proportional to the rate of flux-cutting is therefore set up along OA′, and current will flow along OA′ and divide and flow round the two circular arcs in inverse ratio to their resistances. Alternatively, we may consider that in each of the two closed circuits an e.m.f. is set up proportional to the rate of change of linkage, and that a current will flow equal to the e.m.f. divided by the total resistance of the circuit. Both calculations will give the same result. As A′ passes round the circle the direction of the current at each point will be reversed, but the direction in AOA′ will be constant.

Exactly the same conditions will hold if the ring is attached at A′ instead of at A. If, then, we replace OA′ and the ring by a solid disc, we can consider the disc to be equivalent to the superposition of a large number of rings and spokes, one of which is shown in Fig. B with an indication of the current flow. The superposition of these rings and spokes will give a final pattern somewhat as indicated in Fig. C, the current being nearly radial at all points except near the rim.

It will be seen that the revolving disc is equivalent to an infinite number of circuits such as that shown in Fig. A. If the resistance of the external path OA is made high compared with that of the disc (e.g. by the use of a high-resistance voltmeter) the measured e.m.f. will be exactly equal to that derived from the diameter and speed of rotation of the disc, but any current will distribute itself across the disc according to Ohm's law. A similar analysis can be applied to any other case.

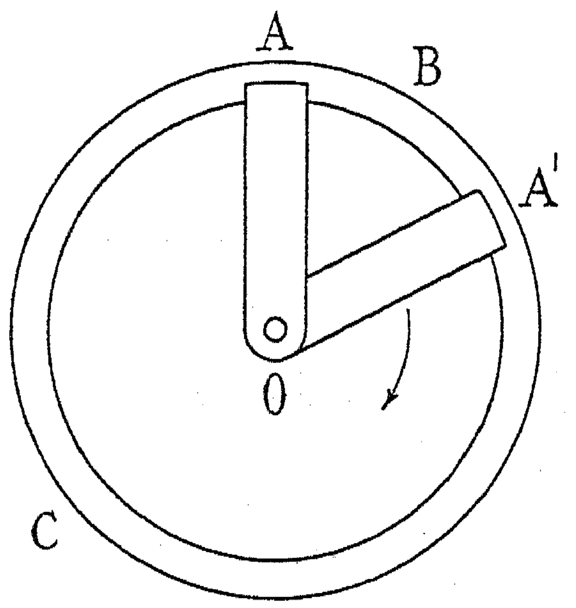

Fig. A

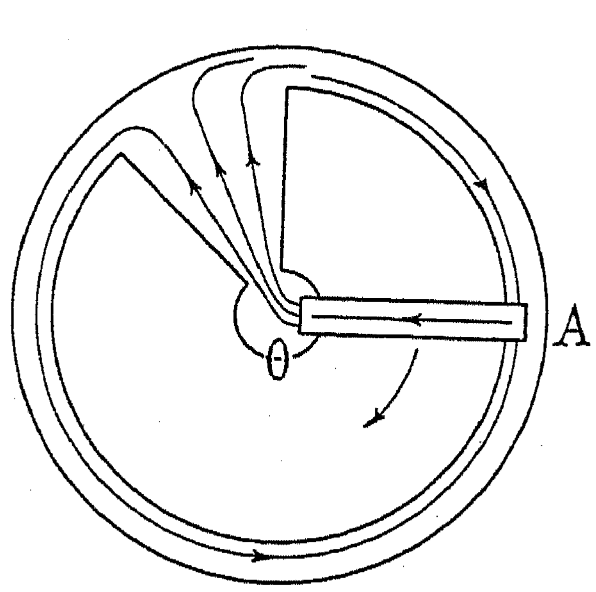

Fig. B

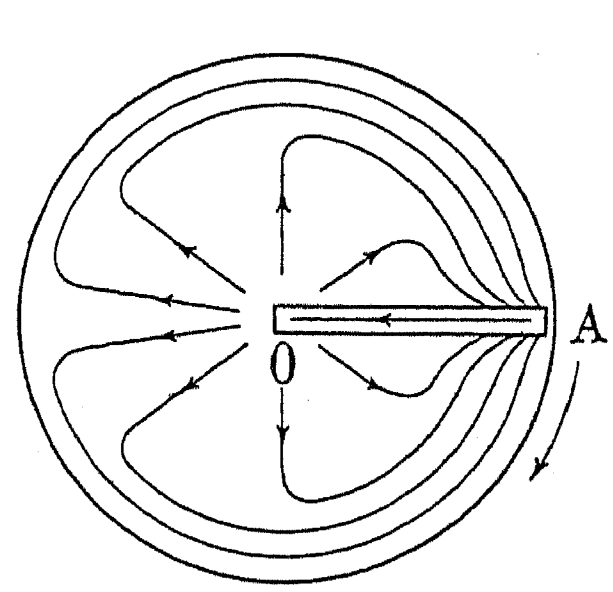

Fig. C

It may be said that there is an e.m.f. even when the circuit is incomplete (though it must be completed if it is to be measured by electrodynamic instruments), and that this case favours the "flux cutting" law. This is true as far as directly demonstrable results are concerned, but it has already been met by Maxwell, who assumed the circuit to be completed by the "displacement current," which again makes the two statements of the law mathematically equivalent, as indeed they must be. The answer appears to be that there is no way of distinguishing between the two equivalent mathematical forms of the same generalization, and, though the cutting theory seems to be less "abstract," both depend on a fictitious though convenient description of magnetic fields as lines of force. In Gray's "Physics," vol. 1, the theory of gravitation is dealt with in the same manner, but no one now finds it necessary to picture the sun's gravitational field as a bundle of lines of force, the mathematically equivalent law of inverse squares being more convenient.

Mr. A. H. Finlay (*communicated*): This paper has been of considerable interest to me and I have not yet com-

* Paper by Prof. W. Cramp and Dr. E. H. Norgrove (see vol. 78, p. 481).

pleted its study. As a student I had visions of a commutatorless dynamo, and a few years later, nearly 40 years ago, I constructed one to give about 12 volts at 1 000 r.p.m. The armature was of a double iron-disc type built up of 24 sections somewhat like a commutator; there were 12 positive and 12 negative slip-rings. May I suggest that further experiments be made with two cylindrical bar magnets end to end having two discs between them, all arranged about a common axis. The magnets and discs could be driven in either direction independently about their axis, and be electrically connected together near the axis. Any electromagnetic action on the leads (which would be twin or concentric), from the peripheries of the discs to the galvanometer, could thus be eliminated. If, as the authors contend, the magnetic flux of a cylindrical magnet remains stationary when the magnet is rotated about its axis, what do they consider to be the motion of the flux of such a magnet spinning at, say, half a right angle to its axis or at right angles to its axis, or revolving about an axis parallel to its own?

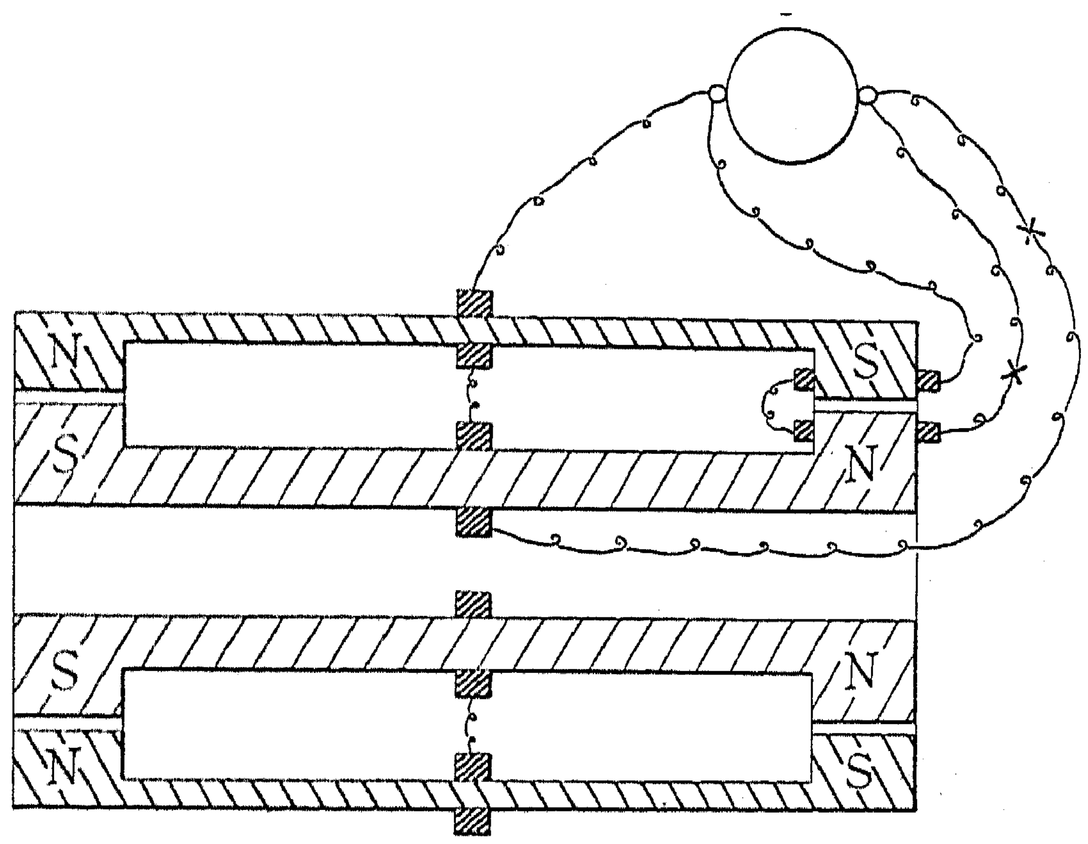

Fig. D

Another suggestion is to employ for further experiments two coaxial hollow cylindrical magnets of equal length and cross-sectional area, free to be driven in either direction (see Fig. D). If electromagnets are employed, each cylinder can be provided with a coaxial coil fixed to it and situated in the space between the two cylinders. If driven at equal speeds (r.p.m.) in opposite directions, there would be, I think, equal e.m.f.'s generated between the inside and outside of each cylinder, and these e.m.f.'s would be in the same direction at the middle. Sliding contacts would be necessary to connect the outside of the inner cylinder to the inside of the outer cylinder, and to connect the inside of the inner cylinder through a galvanometer to the outside of the outer cylinder. If the outer cylinder were fixed and the inner one rotated at twice the speed they were rotated at in opposite directions, the e.m.f. would remain as before, but how much would be generated in the moving cylinder and how much in the stationary one is not clear. I think they would still be equal, but to test this is difficult.

Another arrangement would be to place the rubbing contacts near the air-gap. This idea was embodied in my Patent Specification No. 23192 of 1897, one object being to neutralize any armature reaction.

As regards the necessity for cutting magnetic lines to generate an e.m.f., a helical coil could be slipped over a bar magnet without generating any e.m.f. in it, but it would have to be, as it were, screwed on, just as a corkscrew is driven into a cork.

With reference to Section (17) and Fig. 12 in the paper, as the magnet makes electrical contact with the yoke, and especially if such contact is over the whole length of the bearing, there will be eddy currents circulating in the magnet and yoke, i.e. along the magnet and back through the yoke at the bearing, when the magnet is caused to rotate. I suggest that the space between the magnet and yoke should be either an air-gap (other bearings being provided outside the yoke) or so well lubricated as to prevent any electrical contact between them, and that a rubbing contact should be applied to the end of the magnet outside the yoke, the circuit being completed through the yoke from the outside to the terminal provided on the inside of the yoke, as shown in Fig. 12.

In conclusion, I should like to ask a fundamental question: Suppose that a cylindrical ring enclosing a long coaxial, stationary, straight rod carrying a constant direct current were moved along the axis; then an e.m.f. would be generated between the inside and the outside of the ring. Would a soft-iron ring generate a greater e.m.f. than, say, a copper one of the same size?

Prof. G. W. O. Howe (*communicated*): On page 482 (vol. 78) the following statement is made as to the object and scope of the paper: "experimental proofs are the only guide in such matters, and therefore, avoiding as far as possible disquisitions upon theory, the following experiments which they have recently carried out are put forward as evidence which will always be of use in any future discussion of the problem." If, however, the problem consists in finding an answer to a question which is, in reality, a meaningless form of words, no amount of experimenting, however carefully carried out, will be of any use in any present or future discussion, except in so far as the consistent failure of successive experiments to furnish an answer may lead the thoughtful reader to examine the assumptions underlying the question to which the experiments are expected to furnish an answer.

As long ago as 1915 I published a paper* entitled "Some problems of electromagnetic induction" in which, dealing with a cylindrical bar magnet spinning about its axis, I said "the magnetic field is certainly undergoing no change either in magnitude or direction. It may be objected that although there is no change in the strength and direction of the field, the lines of force may still be rotating around the axis of the magnet. This appears to imply that the lines of force have an individuality and are capable of being earmarked like vortex smoke rings in air." I concluded that "there can be no definite answer to the question whether or not the lines rotate with the magnet, because it has no definite meaning."

The space around a cylindrical uniformly-magnetized bar magnet is in a peculiar condition, which we refer to as a magnetic field. The strength and direction of this field can be determined at every point and can be

* *Electrician*, 1915, vol. 76, p. 169.

represented graphically by lines either actually drawn or —more frequently—imagined. The number of such lines is purely arbitrary; one can draw—or imagine—a hundred, or a million, or a million million lines in any given case, but it is convenient to adopt the convention of assuming the number of lines per cm^2 at every point to be equal to the strength of the field at that point; one can then refer to the strength of the field as so many lines per cm^2. It must not be forgotten, however, that one is then making use of a conventional method of drawing or imagining abstract geometrical conceptions. The temperature distribution around a cylindrical body heated at one end and cooled at the other can be similarly represented by lines of temperature gradient in the medium in which it is embedded. If, now, the bar magnet or hot cylinder is rotated, do the lines rotate with it? That is the question which Prof. Cramp and Dr. Norgrove set out to determine in the case of the magnet, but it requires very little consideration to see that, as the rotation of the magnet or hot cylinder causes no change whatever in the magnetic or thermal condition of the surrounding space, it is meaningless to inquire whether the lines by which we represent this condition have rotated or not. If one cares to picture the lines as rotating around the magnet, even when the magnet is at rest, or as at rest when the magnet is rotating, one can do so, for since they are only geometrical abstractions, such assumptions can have no physical import and do not alter the fact that, whether the magnet is at rest or rotating, its magnetic field is undergoing no change whatever either in magnitude or in direction.

In the opening paragraph of their paper the authors repeat what I have already described as an unfair criticism of Faraday. After quoting Faraday they say "he gives no details of the position of his connections at all." Why should he, since they were in a magnetic field which was constant everywhere in magnitude and direction? Then the authors say "these words confirm the opinion that Faraday supposed the magnetic curves to stand still when the magnet revolved, for only on those conditions could charges be produced on the magnet." This, in my opinion, is an unwarranted imposition of their own ideas on Faraday, as is also the statement that "this comment evidently implies a belief that the 'magnetic curves' remain stationary while the magnet spins." Faraday was in the habit of stating his beliefs clearly, and there is no hint in the quotations given that Faraday attached any meaning to the rotation or non-rotation of a field which was undergoing no change.

Again, after admitting that "the alternative possibility of a flux of curves rotating with the magnet and of relative motion between them and the stationary connections is never mentioned by Faraday," it appears unfair to say, as the authors do on page 482, that Faraday *inclined* to the other view, thus suggesting that he had weighed them both up and hesitated between them.

It is convenient to picture the lines of force as at rest with regard to the observer—whether the magnet is spinning or not—and not to introduce any unnecessary movements, even if they be only postulated rotations of geometrical abstractions, since movement of a conductor in a magnetic field can then be expressed indiscriminately with reference either to the observer or to the lines of force. This is a mere convenience, making for simplicity of description and calculation, and does not involve any physical supposition. Faraday himself used this convention when he concluded that "whenever a closed conductor moves near a magnet in such a manner as to cut across the magnetic curves, an electric current flows in the conductor." This was, to Faraday, equivalent to specifying that the conductor must have a component of motion in a direction at right angles to the magnetic field.

The most fundamental question which arises out of the paper is the following: On page 482, in discussing the production of an e.m.f., the authors say: "Shall we say that the revolving magnet cuts through its own stationary tubes or that these tubes rotate with the magnet and cut the stationary connections which complete the circuit?" The very statement of this question involves the assumption of a new law of electromagnetic induction, which can be set out as follows:—

A conductor at rest relatively to the observer in a magnetic field which is undergoing no change whatever either in magnitude or in direction can have an e.m.f. induced in it by the assumed movement of the geometrical conceptions—or arbitrary conventions, as the authors call them—known as magnetic curves.

It must be emphasized that the experiments were not designed to test the validity of this law, but that the law was assumed in designing the experiments to determine whether the magnetic curves rotate or not. If one does not assume this law, the question set themselves by the authors, and the experiments designed to answer it, become meaningless. In my opinion this assumed law is an unjustifiable addition to the teachings of Faraday and Maxwell. It is stated in the paper that "authors of repute contradict one another as to the view that the student should take." It would clarify the situation if the authors of the paper would give references to the works of any authors of repute who hold or teach this assumed law of electromagnetic induction, and, above all, if they can find any justification for it in the writings of Faraday or Maxwell.

Although it is 21 years since I published my first paper on this subject, I have never made a single experiment in connection with it because I have always maintained that such experiments are merely attempts to obtain from Nature an answer to a meaningless question. This is not to say, however, that my point of view is without experimental confirmation, for the paper by Prof. Cramp and Dr. Norgrove, with its inconclusive experiments and the former's admission that they have made about 50 experiments with the same result, must be regarded as experimental evidence that the object of their search has no real existence.

The experimental results obtained by the authors are all such as one would predict from the laws of electromagnetic induction as laid down by Faraday and Maxwell. I have discussed most of the experiments elsewhere at various times, assuming the experimental facts to be obvious, and showing that any surprise which the observer may feel at the result is due to some such

misconception of the laws of electromagnetic induction as that which pervades the paper under discussion. The paper records several experiments which have not been previously published; these will be discussed fully elsewhere. In my opinion it might have been seen from the outset that they were not capable of furnishing an answer to the question propounded by the authors.

Prof. W. Cramp and **Dr. E. H. Norgrove** (*in reply*): We wish to thank Mr. Fordham Cooper for his appreciative comments. We are familiar with his analytical explanation of the disc, but we do not find it easy to accept for the following reason. If θ is the angle between OA and OA′ in Fig. A, and if the induction is normal to the ring, then when $d\theta/dt$ is negative, the flux through the area OABA′O increases at the rate $\frac{1}{2}(OA)^2 d\theta/dt$. In Fig. C, however, where is the angle θ? We do not think that anyone knowing the behaviour of Fig. A would have been bold enough to predict dogmatically and without trial the results obtained with the disc. We admit that the two statements of the e.m.f. law are identical in Figs. A and B; but we cannot admit their identity in Fig. C unless we can be told what area it is that increases by $\frac{1}{2}OA^2 d\theta$ in the time dt. In Fig. A the two current-carrying strips have a relative motion. Is there any such relative motion in Fig. C?

We have also been much interested in Mr. Finlay's designs for a commutatorless dynamo, but in connection with his proposed experiments we would say that there is no reason now to suppose that the position of the leads from the disc to the galvanometer has any effect upon the electromagnetic action.

The question that he raises concerning the motion of the magnetic flux due to a magnet revolving about an axis parallel to its own is one to which we have given attention. Our view is that the cylindrical magnet carries its magnetic effect about with it, and therefore, like a solenoid, establishes its magnetic circuit at each new position in space which it may occupy. (See footnote § on page 348.)

We have also done some experiments with hollow cylindrical magnets and, while agreeing that the tests upon an apparatus like that shown in Fig. D would be interesting, we fear that the mechanical difficulties in the way of construction and arrangement are too great to make the experiment possible.

We agree with Mr. Finlay that at the bearing of Fig. 12 some eddy currents will exist and are difficult to avoid; we did, however, use the extra brush which Mr. Finlay suggests, working upon a brass slip-ring attached to the magnet. It can just be seen at the right-hand end in Fig. 12.

In answer to his fundamental question, we are of the opinion that the cylindrical ring would have a greater e.m.f. if made of soft iron than if made of copper.

Prof. Howe makes two charges against us. The first is that we are endeavouring to find an answer to a meaningless question. If this question be put in another form, Prof. Howe may understand our position better. In a circuit of any kind where the existence of energy is manifested, it is reasonable to ask in what portion of the circuit the source of this energy lies. Thus, if an annulus has a current of fluid passing continuously round it, the engineer would be justified in asking where the propeller was situated and how driven. In the magnetic circuit, the ampere-turns of the exciting coil are regarded as the source of the flux of induction; in a simple electric circuit the position of the battery or generator is known. So too here, it is legitimate to ask whether the source of e.m.f. is in the magnet or in the external conductor. And since in this instance the generation of the e.m.f. is thought to be due to the relative motion of field and conductor, the foregoing question is tantamount to asking where this relative motion takes place. There are circuits so symmetrical that the e.m.f. and p.d. are everywhere uniformly distributed, but the cylindrical magnet with its external wire is surely not one of these.

Prof. Howe then charges us with unfairness towards Faraday. We yield to no man in our admiration of that great physicist, and if we have been unfair, it is due to the desire to reduce our paper to the smallest possible compass. The quotations were limited to the "Experimental Researches," vol. 1,* but our opinion of Faraday's views rests also upon the Diaries, e.g. Section 267, December, 1831.

We agree with Prof. Howe's statement of the fundamental question at issue, but if this involves (as he says) a new law of electromagnetic induction, then it is a law which must be ascribed to Faraday and not to us. For at Tynemouth in 1851 Faraday re-examined this whole question not once, but thrice, and he stated the problem with his usual clearness in the Diary of July 14, 1851. Under that date we read:—

> "11345. When the magnet is still and the wire is moving, it seems unlikely that the current should be generated anywhere else than in the moving wire; for its motion or quiescence makes all the difference. But then, when the magnet is moving, where is the current then generated? In the wire across which the curves, that may be supposed to move with the magnet, are passing? Or in the magnet, which may be supposed to be moving (as the wire did) whilst the curves are considered as still? Do the lines of force revolve with the magnet or do they not?

It would seem from this work of 1851 that Faraday did not agree with Prof. Howe that "such experiments are merely attempts to obtain from Nature an answer to a meaningless question."

Prof. Howe also considers that Faraday could not be expected to give any details of the position of his connections. Faraday himself was not of this opinion, as will be seen from the Diary of July 12, 1851, when he took care to examine this question (see Section 11331). If we have been unfair to him, it is not in Prof. Howe's sense, but in limiting ourselves to quotations from the "Experimental Researches" of 1832. We think that anyone examining the Diary will see that our thoughts have run parallel to those of Faraday throughout, and that we have carried out our intention of extending his investigations.

Prof. Howe finally asks for information concerning writers of repute who have expressed conflicting views involving his new law of electromagnetic induction. We have shown that Faraday was one of these, but we

* Because, though by the courtesy of Sir William Bragg we had seen the diaries, they had not then been published and so were not generally available.

may add that Steinmetz held that in an electrical circuit consisting of a cylindrical rotating magnet and stationary conductors the e.m.f. was generated in the magnet itself; Sir Oliver Lodge took the opposite view; and Andrew Gray wrote " when the magnet moves, the field of force moves with it."

The references appropriate to these opinions are given in the paper, but our attention has recently been called to certain other papers, which, by confirming our own conclusions, leave no reasonable doubt that this " meaningless question " has now been solved. Among these are the experiments of Barnett,* of Kennard,† and of Pegram,‡ which are all consistent and show that the seat of the e.m.f. in unipolar machines is in the moving conductor and is independent of the rotation of the magnetic field. Pegram shows that this conclusion is in accord with the theory of relativity. Finally, John T. Tate§ in an exhaustive theoretical investigation has reached the following conclusions:—

" The electric field in the neighbourhood of any symmetrical magnetic system spinning about its axis of symmetry is accounted for by the Maxwell–Lorentz theory to an extent determined by the correctness of the assumptions which it is necessary to make. The field so calculated is in complete accord with all known experimental facts.

" A theory which postulates that the lines of magnetic induction rotate with the magnetic system gives incorrect results in general, but may be used to calculate the integrated value of the e.m.f. around a closed conducting circuit, part of which is rotating and part stationary.

" A theory which postulates that the lines of induction stand still while the magnetic system rotates through them will yield correct results if the magnetic system is a conductor, but incorrect results in general if it is a dielectric."

Since in the case of the spinning magnet examined by us the magnetic system was always a conductor, it will be seen that the theory of Tate confirms our experimental conclusions. We are not clear as to what is meant by a magnetic system which is a dielectric.

* *Physical Review*, 1912, vol. 35, p. 324; and Ser. 2, 1913, vol. 2, p. 323.
† *Ibid.*, Ser. 2, 1913, vol. 1, p. 355; and Ser. 2, 1916, vol. 7, p. 399.
‡ *Ibid.*, Ser. 2, 1917, vol. 10, p. 591.
§ *Bulletin of the National Research Council*, Dec. 1922, p. 95.

DISCUSSION ON "NOMOGRAMS IN ELECTRICAL ENGINEERING"*

Mr. G. S. J. Read (New Zealand) (*communicated*): It is a pity that a larger proportion of engineers do not possess a working knowledge of nomograms, which, when once constructed, are simple to use, so that calculations formerly carried out by skilled and responsible persons can be safely delegated to unskilled subordinates. In other cases, where a large number of similar calculations are required for statistical returns, a great deal of time may be saved by their use. I had an instance of this where, for coal-consumption analysis of locomotive stock, a weekly return was instituted involving about 100 similar calculations to reduce the total coal consumption to lb. per engine-mile. The clerical staff in each district office were already fully employed and, as the return for each district took 7 hours to compile, they protested that the extra work involved was too great. A nomogram was devised, and the time taken was reduced to $1\frac{1}{2}$ hours. I feel sure that a good deal of the routine calculations carried out by statistical clerks could be eliminated if engineers would study the clerical problems and devise suitable nomograms. Their actual use is quickly learned by those who have no idea of the manner in which they are constructed. If a straight-edge is used, important figures are often obscured; a celluloid strip upon which is engraved a clear black line is therefore preferable. Prof. Kapp's paper should go a great way in arousing interest in a subject which has been sparsely treated by English writers. It is in the main a product of French mathematical genius, and those who intend to study the subject more fully could not do better than refer to the works of D'Ocagne. It is to him that the name " nomogram " is due.

* Paper by Prof. R. O. KAPP (see vol. 78, p. 567, and vol. 79, p. 227).

FLUCTUATION VOLTAGE IN DIODES AND IN MULTI-ELECTRODE VALVES*

By F. C. WILLIAMS, M.Sc., Student.

(*Paper first received 7th January and in final form 17th March, 1936.*)

SUMMARY

This paper is concerned with the spontaneous fluctuation voltages generated in the anode circuit of thermionic valves. The general behaviour of commercial valves is examined and is shown experimentally to be consistent with the existence of shot and flicker effects: an expression, equation (7), is put forward that is found to be satisfactory experimentally, provided a correction factor A is introduced. The general form of this correction factor is examined; when the operating conditions are such that flicker effect is unimportant, A is found to be independent of the type of valve used, and of the nature of the cathode surface. Under such circumstances the value of A is unity when the current is very small, and is again unity when temperature limitation is reached; it has a minimum value of about $0 \cdot 1$ between these limits. This minimum is independent of frequency with thoriated tungsten filaments: with oxide-coated filaments the minimum increases, and occurs at a lower current value, as the frequency of operation is decreased; this is due to flicker effect. With oxide cathodes, except at very high frequencies, A exceeds unity as temperature limitation is approached; with thoriated tungsten, however, this does not occur. At low currents the value of A is always unity, and provided the operating conditions are chosen such that A decreases as I increases, flicker effect can be ignored. The optimum operating conditions as regards signal/noise ratio usually lie in this region, and indirectly-heated oxide-coated cathodes are therefore found to be the most satisfactory on account of the high mutual conductance that can be obtained.

Thus in general use the shot effect is the important limit to amplification set by the anode-circuit fluctuations. Thermal agitation in the anode stream is shown to be non-existent or negligible; interpretation of the experimental results on such a hypothesis is not self-consistent, nor is it quantitatively accurate. Fluctuation voltages due to collision ionization are not apparent, and the whole fluctuation observed experimentally can be reconciled with the existence of shot and flicker effects alone.

Multi-electrode valves are also considered, and it is shown that the value of A relevant to the current leaving the cathode is sensibly independent of the electrode arrangement, and of the subsequent distribution of current between the electrodes. When the cathode stream is shared between several electrodes the current arriving at any one electrode does not necessarily yield the same value of A as that arriving at other electrodes, but in general the values are of the same order: this is explained by the existence of a fluctuating current travelling round the circuit connecting any pair of current-sharing electrodes. When such sharing occurs it is important to make the current to the actual anode a large fraction of the total current in order to preserve a high value of g^2/I, and give a good signal/noise ratio. The connection of similar valves in parallel improves the signal/noise ratio.

* The research described in this paper was carried out during the author's tenure of the Ferranti Scholarship.

(1) INTRODUCTION

It is found that the anode current of a diode valve contains minute spontaneous fluctuations, for experience shows that there is in general a small fluctuating potential-difference across a resistance through which the anode current is flowing. The mean square value of this fluctuating voltage can be measured by means of a suitable valve amplifier; in general this voltage is of the order of, say, 10 μV squared. If a current from a battery flows through the resistance, no such fluctuating voltage is observable, and therefore the effect arises in the diode; it is associated with the pattering of electrons on the anode. Such fluctuating voltages are usually called "shot voltages." If the arrival of electrons has a true random distribution with respect to time, it can be shown that the mean square value of the shot voltage is expressed by the equation

$$|v|^2 = 2IeR^2df \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I is the mean value of the anode current, e is the electronic charge, R is the value of the resistance carrying the anode current, and df is the frequency range over which the associated amplifier is able to respond. The limitations of the equation (1), imposed by the amplifier, have often been discussed previously,† and need not be detailed here.

The derivation of equation (1) assumes that the arrivals of individual electrons have a random distribution in time. The effect of this basic hypothesis on equation (1) is to make the expression independent of f, where f is the actual frequency corresponding to the middle of the range df. If for a given value of df the observed values of $|v|^2$ are not independent of f, then the disturbances are either not distributed in a truly random manner or else occur at a mean frequency comparable with f.

Equation (1), or a form of it modified to suit the amplifier characteristic, can be used to deduce e (the value of the electronic charge) from the observed values of $|v|^2$. Many such measurements have been made for this purpose, but usually the small frequency range df has been located at a radio frequency. Such measurements have yielded the correct value of e. For example, Williams and Vincent‡ derived a value for e which agreed with Millikan's value to 1 part in 1 000; in their measurements df was located at 120 kc.

In 1925, J. B. Johnson§ made similar measurements for values of df located at frequencies from 6 kc down to 8 cycles per sec., and found that the observed values of the shot voltage were not independent of f. As f increased, the apparent value of e fell asymptotically to a limiting value approximately equal to the true one; at

† See Bibliography, (3) and (4). ‡ *Ibid.*, (1). § *Ibid.*, (2).

10 cycles per sec. the apparent value of e was sometimes 100 times too large. He found that the ratio of e' (the apparent charge) to e (the true charge) was markedly dependent on the type of cathode. Thus for a tungsten cathode, e'/e had reached its limiting value—which in fact was 0·7 and not unity—at a frequency of 500 cycles per sec., whereas with a coated filament e'/e did not reach its limit till a frequency of 6 kc. With a tungsten filament, e'/e was independent of the current, whereas for a coated filament it was not. The comparatively narrow frequency range df was obtained by a selective stage in the amplifier; Johnson quotes experiments which show that at a given resonant frequency of the selective amplifier, e'/e is independent of the values of L, C, and R, for the selective circuit, provided always that LC is constant.

These interesting experiments of Johnson show that there is a component of shot voltage due to some cause other than pure random emission from the cathode and an effect which is dependent on the character of the emitting surface. It is perhaps associated with ionization in the valve or with changes in the surface contamination of the cathode. This additional component will be termed "flicker effect." In parenthesis it should be stated explicitly that Johnson's measurements were made on valves in the condition of temperature limitation. Johnson's measurements do not appear to have been repeated.

Equation (1) is valid only for valves in the condition of temperature limitation, for otherwise the fluctuating potential of the anode will dictate the release of electrons from the potential minimum in the space charge, and the true condition of randomness will be vitiated. Since a valve cannot be used as an amplifier in the region of temperature limitation, the problem of discovering the whole mechanism of the shot voltage is of technological importance as well as of scientific interest, for the shot voltage may set a lower limit to the voltages which can be amplified usefully.

This paper is concerned mainly with the measurement of shot voltage in valves where space-charge limitation obtains, and it also makes some attempt to discuss the meaning and interpretation of the observed results.

In the main the amplifier and the experimental procedure were substantially the same as those described in detail by Moullin and Ellis.* For certain portions of the work it was advantageous to modify this technique, and it seems best to describe such modifications in the sections of this paper dealing with the special measurements where they were used. Sometimes the amplifier was provided with a selective stage in order to restrict the magnitude of df. When such a stage was used, the response rose to a sharp maximum which could be arranged to occur at any one of the following frequencies: 250, 1 850, 3 200, 5 250, 15 800 cycles per sec. For the 250-cycle maximum, the filter was of the band-pass type and cut off sensibly at 100 and at 1 000 cycles per sec. For the other frequencies simple resonant circuits were used, and the fractional widths at $1/\sqrt{2}$ of the maximum height were 12, 13, 19, and 20 per cent respectively. Also there was used a low-pass filter which cut off sensibly completely at 10 kc.

* See Bibliography, (3).

(2) MEASUREMENT OF ELECTRONIC CHARGE IN TEMPERATURE-LIMITED DIODES

Measurements were made with an LS5 valve, having a thoriated tungsten filament, with grid connected to cathode; the filament temperature was reduced until temperature limitation was obtained. For these determinations it was of course necessary to use a known calibrating voltage and to know the response characteristic of the amplifier. Ten determinations of e were made, with various currents and at various times, using the filter which cut off at 10 kc. Five determinations were made with the filter having a maximum response at 5·25 kc, three determinations with the filter giving maximum response at 3·2 kc, and four with the filter which had maximum response at 1·85 kc. The mean

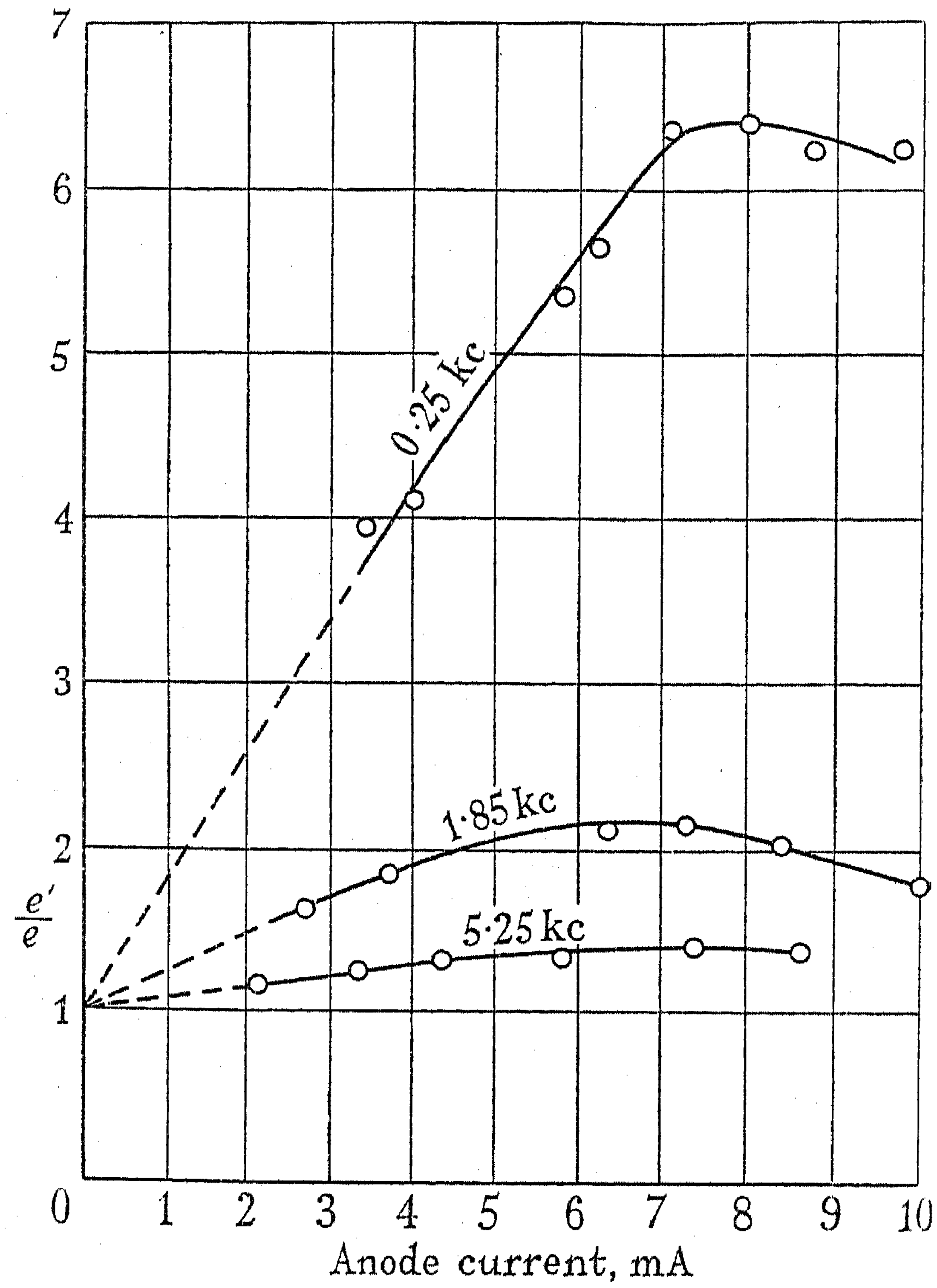

Fig. 1.—VMP4 valve. $R = 6\,\mathrm{k}\Omega$; $E_a = 200$ volts; external resistance. Fluctuations for various filament currents at 0·25, 1·85, and 5·25 kc.

of these 22 determinations yielded a value for e which was 0·55 per cent higher than the correct value. The greatest divergence from the mean was 5 per cent, and the greatest divergence between the mean at any frequency and the mean of the total was + 0·5 per cent. These experiments show that the thoriated tungsten filament of an LS5 valve yields a sensibly correct value of e over the whole acoustic range. Hence this valve can be used as the "calibrating diode." It was found that the value of e deduced from an oxide-coated filament in the temperature-limited condition depended on the frequency. This is illustrated by Fig. 1, which shows the value of e'/e plotted against anode current for a certain valve with oxide-coated cathode. The valve was a pentode (VMP4) having anode, suppressor grid, and

screen-grid connected together, and the control grid connected to the separately-heated cathode. The three curves shown were obtained with the amplifier having its dominant response at the three frequencies marked. It may be seen that the value of e'/e is a function of the anode current and also decreases as the frequency increases. This is due to flicker effect, believed to be caused by bursts of emission arising out of structural variations of the cathode surface.* The effect is known to depend on many variables, and does not seem amenable to quantitative discussion.

(3) SHOT EFFECT IN DIODES WHICH ARE SPACE-CHARGE-LIMITED

It is well known that the fluctuation voltage produced by a given current is much less when it is space-charge-limited than when it is temperature-limited. It has been stated already that equation (1) is not applicable to the space-charge-limited state, because the potential fluctuations of the anode tend to dictate the release of electrons from the region of potential minimum near the cathode. Conditions then obtaining are extremely complex, and a rigid statistical discussion of the problem does not seem to have been made.

It has been argued by F. B. Llewellyn† that equation (1) should be replaced by the following expression:

$$|v|^2 = 2I_c\left(\frac{\partial I}{\partial I_c}\right)^2 e\left(\frac{R\rho}{R+\rho}\right)^2 df \quad . \quad . \quad . \quad (2)$$

where ρ is the slope resistance of the valve, I is the anode current, and I_c is the total current evaporated from the cathode at the relevant temperature.

On the other hand, Moullin and Ellis‡ have argued that equation (1) should be replaced by the expression

$$|v|^2 = 2Ie\left(\frac{R\rho}{R+\rho}\right)^2 df \quad . \quad . \quad . \quad . \quad (3)$$

In the absence of flicker effect, experience shows that the fluctuation voltage is appreciably less than would be predicted from equation (3), and there now seems no doubt that this expression is wrong, or perhaps incomplete. The experimental determination of the term $\partial I/\partial I_c$ in equation (2) presents considerable difficulty, but Llewellyn and others find that when the determination has been made the observed fluctuation voltage with complete space-charge limitation is much more than would be predicted by equation (2). Llewellyn attributes the discrepancy to the simultaneous existence of another effect. He considers that the anode stream is also the seat of a thermal agitation voltage; a fluctuation voltage whose magnitude is equal to that which would exist across a metallic resistance of value ρ if it were at the cathode temperature. Moullin and Ellis state, however, that they cannot understand the existence of an agitation voltage in the anode stream. If the total fluctuation be the sum of two distinct effects, it is extremely difficult to analyse the contribution made by true shot effect. To elucidate this consideration, Pearson§ has published lately some measurements on valves working in circumstances where the contribution predicted from equation (2) should be negligible. He shows that the observed fluctuation voltage has the form of an agitation voltage, but that the apparent temperature is about half that of the cathode. More recently the author* has shown that the exponential form of the characteristic in the retarding-field region makes Pearson's results conform precisely to equation (3). In short, in this region I and ρ are related to the cathode temperature in such a way that the value of $|v|^2$ is then equal to the agitation voltage in a metallic resistance at half the cathode temperature. Such further experience of the author confirms his belief that Moullin and Ellis are correct in holding that agitation voltage does not exist in the anode stream. Further evidence of this will be given in the paper. Accordingly this paper will proceed on the belief that agitation voltage does not exist and that the whole fluctuation is due to shot effect. On this basis, equation (2) bears no relation to the results observed in practice. Accordingly the author has used throughout the incorrect equation (3) as a basis of comparison. It is therefore appropriate to discuss first the extent to which this equation is of the correct form.

Recently W. H. Aldous and N. R. Campbell† have made an analytical study of an associated problem. Their statistical reasoning leads them to conclude " that all known theories of fluctuations are inadequate when ρ is finite, and that an adequate theory is far beyond our present reach. Until it is available, all conclusions concerning experiments in space-charge-limited conditions are precarious." In the face of this conclusion, it seems perhaps inappropriate to consider further the use of equation (3) as a basis of comparison. But even though the difficulties of a sound theoretical approach to the problem seem overwhelming, it is not futile to search for an ordered and consistent behaviour of experiments. There is much to recommend an attempt to maintain an expression based on (3), because the factor $[R\rho/(R+\rho)]^2$ will also appear in the expression for the signal output of an amplifying stage. Unless experiment shows that it is quite unsuitable to retain such a factor, the technological convenience thereby gained does much to outweigh its flimsy theoretical basis. Moullin quotes measurements which show that if I is constant, then $|v|^2$ varies as $[R\rho/(R+\rho)]^2$: the present author has repeated and confirmed such measurements, and some of these appear incidentally in this paper. If such a factor is to be retained, it is desirable to be able to measure it experimentally rather than to rely on values of ρ derived from the static characteristic. The author has described‡ recently a simple method of doing this. Since then, a further modification of technique has been made which makes measurements possible in the circumstances when the anode current is so small that the shot voltage it produces is less than the agitation voltage in the load resistance R.

Thus, referring to Fig. 2, the scale of the output galvanometer was first set to read zero with the resistance R short-circuited. Then, with the valve cathode cold and hence with ρ infinite, the deflection θ_1 was noted,

* This explanation was put forward by Schottky. See Bibliography, (9).
† See Bibliography, (4). ‡ *Ibid.*, (3). § *Ibid.*, (5).

* See Bibliography, (6). † *Ibid.*, (7). ‡ *Ibid.*, (6).

due to thermal agitation in R and expressible as a voltage $|v_1|^2 = 4RkTdf$. Then, with ρ still infinite, a known calibrating voltage of value v_2 was introduced at S, in series with R, and there resulted a deflection θ_2. The known calibrating voltage v_2 was then expressed in terms of an equivalent fluctuating current I_2 by means of the equation

$$v_2^2 = 2I_2eR^2df \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

From the known values of R, k, and T, it is also possible to express $|v_1|^2$ in terms of an equivalent current I_1 such that

$$4RkTdf = 2I_1eR^2df = |v_1|^2 \quad . \quad . \quad . \quad (4)$$

Hence
$$\frac{\theta_1}{\theta_2 - \theta_1} = \frac{|v_1|^2}{v_2^2} = \frac{I_1}{I_2} \quad . \quad . \quad . \quad . \quad (5)$$

In practice it is found that the values of I_1 deduced from (5) differ slightly from those calculated from (4). The discrepancy is due to a trace of grid current in the first stage of the amplifier. This discrepancy does not matter provided the value of I_1 which is used is the apparent value deduced from (5), since the disturbing trace of grid current remains unchanged.

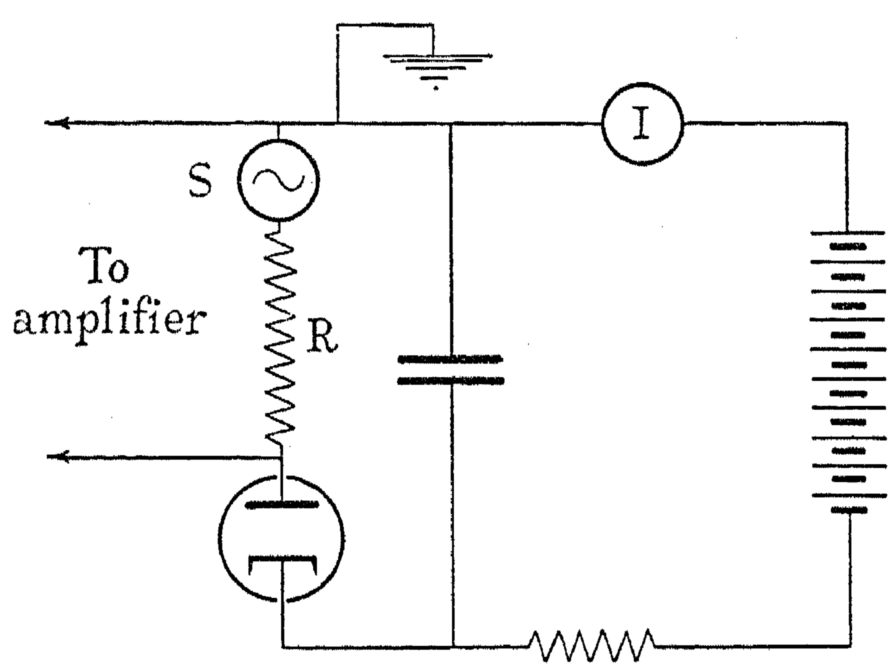

Fig. 2

The experimental valve was then switched on and adjusted to the desired condition of operation, and the deflections θ_3 and θ_4 respectively were recorded which then resulted without and with the calibrating voltage at S. The valve slope resistance will, in general, be finite and it follows from the circuit of Fig. 2 that

$$\frac{\theta_4 - \theta_3}{\theta_2 - \theta_1} = \left(\frac{\rho}{R + \rho}\right)^2$$

and thus $\rho/(R + \rho)$ is found experimentally.

Let I_3 be that apparent current which, according to equation (3), would produce the observed deflection θ_3. Then

$$\frac{2eI_3\left(\frac{R\rho}{R + \rho}\right)^2 df}{2eI_2R^2df} = \frac{\theta_3}{\theta_2 - \theta_1}$$

Or
$$I_3 = \frac{\theta_3}{\theta_2 - \theta_1}\left(\frac{R + \rho}{\rho}\right)^2 I_2$$
$$= \frac{\theta_3}{\theta_4 - \theta_3} I_2$$

Since equation (3) is known to be incomplete, we now modify it by writing

$$|v|^2 = 2AIe\left(\frac{R\rho}{R + \rho}\right)^2 df \quad . \quad . \quad . \quad (6)$$

where A is a correcting factor and is the same as the ratio e'/e already used.

The thermal agitation voltage $|v_1|^2$ in R results from a mean square current $|i_1|^2$ such that

$$|i_1|^2 = \frac{4kTdf}{R}$$

Again the disturbance from the anode current I can be expressed in terms of a mean square current $|i_2|^2$, such that

$$|i_2|^2 = 2AIedf$$

When the resistance R and the valve (of slope resistance ρ) are in parallel, there is a current $|i_1|^2 + |i_2|^2$ flowing through the joint resistance of R and ρ in parallel.

Hence
$$|v|^2 = \left(\frac{R\rho}{R + \rho}\right)^2 \left(\frac{4kT}{R} + 2IeA\right) df \quad . \quad . \quad (7)$$

But
$$|v|^2 = 2eI_3\left(\frac{R\rho}{R + \rho}\right)^2 df$$

$$\therefore \; I_3 = \frac{2kT}{Re} + AI$$
$$= I_1 + AI, \text{ from (4).}$$

Thus the factor A can be determined. It is only when either I or R is very small that I_1 is comparable with AI.

By this means it has been possible to determine A for values of I down to a few microamperes. Fig. 3 shows the values of A so deduced from measurements on a valve with a thoriated tungsten filament (LS5): in this figure it has been found convenient to plot log A against log I.

The two curves in this figure relate to measurements with the grid connected to cathode and also with it connected to anode: such change involved about a five-fold alteration of anode voltage for a given current. It may be seen that A tends to unity when the current tends to zero. Hence equation (3), unmodified by the factor A, is evidently a limiting form which is correct when all the current evaporated passes to the anode and also when a very small fraction of it passes to the anode. It should be recorded that even in the low-current range where A is unity, ρ was still found comparable with R. That A tends to unity has already been pointed out by the author.

It may be seen that A falls to a minimum value of about $\frac{1}{20}$ and that the value of A for a given current is substantially independent of the electrode spacing, as represented by changing the connection of the grid. The linear branch of Fig. 3, which extends from about 0·02 mA to 2 mA, has a slope of $-$ 0·5. The logarithmic plot of the characteristic is also shown on the figure, and its slope is sensibly 1·5. Hence it follows that on the linear branch $AV^{\frac{3}{4}}$ is sensibly constant.*

Fig. 4 exhibits similar curves for a diode having a

* Fig. 3 also gives the values of R used in determining A. The curve is continuous, and the value of A is evidently independent of that of R; the use of the term $[R\rho/(R + \rho)]$ in equation (6) is therefore justified experimentally.

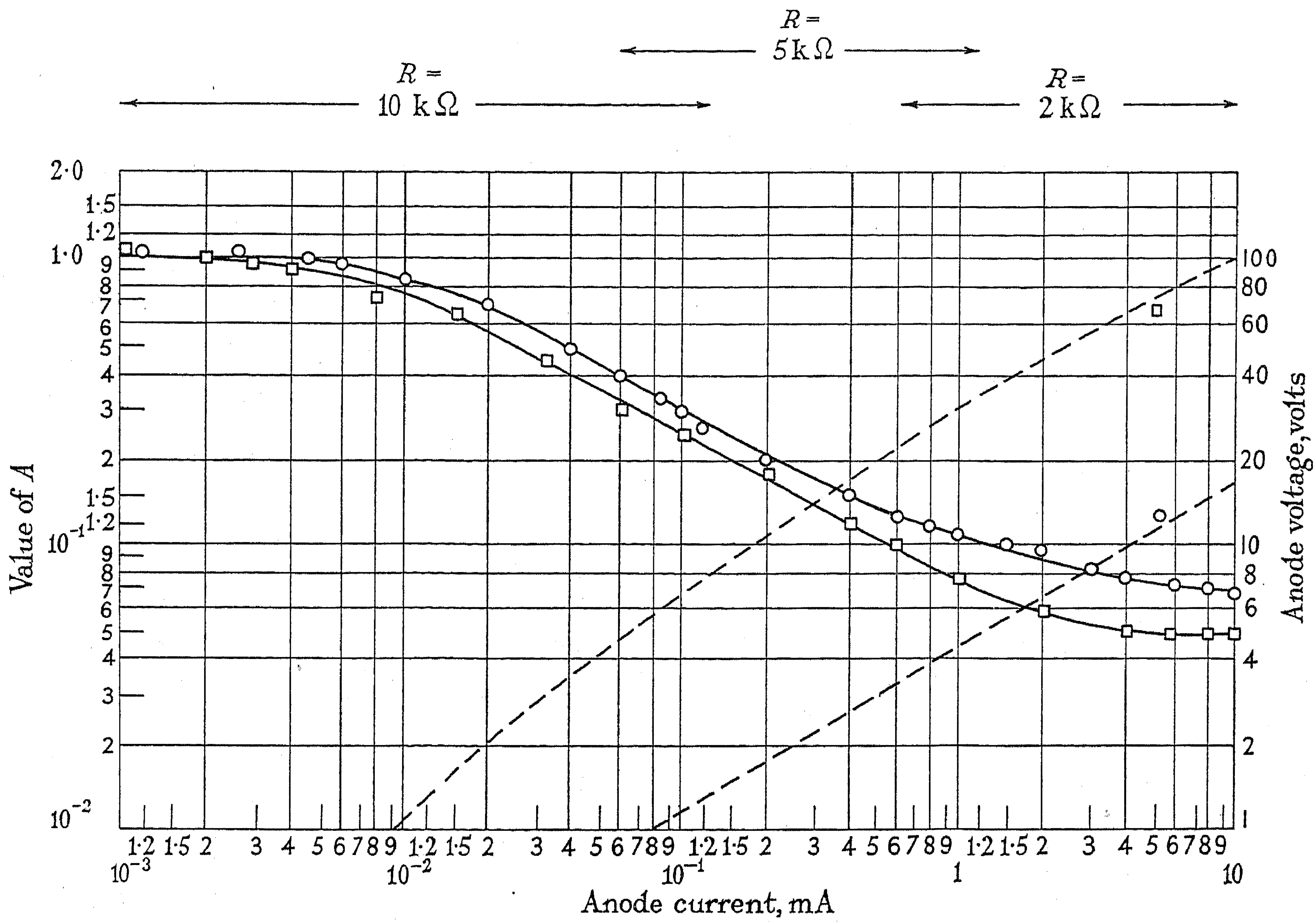

Fig. 3.—Dependence of factor A on anode current.

□ With grid connected to filament negative.
○ With grid connected to anode.
— — — Valve characteristic.

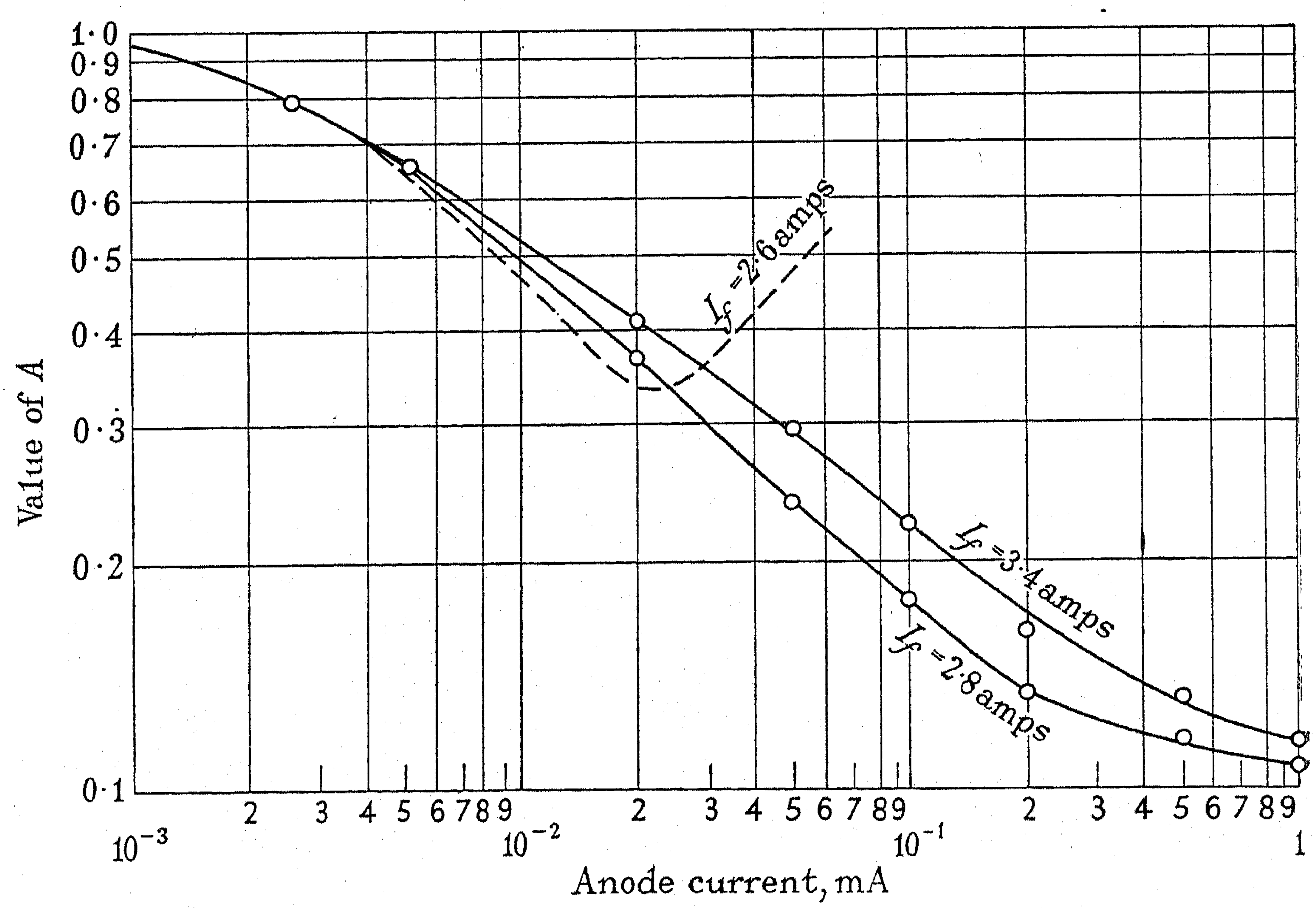

Fig. 4.—Special diode valve. Variation of A with anode current, for $I_f = 2 \cdot 6$, $2 \cdot 8$, and $3 \cdot 4$ amps.; $5 \cdot 25$-kc filter.

thoriated-tungsten filament, which had been constructed for a special purpose. This figure shows that, for a given current, A increases slightly with the temperature of the filament. It should be noted that the linear portion of the curves again has a slope of the order of $-0\cdot5$.

If Llewellyn's equation (2) is correct, the true shot effect is negligible throughout the scope of Fig. 3, and he would interpret the observed fluctuation in terms of thermal agitation. Accordingly he would express the fluctuation by the equation*

$$|v|^2 = \left(\frac{R\rho}{R+\rho}\right)^2\left(\frac{4kT_R}{R} + \frac{4kT_C}{\rho}\right)df \quad . \quad . \quad (8)$$

whereas the author would express it as

$$|v|^2 = \left(\frac{R\rho}{R+\rho}\right)^2\left(\frac{4kT_R}{R} + 2AeI\right)df \quad . \quad . \quad (7)$$

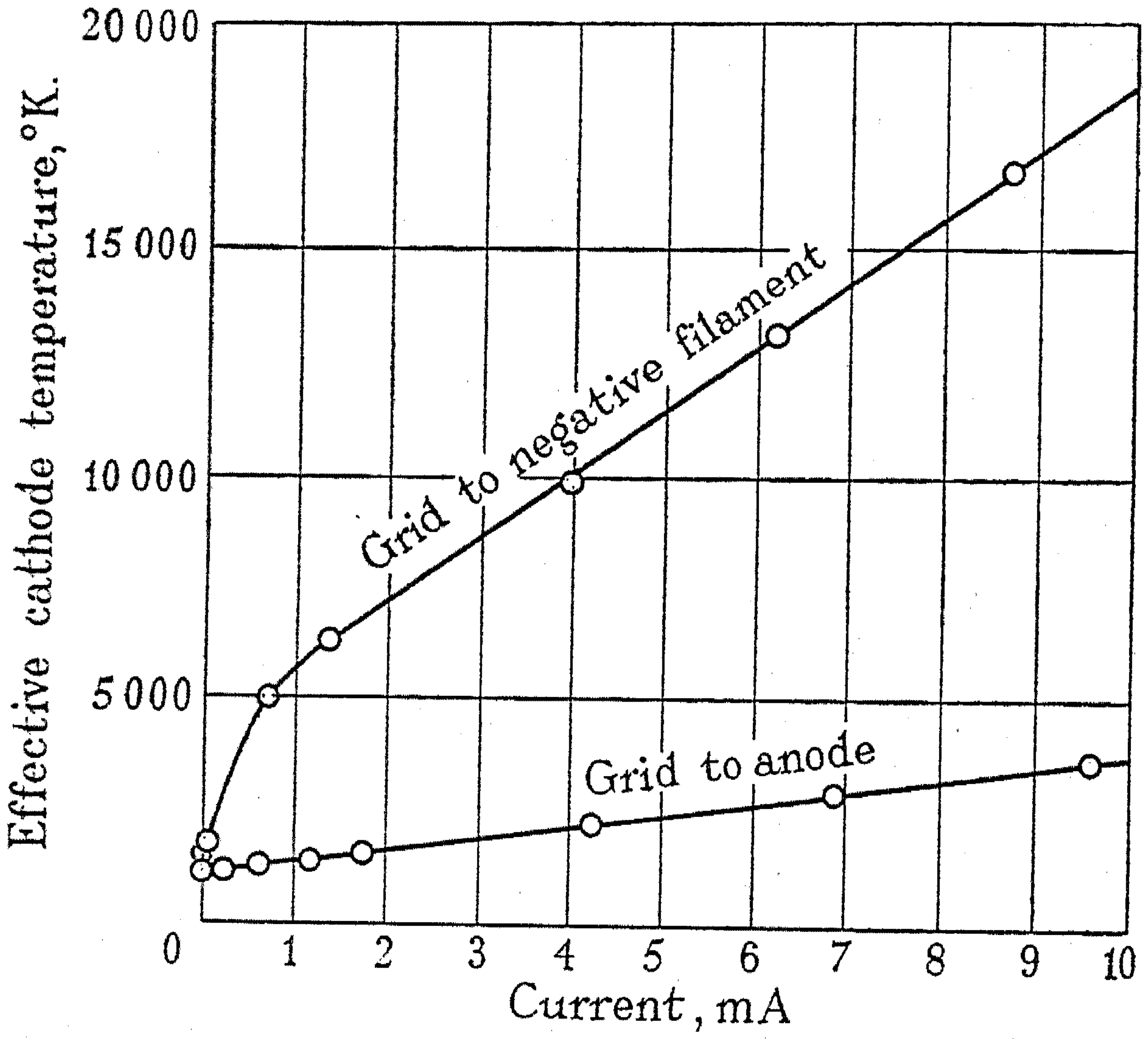

Fig. 5.—Effective filament temperature of LS5 valve with $I_f = 0\cdot8$ amp. and different electrode connections; 5·25-kc filter.

Comparing (8) and (7), it follows that

$$T_C = \frac{2AeI\rho}{4k}$$

By this means, apparent values of T_C have been deduced, and are exhibited in Fig. 5. It may be seen that the apparent temperature varies enormously with the anode current, though the filament current was constant, and also is a function of the electrode arrangement. It should be noted that when the anode current is very small, in the retarding-field region of the characteristic, the apparent temperature for either arrangement of electrodes tends to the same value, which is about 1 000° K. Thus, as in the previous paper,† the apparent temperature is about half the true temperature, as is in fact demanded by the exponential form of the characteristic. That the apparent cathode temperature depends enormously on the electrode arrangement and on the current, seems to the author to prove that the hypothesis of thermal agitation in the anode stream is untenable. The observed fluctuations do appear consistent with a shot effect and are certainly not consistent with an interpretation in terms of thermal agitation: be it noted that such an interpretation requires a temperature which is some six times the melting point of tungsten (see Fig. 5).

* See Bibliography, (4). This equation can be more easily deduced, however, by a method similar to that used in obtaining equation (7).
† See Bibliography, (6).

Sometimes it is held that the action of ions on the space charge make an important contribution to the observed fluctuation voltage. Ballantine* has shown that these effects may rise to sudden maxima at certain frequencies, but show no steady change with frequency such as is found with flicker effect. The author believes that such effects do not constitute an appreciable source of noise in commercial valves, and that such noise is substantially all due to shot and flicker effect. Fig. 6 relates to tests on an indirectly heated oxide-cathode pentode valve, used as a diode. In this valve the flicker effect was marked, in that the observed fluctuation voltage was dependent on the position of the frequency range df. Fig. 6 shows, to a base of filament current, the ratio of fluctuation voltage at 0·25 kc to that at 5·25 kc and also the ratio at 1·85 kc to that at 5·25 kc. For filament currents below about 0·87 amp. the valve is temperature-limited and space charge is not present. Hence in this region ionization effects cannot make a contribution. Shot and ionization effects are independent of frequency, and flicker is dependent on frequency. Hence the appearance of ionization would be expected to cause a decrease in the ratio of the fluctuations observed at two different frequencies; for then the component dependent on frequency would be relatively less important. But the curves of Fig. 6 are continuous from the temperature-limited to the space-charge-limited region, and also show an upward tendency. From this it is argued that no appreciable ionization effect is present.

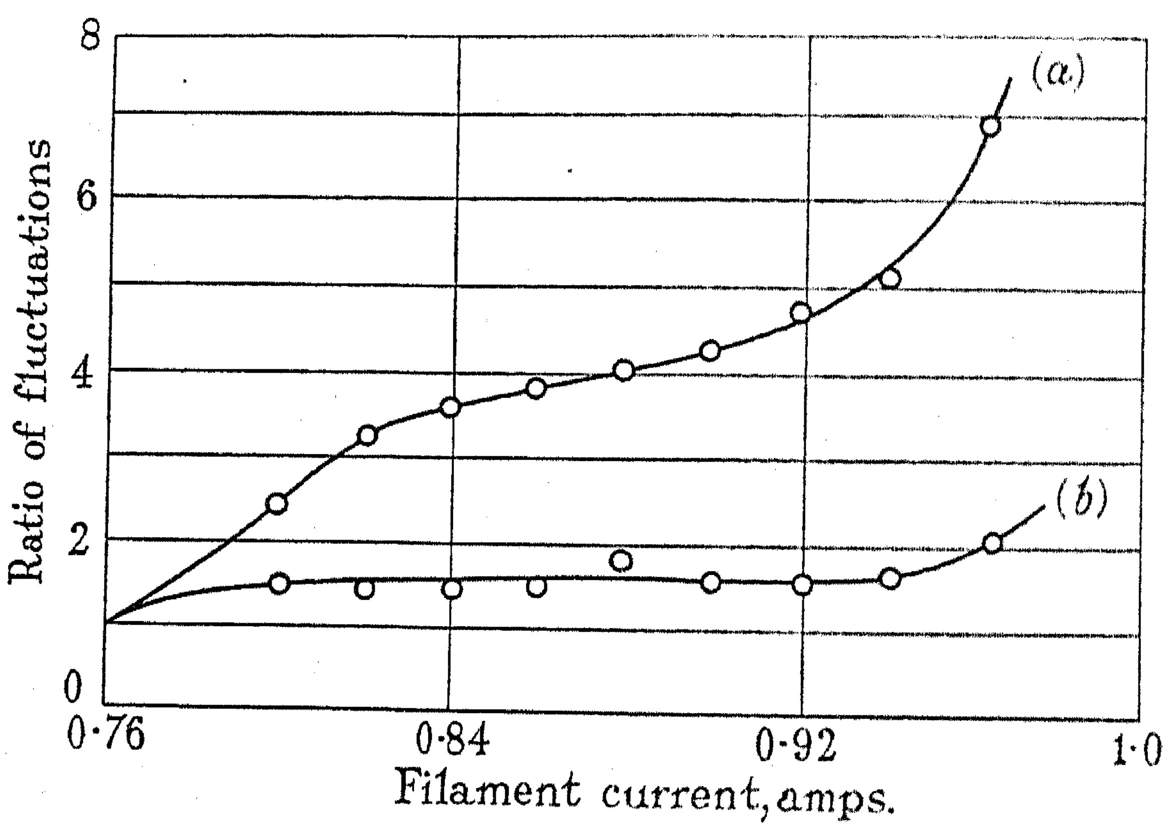

Fig. 6.—VMP4 valve. $R = 6\ \text{k}\Omega$; $E_a = 120$ volts; external resistance.
(a) Ratio of fluctuations at 0·25 kc to those at 5·25 kc.
(b) Ratio of fluctuations at 1·85 kc to those at 5·25 kc.

The dependence of fluctuation voltage on the position of the frequency range df is well illustrated by Fig. 7. This figure relates to measurements of $|v|^2$, at various frequencies, on three different valves operated in given

* See Bibliography, (8).

constant conditions. The valves were the LS5 described already, a separately heated triode (MH/D4),* and a separately heated pentode (VMP4). In all three valves the control grid was connected to cathode, and the remaining electrodes were connected together to form a composite anode. In each valve the anode potential was adjusted to give a current of 5 mA. Fig. 7 shows the quantity A, appropriate to this current, plotted against frequency.

It may be seen from the figure that, for the LS5 valve, A remains constant for frequencies between 1·8 kc and 15·8 kc. Accordingly this valve is said to be free of flicker effect: for if A is found to be a function of frequency, then flicker effect is said to be present.

The curve for the MH/D4 valve shows that, over the same range of frequency, A varies as some power of the frequency. The curve for the VMP4 valve relates to frequencies between 0·25 kc and 5·25 kc, and here again A varies as some power of f. It should be noted that the slope of the line relating $\log A$ and $\log f$ is a function of the anode current.

Fig. 7 shows that, at anode currents of this order, in valves with oxide-coated cathodes, the flicker effect is much greater than the true shot effect over the whole acoustic range of frequencies. Consequently such valves are unsuited to a study of true shot effect; also in technical applications of such valves the true shot effect is unimportant in the low and middle acoustic range. The curves of Fig. 7 bear out common experience that flicker effect decreases with frequency, and hence presumably the true shot effect from an oxide cathode could be studied if measurements were made at a sufficiently high frequency.

* The additional double-diode anodes in this valve were ignored.

Figs. 8 and 9 show $\log A$ plotted against $\log I$ for two valves having oxide cathodes, namely the MH/D4 and the P220. These figures should be compared with Figs. 3 and 4 for valves with thoriated tungsten cathodes. Figs. 8 and 9 are similar in general form to Figs. 3 and 4, in that the value of A starts at unity and falls by a linear branch to a minimum value. The slope of the linear branches is once more of the order $-0\cdot5$; but now the minimum value is dependent on frequency, whereas with thoriated tungsten it is not. The linear branches are not very dependent on frequency, because in this range the current is less than 1 mA. Since the flicker effect is proportional to some power of the current, it is not dominant over the linear branch. The flicker effect tends to zero with I. To investigate this, 10 values of A were deduced for various currents less than 20 μA: the mean value so obtained was 1·03, and individual values ranged between 0·95 and 1·07.

Again, it should be noticed that for any given current the value of A is sensibly independent of the electrode arrangement as represented by changing the connection of the control grid: a change which involved a 40 to 1 change of electrode potential in the MH/D4 valve.

It is established that A tends to unity when the current tends to zero and also when it tends to the saturation limit. Hence A must pass through a minimum for some value of I: this is illustrated by Figs. 3, 4, 8, and 9. It would be of interest to know what regulates the value of this minimum. Figs. 8 and 9 show that the minimum value for an oxide cathode depends on the frequency, and hence the true limiting value, if it exists, could be studied only at radio frequencies. The available high-gain amplifier was not suitable for such an investigation.

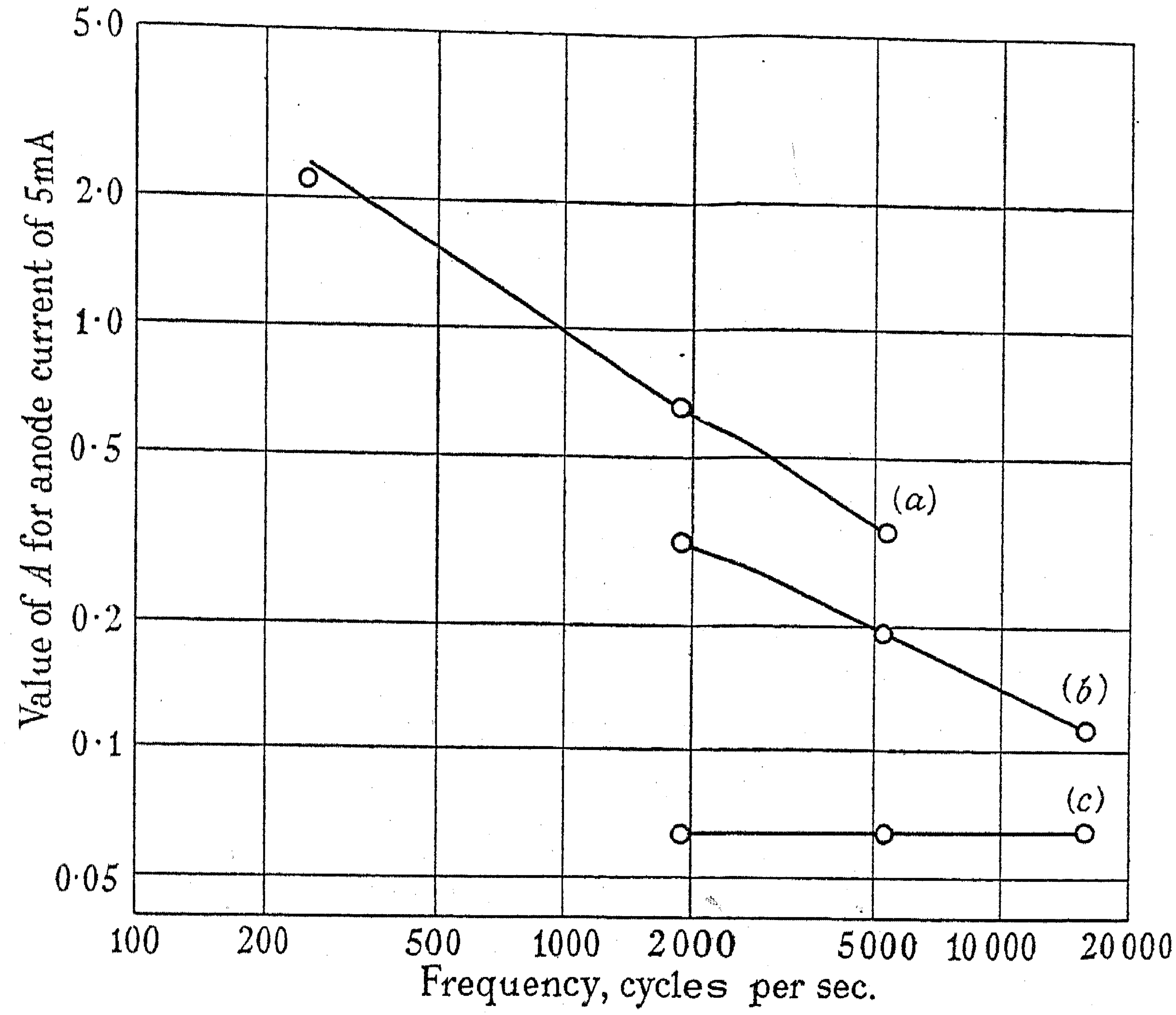

Fig. 7.—Variation of A with frequency for two oxide cathodes (VMP4 and MH/D4) and thoriated-tungsten filament (LS5).
(a) Separately-heated pentode (VMP4). (b) Separately-heated triode (MH/D4). (c) LS5 valve.

Pure tungsten cathodes are not admissible because, as is well known, they emit positive ions which produce a fluctuation voltage other than that being considered

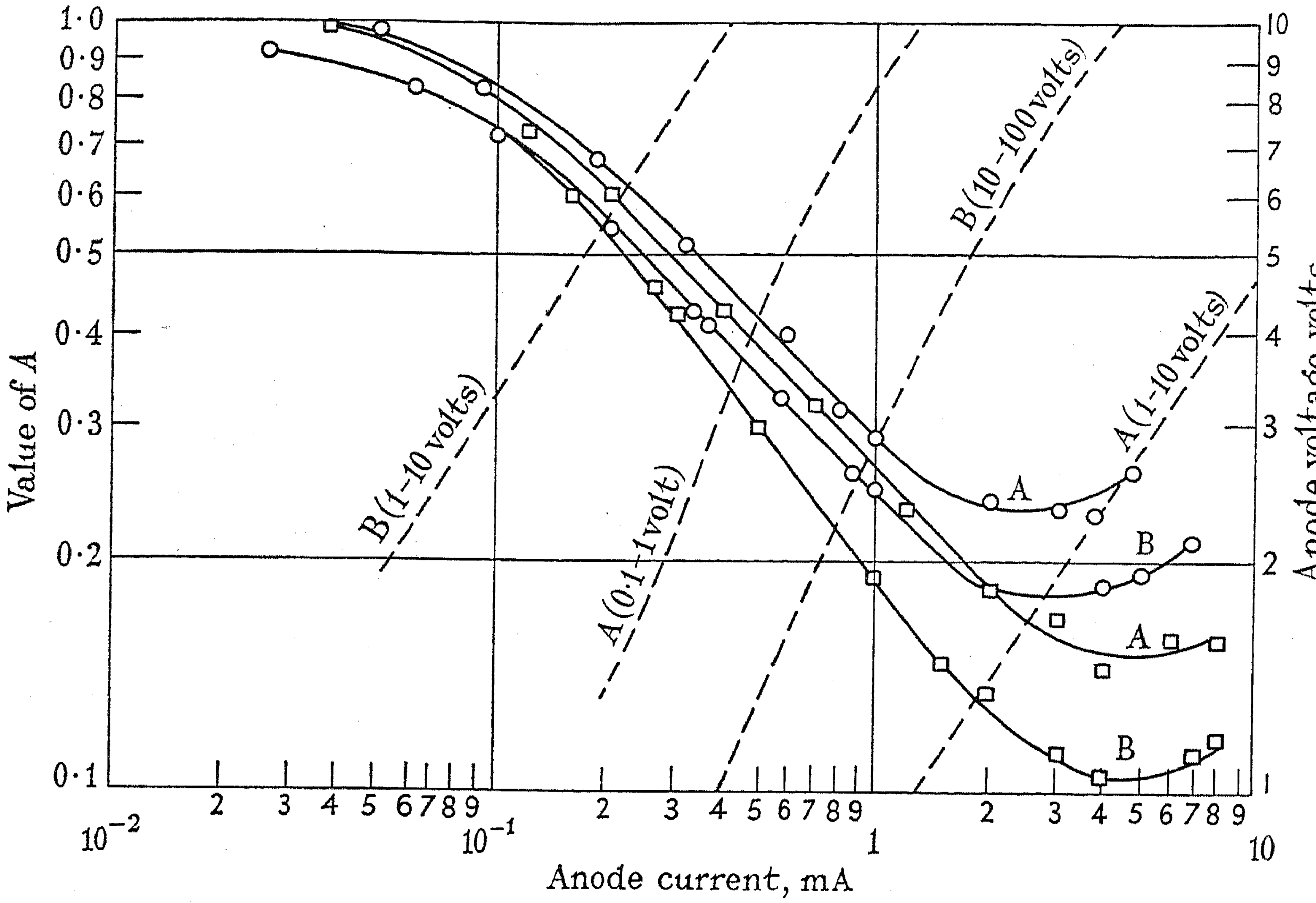

Fig. 8.—Marconi MH/D4 valve.

Curves A with grid to anode. Upper curve of each pair at 5·25 kc (○).
Curves B with grid to cathode. Lower curve of each pair at 15·8 kc (□).
— — — Valve characteristic.

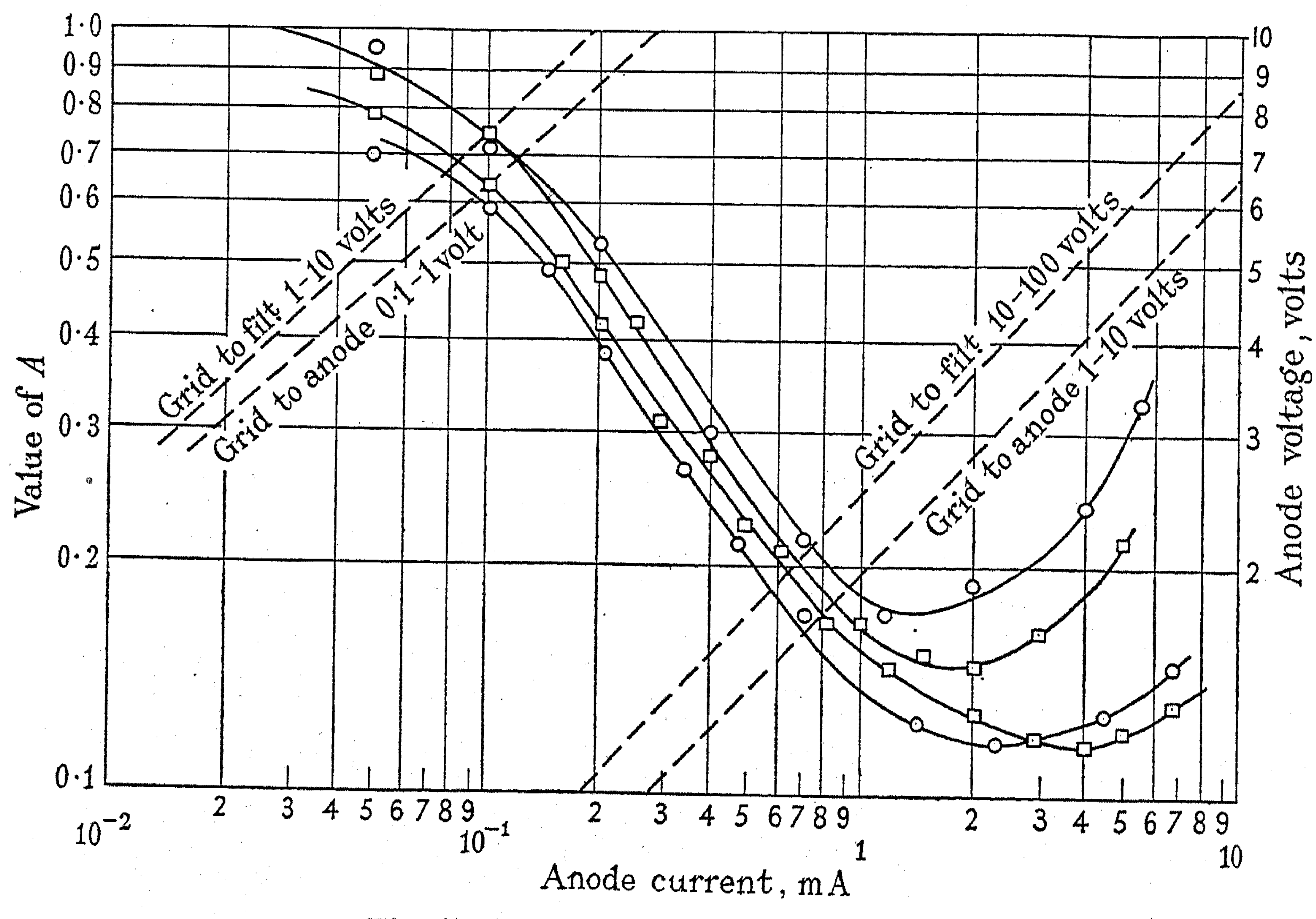

Fig. 9.—Mazda P220 oxide-coated filament.

Upper two curves "grid to anode"; □ 15 kc, ○ 5 kc. Lower two curves "grid to filament negative"; □ 15 kc, ○ 5 kc.

at the moment. Thus it has been possible to observe the "shot minimum" of A only from thoriated tungsten cathodes, and for this only two valves were available, an LS5 and a special diode. For these two valves, $A_{min.}$ (see Figs. 3 and 4) lay between 0·05 and 0·11. These curves, and other measurements, have shown that $A_{min.}$ decreases as the cathode temperature is increased. A full investigation could be made only if a wide range of valves with thoriated filaments were

available. Further, such filaments are always directly heated; the importance of "cool end effect," and of the potential drop along the filament, cannot be estimated.

(4) SHOT EFFECT IN MULTI-ELECTRODE VALVES

This paper attempts to relate the observed fluctuation voltage with an expression of the form of equation (3) or equation (6): an essential feature of these equations is that $|v|^2$ is proportional to I and is independent of the electrode potentials except so far as these dictate the relevant value of ρ. Accordingly it is of interest to see whether fluctuation voltage is affected by sharing a current I between several electrodes. For this purpose a pentode valve was employed, which could be used as a diode by connecting the screen grid, suppressor grid, and anode, together, or as a pentode. The control grid was connected to the cathode in both arrangements. The output resistance R was connected between the cathode and the common connection from the batteries supplying the various electrodes: thus it carried the total current emitted from the cathode and collected by the electrodes. Fig. 10 shows the apparent current plotted against the actual current. The curves relate to the valve used as a diode and also as a pentode with three different anode potentials. The total current was varied by means of the potential of the screen grid. It may be seen from Fig. 10 that the apparent current is sensibly independent of the manner in which a given current I is shared between several electrodes. The same valve was used as a triode, and the total current varied by adjusting the negative potential of the control grid. With this arrangement A was slightly larger than is shown by the diode curve of Fig. 10, the discrepancy increasing with the negative bias, but never exceeding 20 per cent. Possibly this effect is due to the control grid wires, which were very close to the separately heated cathode, producing just underneath them a weak or even negative field. It is worth noting that, in the pentode use, the value of ρ is not infinite: the relevant value was found experimentally as explained

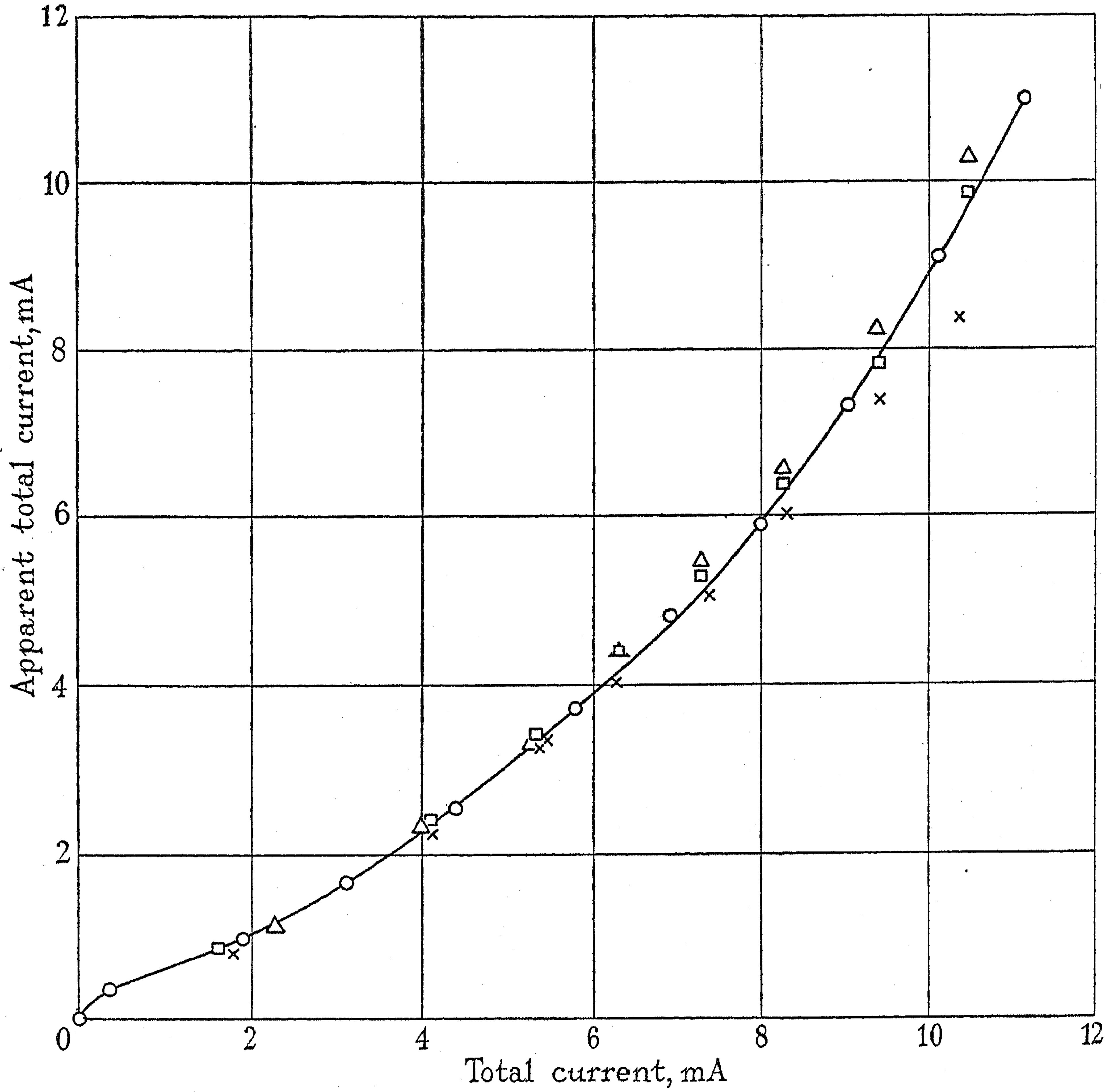

Fig. 10

Points ○ refer to diode connection with resistance in anode lead.
Points × refer to pentode connection with resistance in cathode lead and $E_a = 0$.
Points □ refer to pentode connection with resistance in cathode lead and $E_a = 18$ V.
Points △ refer to pentode connection with resistance in cathode lead and $E_a = 80$ V.

in Section (3), and is dominated by the screen-to-cathode slope resistance.

But although the total fluctuation voltage produced by a given current seems to be independent of the potentials of the electrodes among which it is shared, yet it appears that the fluctuation due to a portion I_1, of a total current I_T, flowing to electrode No. 1, is not equal to I_1/I_T of the whole. Thus consider first Fig. 11, which shows apparent anode current plotted against measured anode current, for three different constant values of anode potential: the anode current being varied by means of the screen-grid potential, and control and suppressor grids being connected to cathode. It may be seen that the ratio of apparent to true anode current increases as the anode potential decreases. This is not in disagreement with equation (6), for we associate A in this equation with the total current drawn from the cathode. Since a given anode current can belong to a wide range of total current, it is to be expected that different values of A are appropriate to a given anode current. But if the total current is recorded when a given anode current is obtained in some specified manner, the appropriate value of A can be found by reference to the curve for the valve used as a diode (Fig. 10). It might then be expected that $|v|^2$ could be predicted from equation (6) if the relevant current was the anode current and the relevant A was for the total current. The value of $|v|^2$ was observed as a function of anode current with the load resistance R in the anode lead and with the screen connected directly to its battery: also with R in the screen lead and with the anode connected directly to its battery. Fig. 12 shows the values deduced for screen or anode current plotted against the total current: it also shows the values deduced from equation (6) using the values of current and of A explained previously: the predicted values are appreciably greater than the true values. A similar curve was obtained at a frequency of $5\cdot25$ kc instead of $0\cdot25$ kc, and then

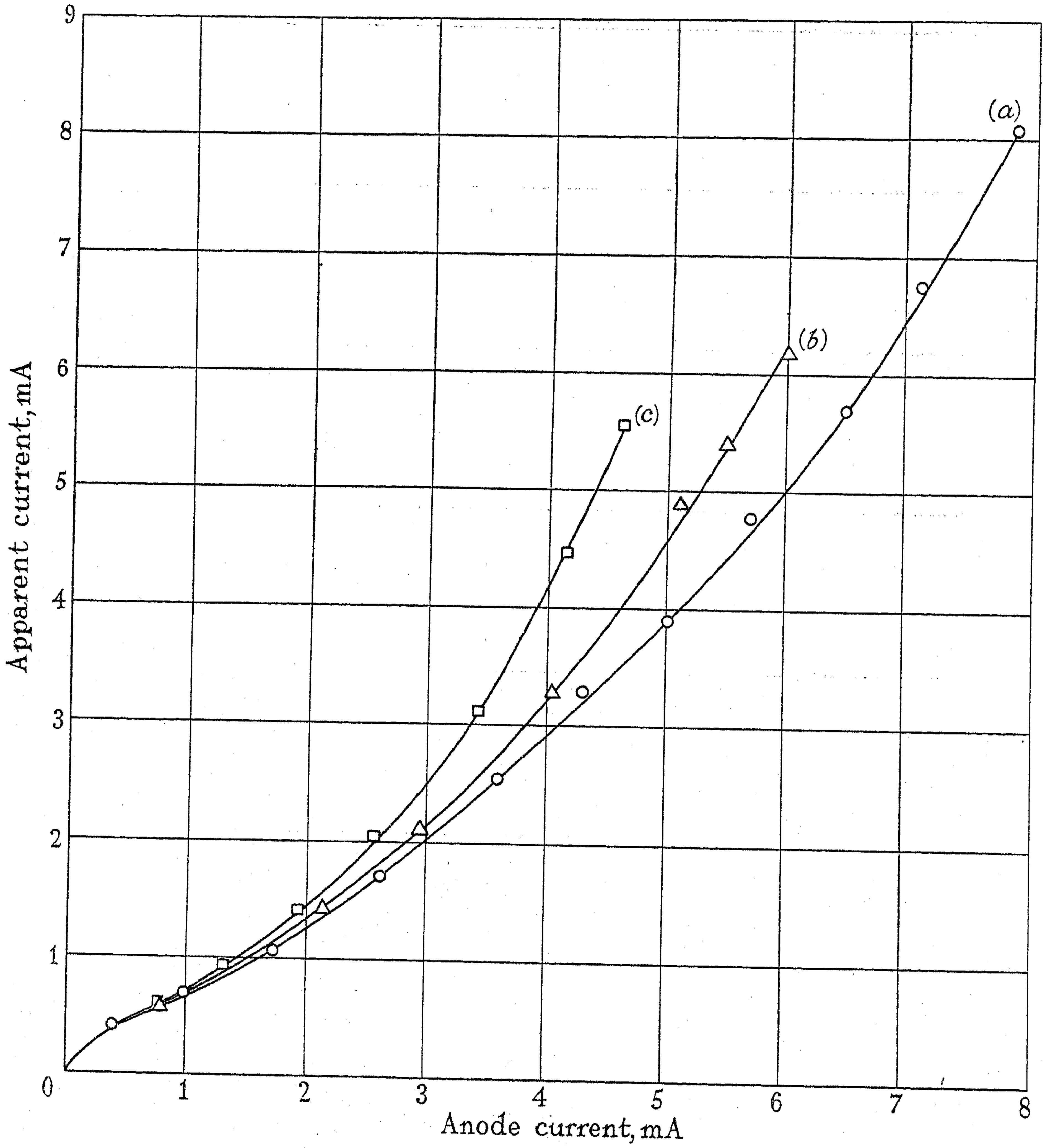

Fig. 11.—Anode fluctuations of VMP4 valve used as a pentode over 0–10 kc band. Current varied by adjusting screen-grid voltage.

Curve (a): $E_a = 190$ volts. Curve (b): $E_a = 30$ volts. Curve (c): $E_a = 18$ volts.

the predicted values were too small (see Fig. 13). But at both frequencies the discrepancy between the predicted and the true screen current, at any given total

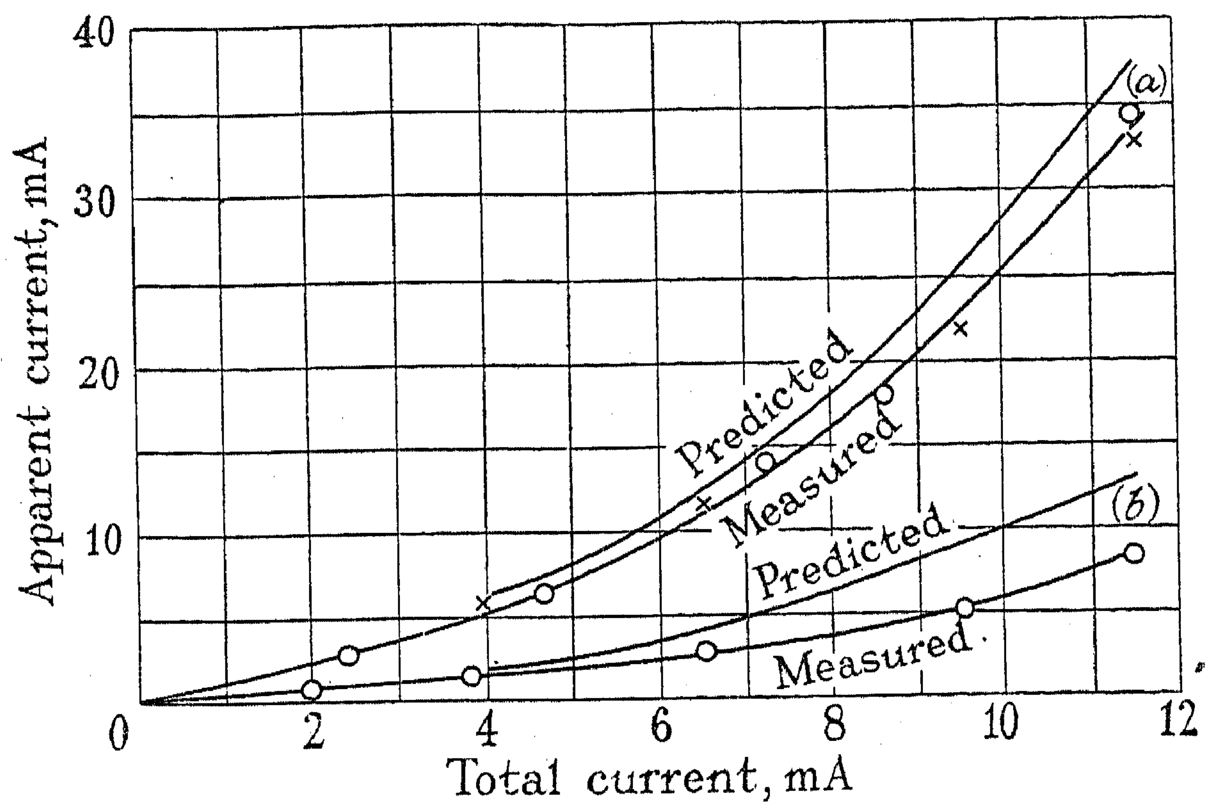

Fig. 12.—VMP4 valve. Fluctuations at 0·25 kc.
(*a*) Measured and predicted values of anode fluctuations.
(*b*) Measured and predicted values of screen fluctuations.

current, is apparently equal to the discrepancy between the predicted and the true anode current. Thus it is as though there were a fluctuation current circulating between screen and anode, and superposed on the fluctuations arising at the cathode. It would seem that if there were no fluctuations arising at the cathode, there might still be fluctuations in the portion falling on the anode. Experiment appears to show that this is correct, for the fluctuations arising at the cathode can be smoothed artificially by suitable use of the control grid, if this is connected as shown in Fig. 14. A resistance R′, of value about 2 kΩ, is connected in the common cathode lead. The control grid is connected to cathode through a resistance of about $\frac{1}{2}$ MΩ, and a capacitance C of about 2 μF is connected as shown in the figure. The fluctuation voltage produced by R′ is imposed on the control grid and thereby excites corresponding anti-phase fluctuations. The screen was connected to anode and the values of $|v|^2$ produced by

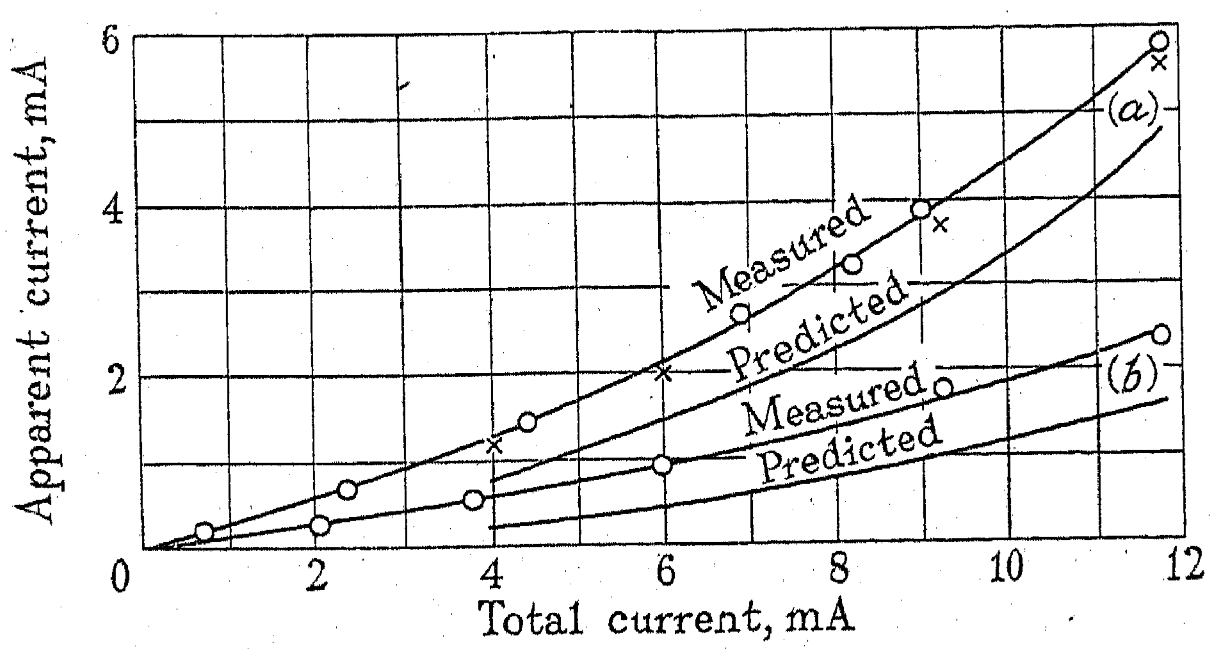

Fig. 13.—VMP4 valve. Fluctuations at 5·25 kc.
(*a*) Measured and predicted values of anode fluctuations.
(*b*) Measured and predicted values of screen fluctuations.

R were observed as a function of anode current. The lowest curve in Fig. 15 shows the values thus deduced for total apparent current plotted against the actual total current. It may be seen that the action of the control grid almost completely removes the fluctuation.

The two pairs of curves show the fluctuation voltage, expressed in terms of equivalent current, obtained by placing the load resistance R in the anode lead or in the

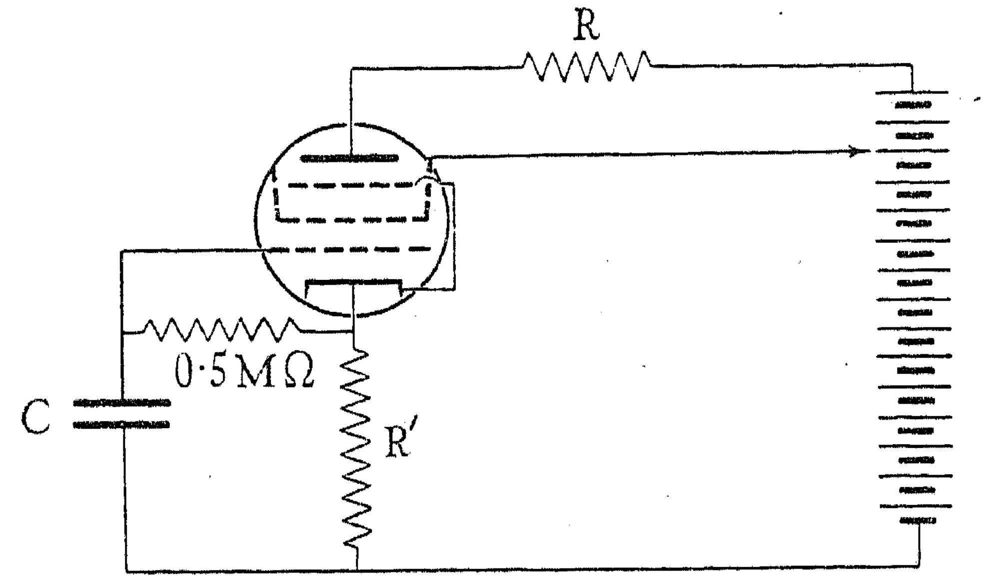

Fig. 14

screen lead for two different constant values of anode potential. It may be seen that sensibly the same result is obtained whether the fluctuations are measured in the screen lead or in the anode lead. Measurement in these two circumstances should not yield precisely the same result, for the fluctuations arising from the cathode

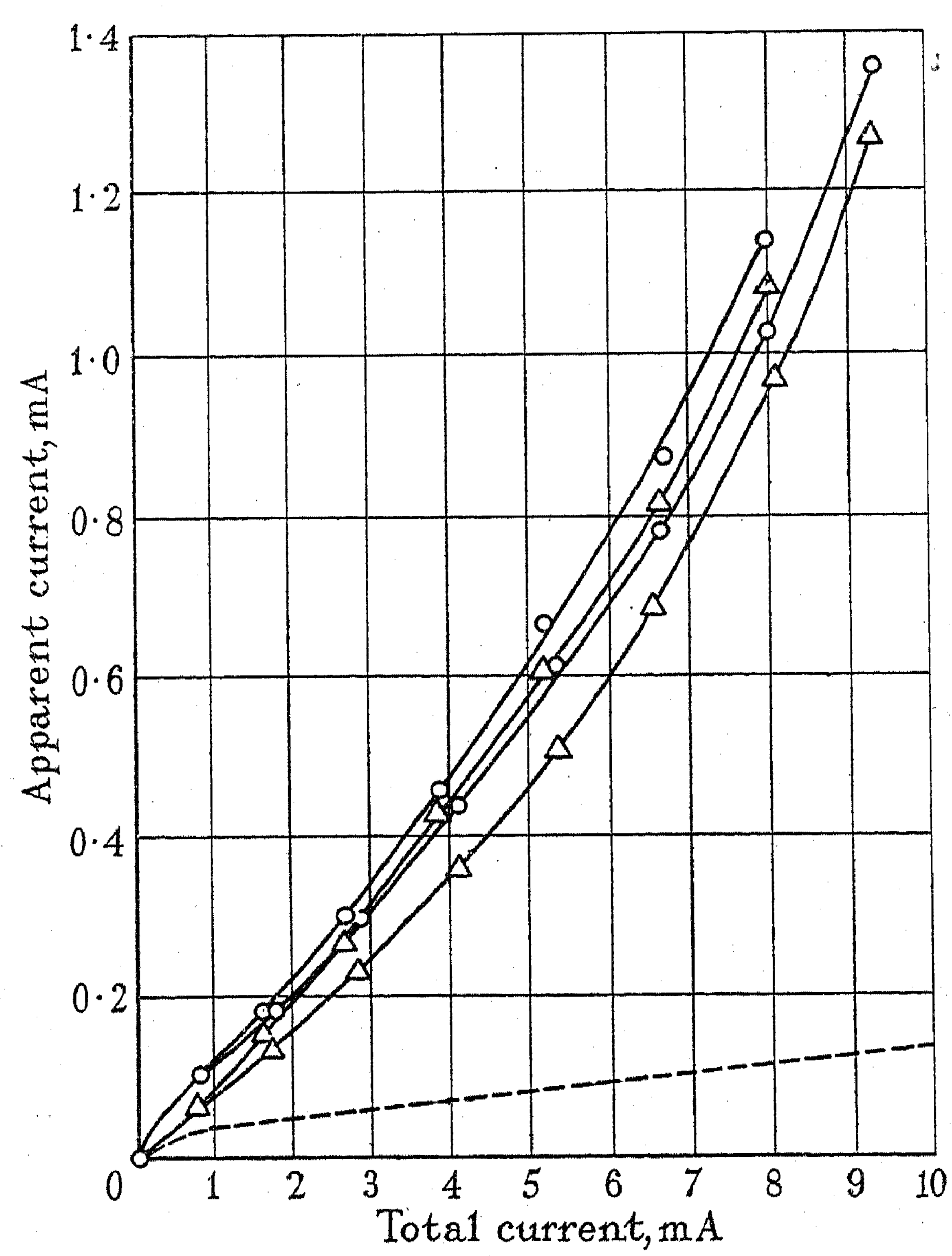

Fig. 15

○ Fluctuations in anode lead. Upper pair: $E_a = 63$ volts.
△ Fluctuations in screen lead. Lower pair: $E_a = 240$ volts.
Dotted line shows fluctuations in valve used as diode, indicating the extent of "smoothing."

have not been reduced completely to zero. Thus, suppose the fluctuation in the anode lead is due to a part C_1 arising from the cathode, and a part B due to the sharing of current between the two electrodes: similarly, that the fluctuation in the screen lead is due

to C_2 and B. So that $F_a = C_1 + B$, and $F_b = C_2 + B$; where it is supposed that $C_1 + C_2 \equiv D$ is equal to the total residue cathode effect measured by the lowest curve in the figure. Accordingly, $F_a - F_b = C_1 - C_2$. It may be seen that $F_a - F_b$ is in general less than D. Further, on the assumption that $C_1 = (I_A/I_T)D$ and $C_2 = (I_S/I_T)D$, it is possible to deduce the curve of B from the observed values both of F_a and of F_b. When this is done, it is found that the two curves of B are indistinguishable. In these measurements steps were taken to make D small compared with B, and hence the results are unsuitable for testing the validity of the method of deducing C_1 and C_2 from D. This is better investigated from Figs. 12 and 13, in which the points marked by a cross are obtained by adding the discrepancy between the calculated and the observed screen fluctuation to the predicted anode fluctuation. The crosses are very close to the curve of observed anode fluctuations. It must be noted that the value of B, as defined previously, is negative at 0·25 kc and positive at 5·25 kc.

(5) SIGNAL/NOISE RATIO

It follows from equation (7) that the ratio of signal to noise is given by the expression

$$\left(\frac{\text{Signal}}{\text{Noise}}\right)^2 = \frac{g^2 V_S^2}{\left(\frac{4kT}{R} + 2IeA\right)df}$$

provided there is no thermal agitation in the grid circuit. Taking $T = 290°$ K., I in mA, and R in kΩ, this reduces to

$$\left(\frac{\text{Signal}}{\text{Noise}}\right)^2 = \frac{3\cdot 1 \times 10^{15} g^2 V_S^2}{IA\left(1 + \frac{1}{20IAR}\right)df}$$

Since A is seldom much less than $\frac{1}{10}$, the term in R will be comparatively unimportant so long as IR is greater than, say, 5.

Experiment shows that the curve relating g^2/I and I always passes through a maximum value. This maximum ranges between 10 in modern oxide-coated cathodes, and 0·1 in thoriated-tungsten cathode valves, such as the LS5 type. Since A falls as I increases, the maximum value of $g^2/(IA)$ is likely to occur at a current greater than that corresponding to the maximum value of g^2/I. Further, we have seen that in general there is a considerable portion of the valve characteristic over which A is roughly proportional to $I^{-0\cdot 5}$; now if the valve obeys the $\frac{3}{2}$-power law we have g^2/I proportional to $I^{-0\cdot 33}$, and therefore $g^2/(IA)$ varies roughly as $I^{+0\cdot 17}$. Thus if the anode current is greater than that giving the maximum value of g^2/I, and lies on the linear branch of the variation of A, the operating conditions will be close to the optimum, which will not be critical. Experiment has shown that there is indeed a considerable range of current over which the signal/noise ratio is sensibly constant.

It might be thought that the predominance of flicker effect with oxide cathodes would preclude their use in circumstances where the signal/noise ratio is not dominated by thermal agitation in the input circuit. This is not so, because they are possessed of a maximum value of g^2/I, which far outweighs the advantage of the smaller values of A associated with the thoriated tungsten, especially since this maximum occurs at small currents, for which flicker effect has been seen to be relatively less important. For example, in the range 0–10 kc the average value of A for a current of 1 mA in an oxide-cathode valve is about 0·5 and the maximum value of g^2/I is about 10, giving an optimum $g^2/(IA)$ of about 20. For a thoriated tungsten valve, such as the LS5, A is about $\frac{1}{20}$ at the maximum value of g^2/I, which is about 0·1. Hence the optimum value of $g^2/(IA)$ is about 2. It is only in circumstances when the amplifier cuts off all frequencies above, say, 100 cycles, that it may be advantageous to use a thoriated cathode; for example, in the normal use of electrometer valves.

In a tetrode or pentode valve, it is important that the anode current should be as large as possible compared with the sum of anode and screen currents. For, with a given total current, g is proportional to I_a/I_T, and can be written $g_0 I_a/I_T$, where g_0 is the value when screen and anode are connected. We have seen that, in general, equation (6) is substantially correct, provided that A has the value appropriate to the total current, and it follows that

$$\left(\frac{\text{Signal}}{\text{Noise}}\right)^2 = 3\cdot 1 \times 10^{15}\frac{g_0^2 V_S^2}{I_T A} \times \frac{I_a}{I_T}$$

and this ratio increases in direct proportion to I_a/I_T.

Possibly this factor may be of importance in multi-grid frequency-changing valves; especially when combined with the fact that the "conversion conductance" of such valves is known to be only about one-fifth of the mutual conductance. It is perhaps worth noting that the signal/noise ratio can always be increased by connecting several similar valves in parallel, for thereby the ratio g^2/I is increased in proportion to the number of valves in parallel.

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THE NON-LINEAR THEORY OF THE MAINTENANCE OF OSCILLATIONS

By Ph. le CORBEILLER, Sc.D.

(*Lecture delivered before the* Wireless Section, *3rd April,* 1935.)

INTRODUCTION

During the last fifty years a remarkably simple and powerful theory of harmonic (or sinusoidal) oscillations has been developed, with which every student of electricity is now familiar and which has enabled engineers to predict with accuracy the behaviour of innumerable machines or networks in the electrotechnical, wireless, and transmission fields. More recently the method has been applied to mechanical, electro-mechanical, and acoustical systems, so that it now forms the technical background of the gramophone and cinema industries, and is beginning to be used in civil engineering.

This fundamental theory presupposes that all the physical phenomena involved in the working of a given system follow "linear" laws (such as Newton's law, $f = m\ddot{x}$; Ohm's law, $e = Ri$; Faraday's law, $e = Ldi/dt$, and so forth); for which reason we shall call it the "linear theory."

Important problems, however, which can be shown to depend upon *non-linear* differential equations, are constantly arising. Since the mathematical theory of these is but little advanced, the study of such problems, desirable as their solution may be, presents great difficulties. It is entirely in keeping with the essential mathematical disparity between linear and non-linear equations that the so-called "non-linear" physical phenomena should present interesting new features, specifically their own. As instances of such phenomena may be quoted the generation of oscillations, synchronization, oscillation hysteresis, frequency de-multiplication, cross-modulation, and so forth. The theory of all of these has been carried up to a certain point, explaining the main characters of the phenomena but still leaving much to be found. The first of all in logical order, and presumably the simplest, should be the phenomenon of the generation of oscillations, and it is the purpose of the present lecture to give a connected account of what has been found in this field, and to point out the problems still awaiting a solution.

INADEQUACY OF LINEAR THEORY.

Let us first present the line of argument by which it is attempted, in elementary textbooks, to explain how oscillations are generated (or "maintained," or "sustained") in a series resonant circuit. This circuit (L, C) will have a resistance R and, if it were alone, disturbances induced in it would die out exponentially according to the linear equation

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now if the equations of the whole generating system are set up, assuming linear relations throughout (in particular, if a triode is present, taking

$$\rho i_a = e_a + ke_g \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for the equation of its characteristics) it is found that equation (1) is replaced by

$$L\ddot{q} + (R - r)\dot{q} + \frac{q}{C} = 0 \quad . \quad . \quad . \quad (3)$$

in which r is an expression which depends upon the constants of the circuit, and has the same "dimensions" as a resistance.

It is seen that the combined effect of the whole sustaining system (for instance, battery, triode, and retroaction circuit) is to introduce into the resonant circuit the so-called *negative resistance* $(-r)$. If, then, we are told, the values of the constants are such that r becomes just equal to R, then equation (3) itself becomes

$$L\ddot{q} + \frac{q}{C} = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is satisfied by a sinusoidal function of time, $q = Q \sin (2\pi t/T)$, of period $T = 2\pi\sqrt{(LC)}$, a mathematical entity apparently adequate to represent a "sustained oscillation."

This mode of reasoning is open to three objections, and it would be difficult to say which of these is the most damaging:—

(*a*) Surely r cannot be exactly equal to R; it must be smaller or greater, no matter by how small a quantity. If smaller, oscillations will die out; if greater, they will increase indefinitely. We should never be able to hit upon "sustained oscillations."

(*b*) Given that r is exactly equal to R, the amplitude of the oscillations satisfying equation (4) would be determined in each case by the "initial conditions" of the movement; physically this would mean that Q, and therefore the power which the generator will put into a given load, can take any value according to the way the generator has been started—surely a distressing consequence.

(*c*) Lastly, our theory does not fit with experiment. For, if we control r by, say, varying the coupling in the feed-back transformer, it is true that the generator will refuse to oscillate as long as r keeps smaller than R; but oscillations will not increase indefinitely when r gets bigger than R, which remark, one might say, answers objection (*a*) as far as practice goes, but at the same time gives the lie to our fundamental equation (3).

Surely, then, the linear theory fails to account for the working of an oscillator. But there is a deeper and simpler reason why it should do so, to which we shall now proceed.

The purpose of an oscillator is to bring power to the load at a definite frequency. If we ask where this power comes from, the answer in the case of a triode is "from the plate battery." Now a battery, if connected directly to a load, will deliver power into it at zero frequency; so that if we consider our oscillator to be composed of three parts (see Fig. 1), where (i) is the battery, (iii) the load, and all the rest of the system forms one intermediate part (ii), we see that the function of the whole part (ii), triode and network, is that of a frequency transformer.

It is not at all surprising, then, that linear equations should fail to account for the functioning of an oscillation generator. It is, in fact, a fundamental property of linear differential equations with constant coefficients, or of the physical transmission systems which they represent, that whenever one of the variables (a velocity, a current, etc.) oscillates harmonically at frequency f, all the other variables will oscillate harmonically at the same frequency. The equations give us then the relative amplitudes and phases of all these synchronous oscillations. Conversely, when the problem is to account

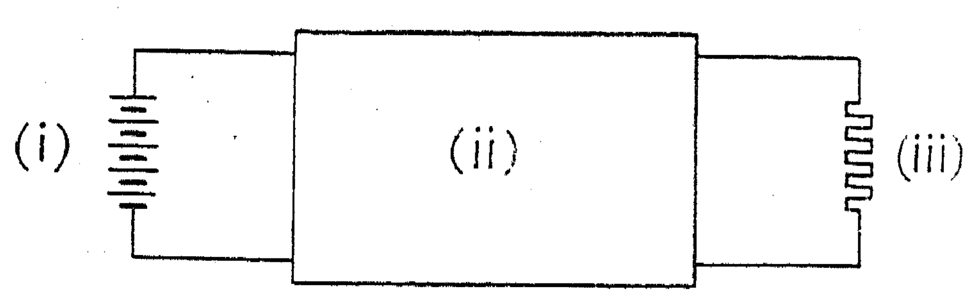

Fig. 1

for a frequency transformer like our triode oscillator, no linear equation or equations whatsoever will be equal to the task. This is indeed what we found of the otherwise promising equation (3), and we see that we need not try out any other linear combination, but should rather plunge resolutely into non-linear theory.

INTRODUCTION OF NON-LINEARITY BY LORD RAYLEIGH

The inadequacy of the linear theory had already appeared to Lord Rayleigh, and with his usual deep and constructive insight he saw at once the correct way out of the difficulty.* Equation (3) can be written as

$$\ddot{q} + K\dot{q} + n^2q = 0 \quad . \quad . \quad . \quad . \quad (5)$$

Lord Rayleigh said "When K is negative, so that small vibrations tend to increase, a point is of course reached after which the approximate equations cease to be applicable. We may form an idea of the state of things which then arises by adding to equation (1) a term proportional to a higher power of the velocity. Let us take

$$\ddot{q} + (K\dot{q} + K'\dot{q}^3) + n^2q = 0 \quad . \quad . \quad . \quad (6)$$

in which K and K' are supposed to be small. The approximate solution of (6) is

$$q = A \sin nt + \frac{K'nA^3}{32} \cos 3nt \quad . \quad . \quad (7)$$

* See Reference (4).

in which A is given by

$$K + \tfrac{3}{4}K'n^2A^2 = 0 \quad . \quad . \quad . \quad . \quad (8)$$

From (8) we see that no steady vibration is possible unless K and K' have different signs. . . . If K be negative and K' positive, the vibration becomes steady and assumes the amplitude determined by (8). A smaller vibration increases up to this point, and a larger vibration falls down to it."

We have in these few words the notion of an asymptotically periodic solution put before us in a remarkably concise way; perhaps even too concise, but we shall come back to it later on. For the present let us remark that while Lord Rayleigh in the above passage has shown what mathematical expression $K'\dot{q}^3$ might suitably be added to the basic equation (5), he has not told us where this term might come from physically. It is of course important to clear up this point and we shall now do so, beginning with the simplest electrical oscillator of the series type.

THE SERIES OSCILLATOR

Let us consider* a circuit (Fig. 2) made up of four elements in series: an ordinary L, R, and C, and a certain dipole which we shall denote by the letter N. Let us

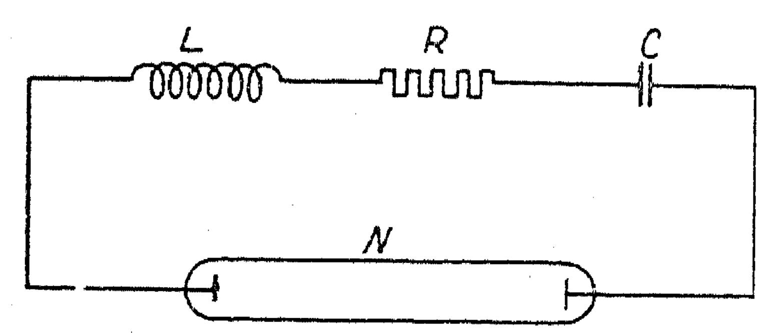

Fig. 2

suppose that N is such that the relation between the potential difference v between its terminals and the current i through it is given by the equation

$$v = -\rho i + \gamma\frac{i^3}{3} \quad . \quad . \quad . \quad . \quad (9$$

Then the equation for the current in the series system will be

$$L\ddot{q} + (R - \rho)\dot{q} + \gamma\frac{\dot{q}^3}{3} + \frac{q}{C} = 0 \quad . \quad . \quad (10)$$

that is, exactly of the type suggested by Lord Rayleigh.

Now can we actually have a dipole with such a characteristic as (9)? The answer is: We can, and in two different ways. In the first place a series dynamo, if we take into account the two possible directions of the current through it, and observe carefully the signs of v and i, has precisely such a characteristic (Fig. 3). Therefore, a series dynamo should be able to sustain oscillations in a series circuit (see Appendix 1). In the second place—and this is much more important in the wireless field—there exist several kinds of gas-filled tubes, e.g. an ordinary neon lamp, whose characteristics at first sight differ from (9), but can be made to serve the same purpose. Such a characteristic will have, let us say, the shape of curve N in Fig. 4. Let us then pass a constant current I_0 through the two branches N and L, R, C, in parallel (Fig. 5). This constant current I_0 will flow

* See Reference (17).

through the dipole N only, and a constant potential difference V_0 will appear at the terminals of N and of C. Let us suppose that I_0 has been so chosen that the point (I_0, V_0) lies on the falling part of characteristic (N). It is possible for an alternating current i to flow in the *series* circuit NLRC, without the continuous regime in

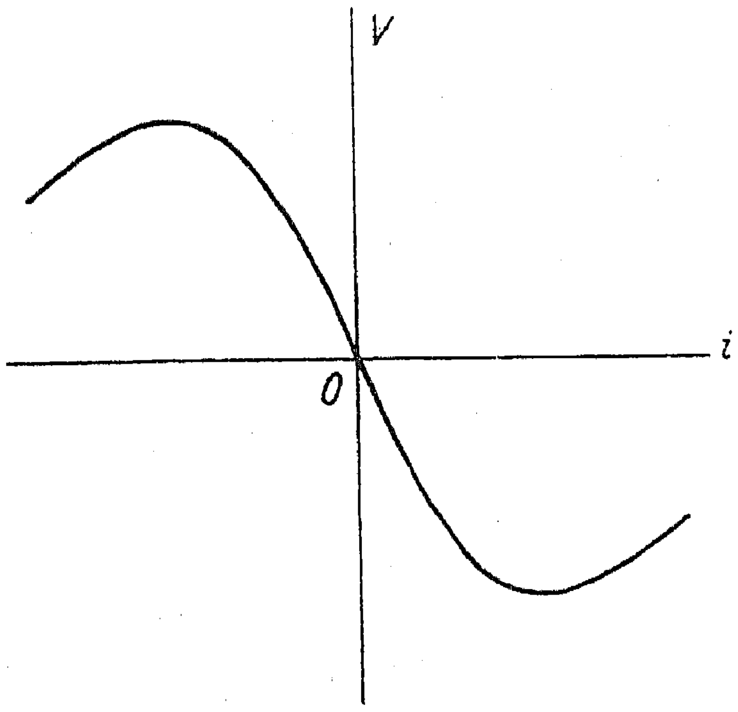

Fig. 3

the external circuit being changed by it. Calling v the alternating p.d. between the terminals of N, which corresponds to i, we shall have, since the p.d. across the closed circuit LRCN should be nil,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt + v = 0$$

A neon-lamp characteristic, after we have thus transferred the origin to a point on its falling branch, is shaped somewhat like that of a series dynamo, only it is not symmetrical. But it can be shown* that such dissymmetries in the characteristic only give rise to dissymmetries in the resulting oscillation, without playing a part in the maintenance process. We may therefore

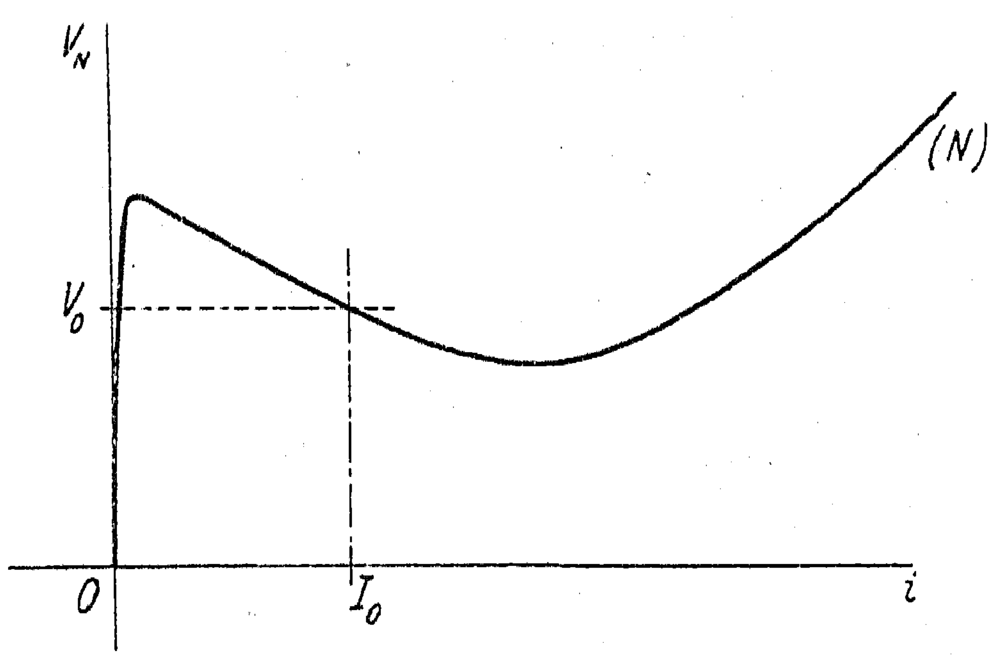

Fig. 4

adopt (9) as describing sufficiently well the relation between v and i, and we get

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt - \rho i + \gamma\frac{i^3}{3} = 0 \quad . \quad (10a)$$

which is the same as equation (10).

We see, then, that if the load R is big, i.e. $R > \rho$, the continuous regime will be *stable* and the whole of the current I_0 will flow through N, no current at all flowing through L, R, C; in other words, the generator will refuse

* See Reference (11).

to oscillate. But, if the load R is sufficiently light, that is, $R < \rho$, the continuous regime will be *unstable* within the system N, L, R, C; an alternating current i will flow through L, R, C, and a pulsating current $(I_0 - i)$ through N. In this case we shall indeed have obtained a frequency transformer.

We can easily get a physical view of what happens in the oscillatory régime. The resistance terms, $-K\dot{q} + K'\dot{q}^3$, can be written $-K'\left(\frac{K}{K'} - i^2\right)i$. We see, then, that they are equivalent to a negative resistance whenever the current intensity $\dot{q} = i$ is small, and to a positive one when it is big, irrespective of the direction of flow. The movement will then build up from the unstable state of rest, to be braked when the current becomes large enough; it will then die down, to be revived when the current has become sufficiently small, and so forth. The next task is evidently to make precise this vague, but essentially correct, view of things, and this is what we shall now proceed to do.

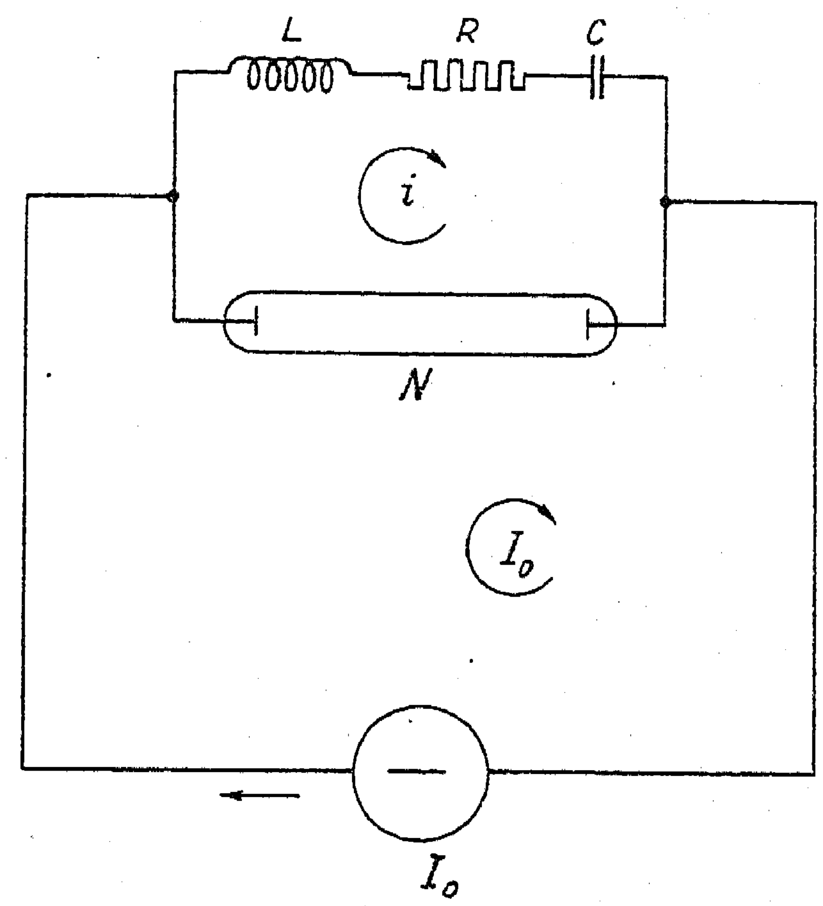

Fig. 5

GRAPHICAL STUDY OF SUSTAINED OSCILLATIONS

A simple mathematical statement, due to A. Liénard,* will enable us to study graphically the behaviour of the charge in a series system, and almost to see its oscillations build up before our eyes.

To begin with, let us change from our physical variables to non-dimensional ones, which will suit the analyst better. We may take $\sqrt{(LC)}$ as unit of time, putting $t = \sqrt{(LC)}x$. Next we may take a certain charge q_0 as a unit, putting $q = q_0 y$; then by a suitable choice of q_0 (see Appendix 3) equation (10) will be brought to the form

$$\ddot{y} - \epsilon\left(\dot{y} - \frac{\dot{y}^3}{3}\right) + y = 0 \quad . \quad . \quad . \quad (11)$$

ϵ is here a non-dimensional quantity, given by

$$\epsilon = (\rho - R)\sqrt{\frac{C}{L}} \quad . \quad . \quad . \quad . \quad (12)$$

This parameter, then, is essentially positive for sustained oscillations.

* See Reference (13).

Let us call z the derivative $\dot{y}$ of y with respect to time. Then equation (10) can be written

$$z\frac{dz}{dy} - \epsilon\left(z - \frac{z^3}{3}\right) + y = 0 \quad . \quad . \quad (13)$$

that is, we now have a non-linear differential equation of the first order.

Let us plot y and its derivative, or speed, z, along two axes (Fig. 6), and construct the curve Γ represented by the equation

$$y = \Gamma(z) = \epsilon\left(z - \frac{z^3}{3}\right) \quad . \quad . \quad . \quad (14)$$

Let MN be the normal to the curve (C) satisfying equation (13) and passing through any point M in the plane. The projection Nm of NM on Oy is

$$\overline{\mathrm{Nm}} = -z\frac{dz}{dy}$$

equation (13) can therefore be written as

$$\overline{\mathrm{Nm}} = \overline{\mathrm{QM}} - \overline{\mathrm{QP}}$$

which means that, given any point M in the plane, all we have to do to obtain the tangent to curve (C) passing

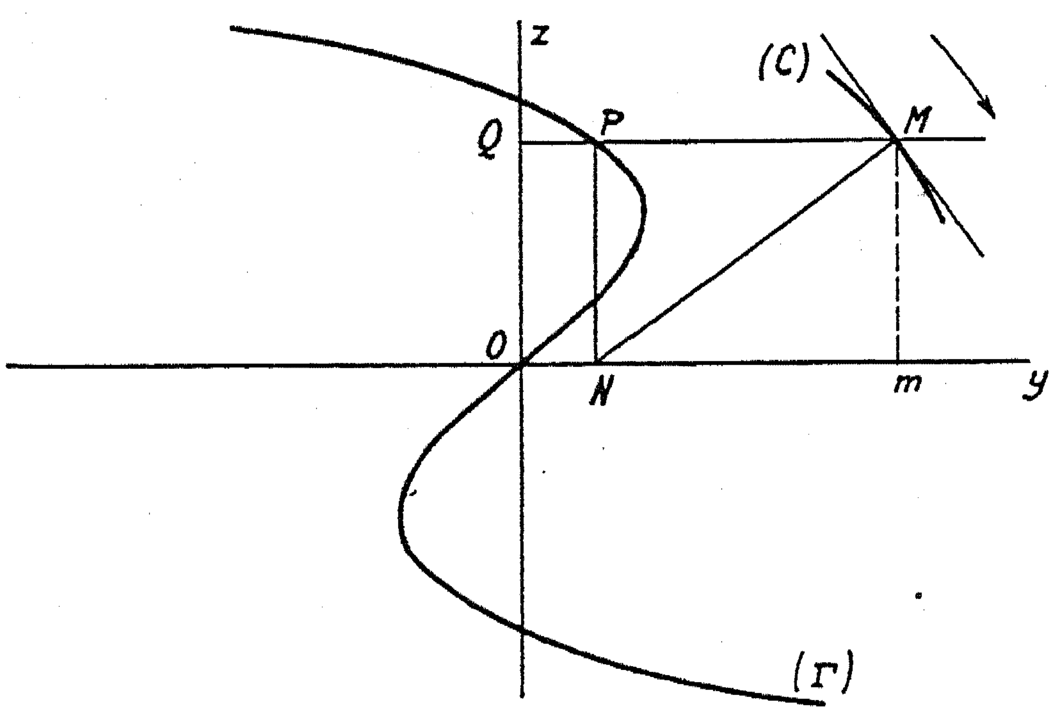

Fig. 6

through it is to draw MPQ horizontal, PN vertical, and join NM: the required tangent is perpendicular to NM.

Now if we regard equation (11) as representing the movement of point m on the straight line Oy, we see that the movement is determined as soon as we give ourselves its "initial conditions," the values of y and of $\dot{y}$ (or z) at time $t = 0$, or, what is the same thing, the position of some point M_0 in the Oyz plane. We can then draw the tangent to the "trajectory" of M according to the above construction, repeat it for a neighbouring point, and obtain eventually the whole trajectory of M as a succession of small segments.

In this way we obtain a family of curves, only one of which goes through any point of the plane. The origin is an exception; all curves start from the origin (which is, as we have seen, a position of unstable equilibrium) and describe around it, in a clockwise direction, increasing spirals.

But a very remarkable fact is that as any one of these spirals grows, its successive volutes get nearer and nearer to one another, so that eventually all the spirals curl asymptotically inside and against a certain closed curve (Fig. 7)—a particular result of a general theory due to H. Poincaré (see Appendix 2).

This has important consequences. First, since the coordinates of a point on one of the spirals are y and $\dot{y}$, both functions of time, a point M describing a closed curve in the plane corresponds to a point m having a periodic movement on the straight line Oy; the period of the movement on Oy being equal to the duration of one revolution of M on the closed curve. But y is only another name for the electric charge q, therefore *the closed curve represents a periodic, sustained oscillation* of our series electrical system.

Secondly, a purely sinusoidal oscillation would be represented in the plane by a perfect circle; our closed curve is not a circle, therefore we have discovered *a periodic oscillation which is not sinusoidal*, something that no linear theory could ever provide for us.

Thirdly, *there is only one such closed curve, no matter from what initial conditions we may have started,* for all the spirals which run away from the origin are asymp-

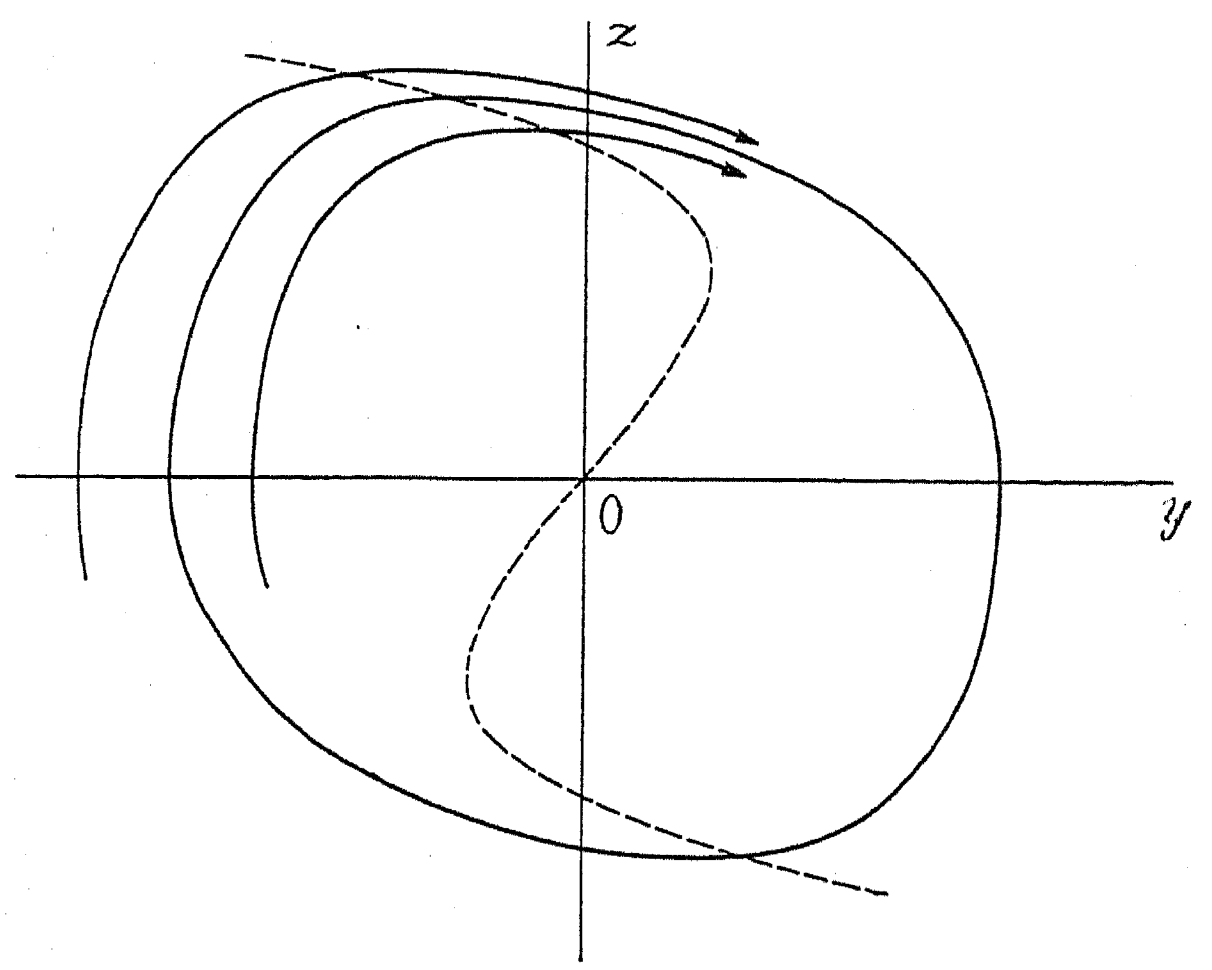

Fig. 7. $\epsilon = 1$.

totic to this one curve, and if we start from any point sufficiently far away we describe a spiral (also clockwise) which diminishes and curls asymptotically outside and against this same closed curve (Fig. 7). This entirely answers all our objections to the linear theory; there is indeed, as we see, only one periodic régime for a given oscillator working into a given load (its "amplitude," or rather maximum elongation, has not much meaning now), no matter how the oscillator was started; therefore the power, efficiency, etc., of the oscillator are unique. This, you will notice, is fully in accord with engineering common-sense, but its mathematical counterpart is much deeper in theory and much more delicate in its working than could have been predicted.

One more point. Our graphical construction is based upon a symmetrical curve given by an algebraic equation of the third degree, a cubic. But the reasoning by which we justified this construction presupposes nothing so specific. The same graphical method would apply to a curve of more or less the same shape, derived from the characteristic of the neon lamp, that is, obtained experimentally, and we need not worry at all whether

this experimental curve is more or less precisely represented by an algebraic equation $y = F(z)$ of this or that degree. This is a very welcome generalization from an engineering point of view. I would invite you to try it out yourselves, drawing a Γ curve quite freely, and you will be interested to see with what relatively great accuracy the asymptotic closed curve is obtained. This,

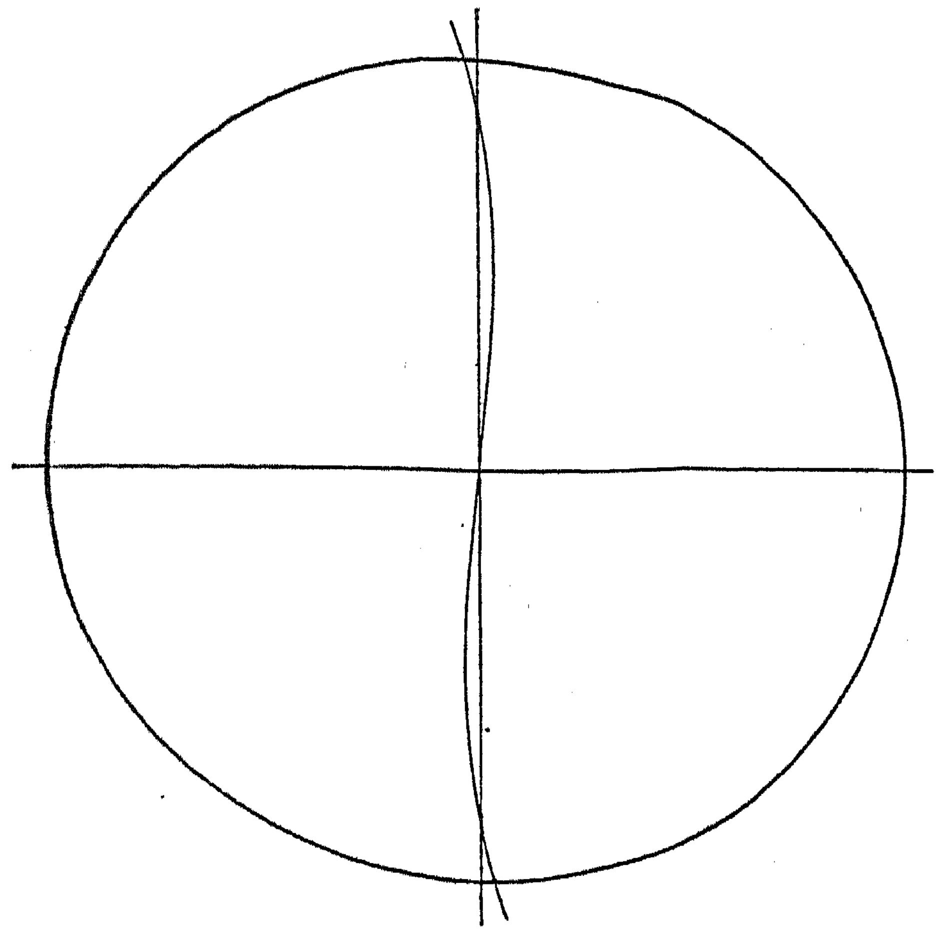

Fig. 8. $\epsilon = 0 \cdot 1$.

by the way, raises the question whether the Γ curve could be drawn entirely at random and still give rise to one or more asymptotic closed curves; this is a difficult mathematical question, to which a partial answer has been given by Liénard.*

Having obtained a closed curve which corresponds to a periodic regime of a series electrical network, we can deduce from it the shape of the oscillation of the electric charge. Because, since we know the value of the time derivative $\dot{q}$ which corresponds to any value of q, we can again build up graphically the curve $q(t)$ as a succession of small segments.

If we do this for the closed curve represented in Fig. 7, we obtain the oscillation represented by A′ in Fig. 12, that is, a sinusoid a good deal out of shape, a curve which, I repeat, we could never have derived from a linear theory.

* See Reference (12).

SINUSOIDAL AND RELAXATION OSCILLATIONS

We shall become much more familiar with this new curve if we succeed in fitting it into a continuous family, evolving from a first antecedent towards a last consequent, the shape of both being simple and accurately known. This is indeed, as I see it, a central point in our subject.

Let us go back to equation (11),

$$\ddot{y} - \epsilon\left(\dot{y} - \frac{\dot{y}^3}{3}\right) + y = 0 \quad . \quad . \quad . \quad (11)$$

and let us suppose, to begin with, that ϵ is very small; that is, $\epsilon << 1$. The Γ curve will then lie very flat against the Oz axis; the corresponding closed curve will be very nearly a circle (Fig. 8), and the corresponding oscillation very nearly a sinusoid.

The idea suggests itself of developing this unknown oscillation in a Fourier series, the successive coefficients of which are likely to decrease rapidly. This is what Lord Rayleigh has very naturally done. His result, given by equations (7) and (8), reads in our present notation

$$y = 2 \sin x + \frac{\epsilon}{12} \cos 3x + \ldots \quad . \quad . \quad (15)$$

(the following terms would contain the successive powers of ϵ). This gives a sinusoid distorted in the direction which we have already found. One correction term, we notice, is quite sufficient for small values of ϵ, since even for $\epsilon = 1$ it reaches only 4 per cent of the fundamental.

It is remarkable that this method gives a definite amplitude, i.e. 2, for the fundamental, whereas the amplitude of the solution of the linear equation (2) could take any value; but then this is only a particular case of the uniqueness of an integral closed curve, which general fact we have already noticed.

We can say, therefore, that we know very well the curves at one end of the chain, i.e. when ϵ is small.

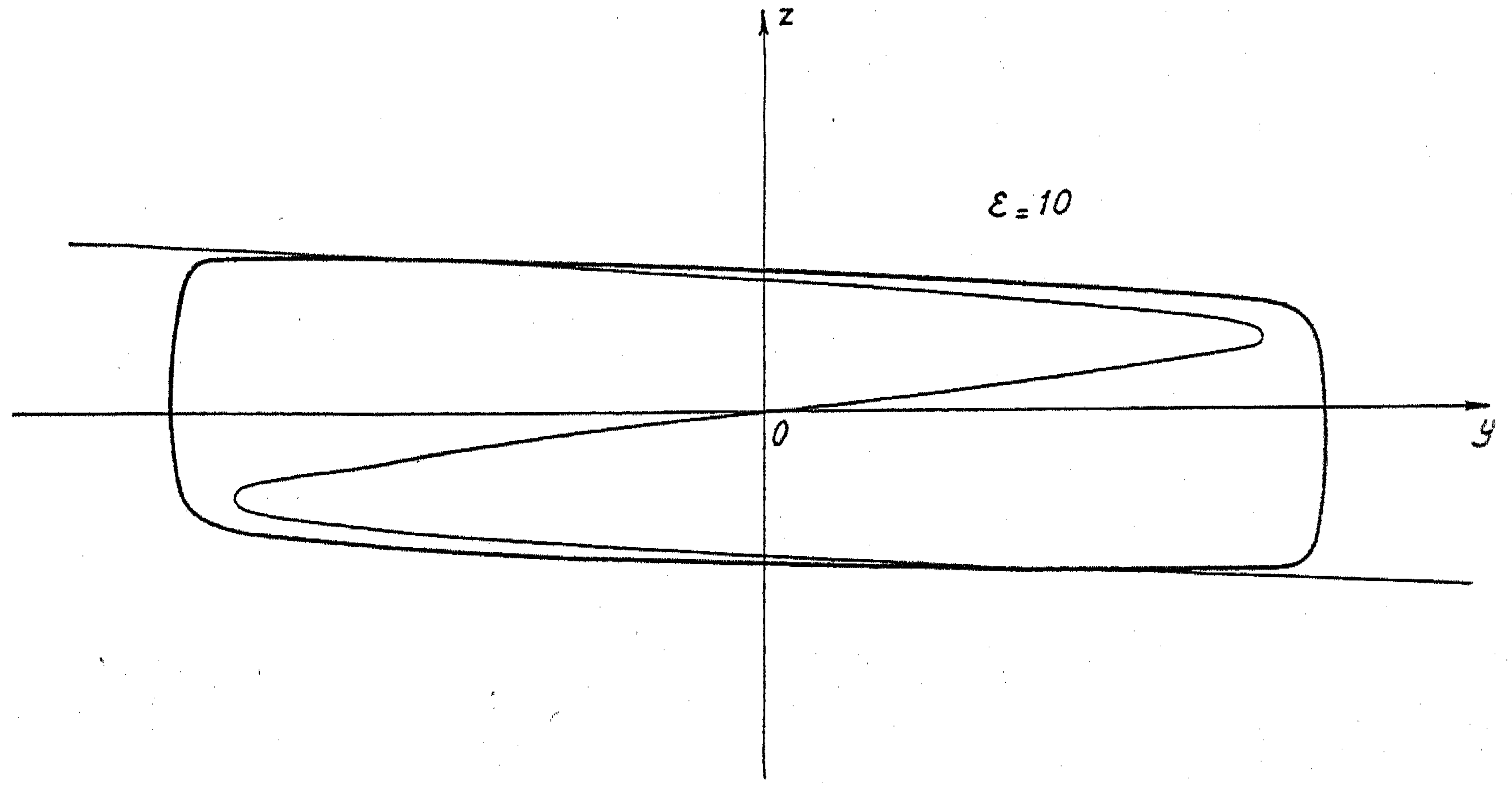

Fig. 9. $\epsilon = 10$.

Let us now see what happens when ϵ is very large ($\epsilon >> 1$).

In this case the Γ curve presents a very ample zig-zag, in the general direction of the Oy axis (Fig. 9). If, in order to get a clearer picture, we put

$$x_1 = \frac{x}{\epsilon}, \quad y_1 = \frac{y}{\epsilon}, \quad z_1 = \frac{dy_1}{dx_1} = z$$

the relation between y and z becomes

$$y_1 = \left(z_1 - \frac{z_1^3}{3}\right) - \frac{1}{\epsilon^2} z_1 \frac{dz_1}{dy_1} \quad . \quad . \quad . \quad (16)$$

and therefore we can neglect the last term, provided dz_1/dy_1 is not very big. We have then

$$y_1 \simeq z_1 - \frac{z_1^3}{3} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This gives us two arcs of our wanted closed curve, that is, arcs D_1B and B_1D of the Γ_1 curve (Fig. 10), but

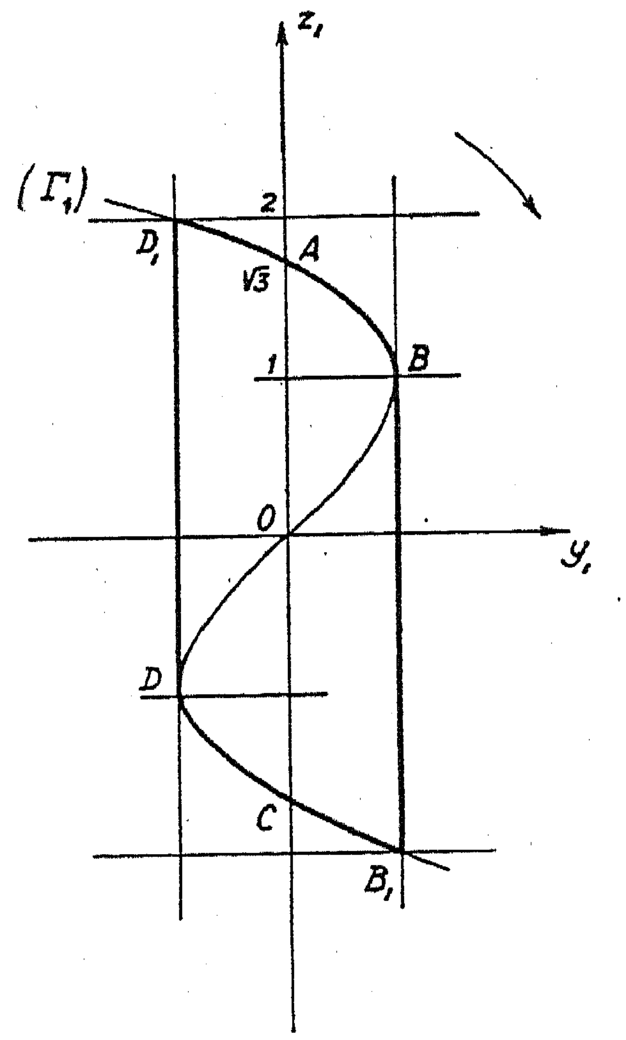

Fig. 10

leaves us completely in the dark at points B and D. However, our knowledge of the closed curve for, say, $\epsilon = 10$, obtained graphically, leaves no doubt as to what happens in the limiting case of ϵ infinite: we should follow arcs D_1B and B_1D right to their ends, and then draw a vertical line connecting the two, BB_1 or DD_1. The derivative dz_1/dy_1 being infinite on these segments, it is quite natural that approximation (17) should fail there.

Now let us take the general case. If ϵ is neither zero nor infinite, we have found our closed curve by graphical approximation, and not because we have arbitrarily chosen to do so. Not only is the analyst unable to give us an equation for that curve (even in the case where Γ is simply a cubic), but he is even unable to suggest any process of approximation the results of which would converge towards the wanted curve (see Appendix 2). You see, then, how very great the mathematical difficulties of our subject must be. But, if ϵ is zero or infinite, that is, at both ends of the chain, the situation is quite different. If $\epsilon = 0$, the closed curve C is the circle $y^2 + z^2 = 4$, the corresponding oscillation is the sinusoid, $y = 2 \sin x$. If $\epsilon = \infty$, we cannot give an equation for the whole of the (C) curve, but, if we know the equation of the Γ curve, we can give the equation of any of the four parts, D_1B, BB_1, B_1D, DD_1. We can, therefore, obtain rigorously the equation, not of the whole of the corresponding oscillation, but of its constituent parts. This derivation is given in Appendix 3 and the shape of the oscillations is then as shown in A (Fig. 11).

We obtain, then, as a limiting case of our periodic oscillations, not a periodic oscillation but a regular succession of non-periodic phenomena, represented by

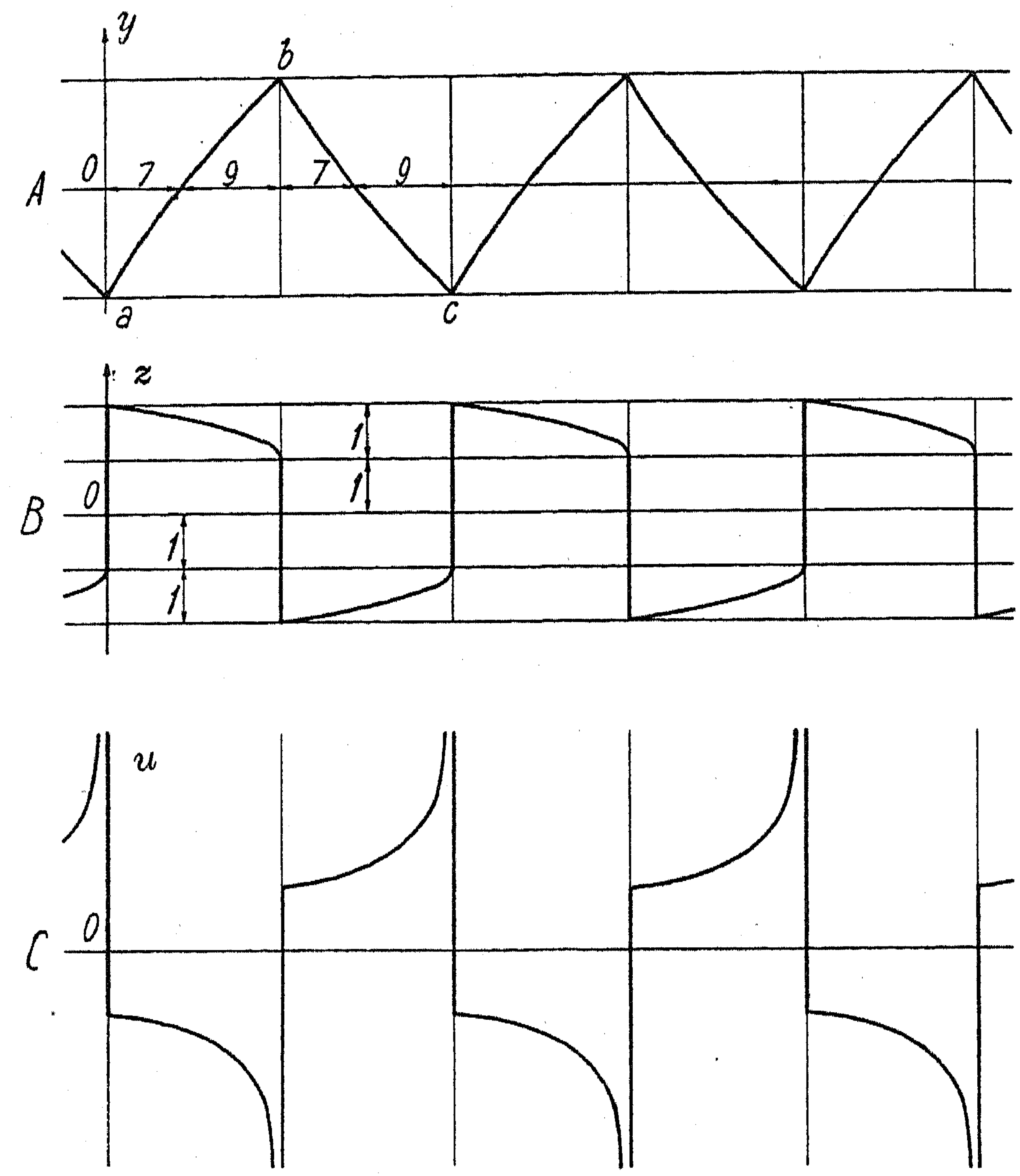

Fig. 11

such arcs as ab or bc, which in virtue of the mechanism of the system succeed each other in alternate sequence. Following van der Pol,* we shall call such oscillations *relaxation oscillations.* We see, then, that when the parameter ϵ varies continuously from zero to infinity, the corresponding periodic oscillations vary continuously from the sinusoidal type to the relaxation type.

This beautiful and simple result, obtained by van der Pol in 1926 (on oscillations of the B family, which we shall consider shortly) thus throws light upon a considerable and, as we shall see, fundamental domain of the theory of sustained oscillations. Oscillations of the relaxation type had been met with earlier by various physicists, and, to quote an early instance, Prof. Righi had in 1902 given a thorough study of the oscillations of a neon lamp.† But it was van der Pol who, on the one hand, made the first study of the oscillations in the general case (that is, for any ϵ) and showed that the sinusoidal and relaxation oscillations were their two extreme limiting cases; and, on the other hand, drew

* See Reference (10). † *Ibid.*, (5).

attention to the frequent appearance of relaxation oscillations in a number of fields of natural science.

A very important property of the two limiting cases of oscillation, the demonstration of which is given in Appendix 3, is as follows:*

Given an electric oscillator of the series type, its period in the sinusoidal regime is $2\pi\sqrt{(LC)}$, *while in the relaxation regime it is* $1 \cdot 6C(\rho - R)$.

We should note here that both sinusoidal and relaxation oscillations are limiting cases of the periodic oscillations represented by equation (10), which are never met in practice. Physically, an inductance L cannot become infinite, and, on the other hand, some residual L will always be present in an electrical circuit (as well as some mass in a mechanical system) and therefore we shall never be able to obtain $\epsilon = 0$ or $\epsilon = \infty$. Mathematically, putting $\epsilon = 0$ in equation (10) gives $\ddot{y} + y = 0$, an equation which, we have seen, cannot represent a sustained oscillation; $\epsilon = \infty$ leads to two distinct differential equations of the first order, neither of which admits of a periodic solution. Whenever ϵ is small or large (the critical value is actually, as with damped oscillations, $\epsilon = 2$) one will usually replace the actual sustained oscillations by sinusoidal or relaxation oscillations, the corresponding theories being so much simpler and leading to numerical computations; but it is well to be aware that this is in every case a physical approximation.

I shall not dwell any longer upon the opposite properties of relaxation and sinusoidal oscillations, as my subject to-day is specifically the generation of oscillations; I shall rather refer you to the list of papers at the end of the lecture for more detailed information.

THREE FAMILIES OF SUSTAINED OSCILLATIONS

The series oscillator which we have been considering offers us three distinct, if intimately connected, families of sustained oscillations.

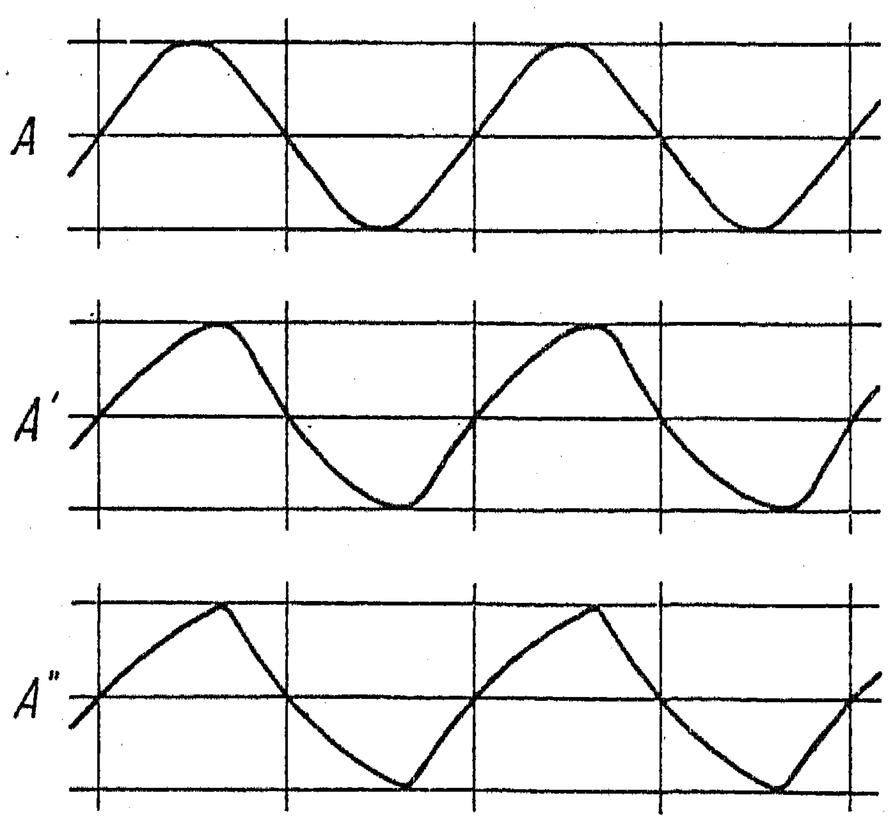

Fig. 12

We have called i the alternating current in the series circuit N, L, R, C. Let us call v_C, v_R, v_L, the potential differences across C, R, and L respectively. We then have

$$v_C = \frac{q}{C}, \quad v_R = Ri = R\dot{q}, \quad v_L = L\frac{di}{dt} = L\ddot{q} \quad . \quad (18)$$

* See Reference (10).

We have seen that $q(t)$, by a proper choice of units, corresponds to the non-dimensional function $y(x)$. Let us put

$$z = \frac{dy}{dx}, \quad u = \frac{d^2y}{dx^2} \quad . \quad . \quad . \quad . \quad (19)$$

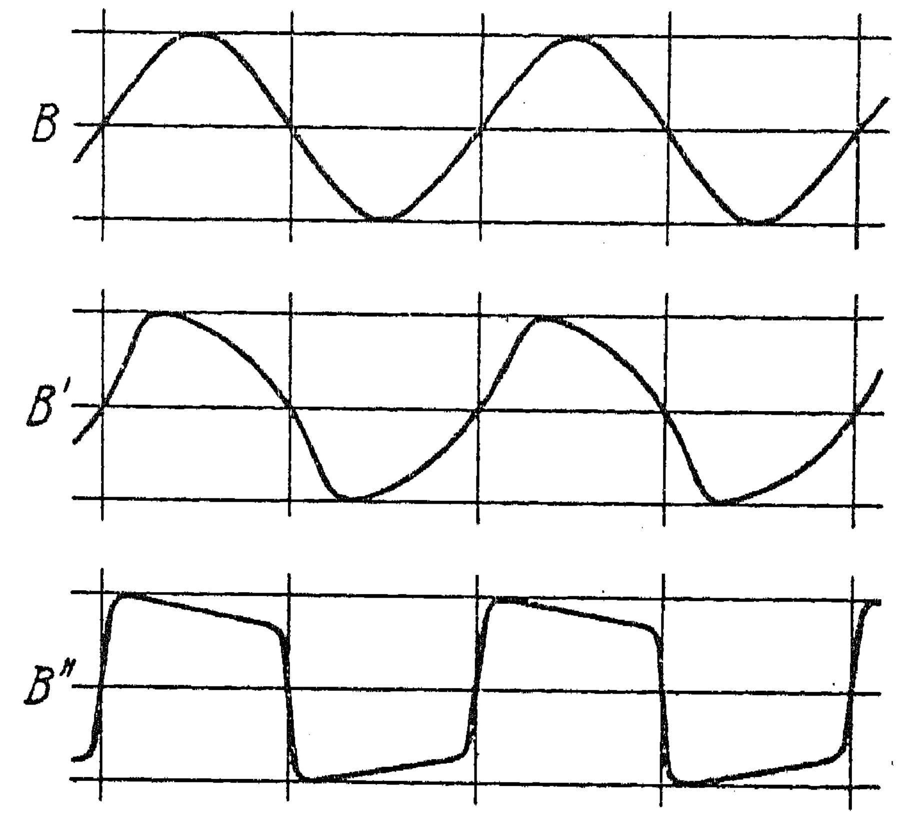

Fig. 13

Then we see that v_C, v_R or i, and v_L, by a proper choice of units, correspond respectively to the three functions y, z, and u, of the non-dimensional variable x.

When ϵ is zero, since $y(x)$ is a sinusoid, $z(x)$ and $u(x)$ are also sinusoids.

When ϵ is infinite, y has the shape given in A (Fig. 11). Therefore, z and u have shapes B and C of the same figure, which can be accurately calculated.

When ϵ varies from zero to infinity, we obtain three

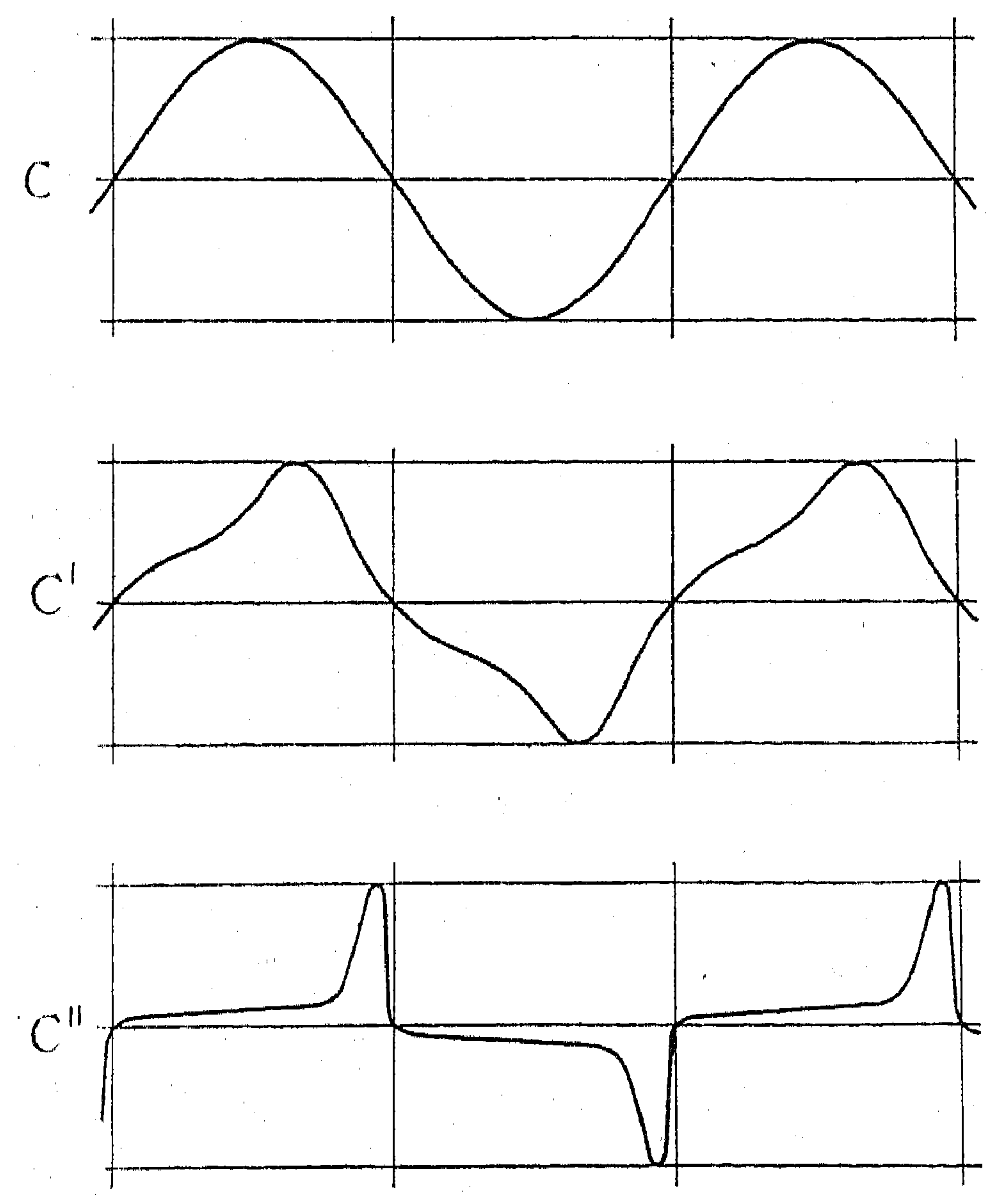

Fig. 14

families of periodic curves, A, B, and C, which vary continuously from the sinusoidal type to the relaxation type. For a given ϵ, $z(x)$ differs more from a sinsuoid than $y(x)$, and $u(x)$ more than $z(x)$. It is easy to observe the evolution of any one family on a cathode-ray oscillograph* (Figs. 12, 13, and 14).

* This was shown during the lecture on a 12-inch cathode-ray tube.

By differentiating equation (11) with respect to x, we obtain the differential equation satisfied by $z(x)$,

$$\ddot{z} - \epsilon(1 - z^2)\dot{z} + z = 0 \quad . \quad . \quad . \quad . \quad (20)$$

It is on this equation that van der Pol's work is based. By differentiating once again we obtain the non-linear equation, satisfied by $u(x)$. It is complicated and need not be repeated here.

I should like to draw your attention to the curves in Fig. 11, which will soon become familiar to all of us, by reason of the increasing number of cases in which relaxation oscillations (i.e. sustained oscillations with great ϵ) are of practical use; for example, scanning in television. The older point of view was that the sinusoidal oscillation, being alone desired, was the only good one; any kind of distortion from it was bad, and its form did not much matter, since it had to be smoothed out anyway. The new point of view is that for certain applications a sinusoidal oscillation is definitely bad; on the contrary, the bigger ϵ one can get, the better, and therefore the oscillation shapes in Fig. 11 are, in these cases, welcome.

A CORRESPONDENCE PRINCIPLE

All that we have already found on the subject of the series oscillator can be very easily translated into similar properties of another type of oscillator, the parallel oscillator, by means of a correspondence principle which we shall now present.

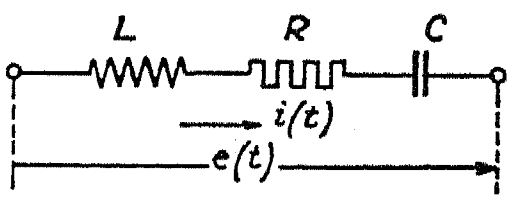

Fig. 15

Let us consider a simple network consisting of an inductance L, a resistance R, and a capacitance C in series (Fig. 15), and apply a sinusoidal e.m.f. v at its terminals. Then the current i passing through it is given by

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = v \quad . \quad . \quad . \quad (21)$$

On the other hand* let us consider a network consisting of a capacitance C, a conductance G and an inductance L in parallel (Fig. 16) and pass a sinusoidal current i through it. Then the potential difference v at its terminals is given by

$$C\frac{dv}{dt} + Gv + \frac{1}{L}\int vdt = i \quad . \quad . \quad . \quad (22)$$

This procedure can be easily generalized, in this way: a network (A) being given, let us replace all the L, R, C's by C, G, L's of the same numerical value in a given system of units, putting in parallel any two elements which were in series in (A) and vice versa; we thus obtain a second network (A′). Then the current through any series combination in (A) will be numerically equal to the potential difference across the corresponding parallel combination in (A′), and a similar equality will hold in the three other cases which you will readily imagine.

* See Reference (15).

This principle, as you see, applies to linear systems, but it is easy to generalize it to apply to our particular non-linear problem. The non-linear dipole, which we

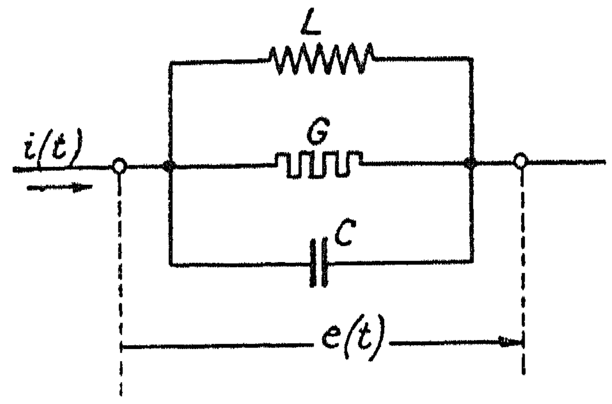

Fig. 16

found was the essential part of a series oscillator, maintained, between the alternating v and i at its terminals, a relation of the type

$$v = -\rho i + \gamma\frac{i^3}{3} \quad . \quad . \quad . \quad . \quad (9)$$

Obviously, then, a non-linear dipole corresponding

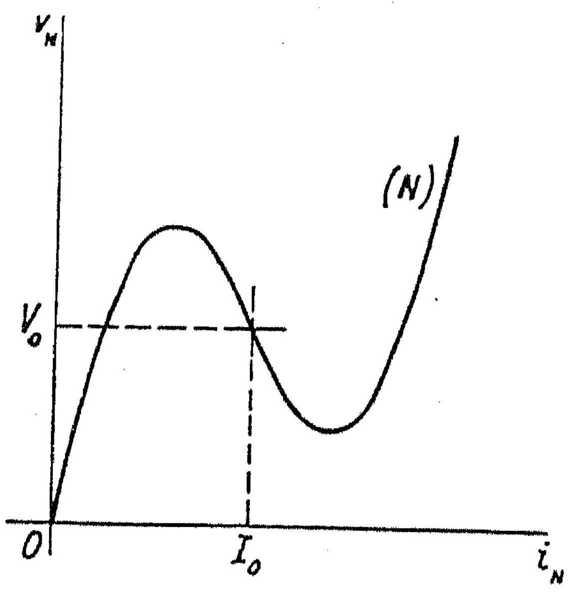

Fig. 17

to this one in the sense of the correspondence principle should maintain between its i and v the relation

$$i = -\sigma v + \delta\frac{e^3}{3} \quad . \quad . \quad . \quad . \quad (23)$$

Now we have seen that instead of a dipole having a characteristic such as (9) (series dynamo, Fig. 3), to

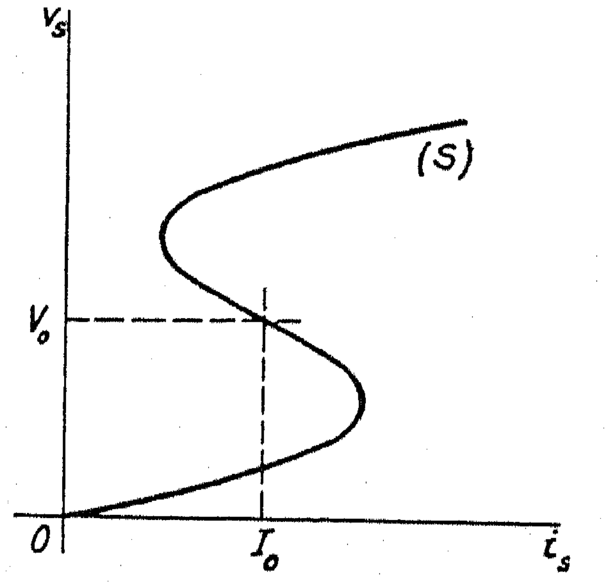

Fig. 18

generate oscillations we may use a dipole having a characteristic such as N (Fig. 17), provided we add a constant-current generator working on a point of the falling branch of the characteristic. Correlatively, then, to generate oscillations we may also use a dipole having a characteristic such as S (Fig. 18), adding a constant-

voltage generator so that it should work on a point of the falling branch of the characteristic. Circuits of that type actually exist; for instance a dynatron, that is, a three-electrode valve with highly positive grid.

THE PARALLEL OSCILLATOR

A generator of oscillations, using a dipole with a characteristic such as S, will then be arranged as in Fig. 19. It will be possible to have an alternating potential difference v at the terminals of the parallel system C, G, L, and a pulsating potential difference $(V_0 - v)$ at the terminals of S. Let us call i_L, i_G, i_C, the currents passing through L, G, and C, respectively. We have then

$$i_L = \frac{1}{L}\int vdt; \quad i_G = Gv; \quad i_C = C\frac{dv}{dt} \qquad . \quad (24)$$

Since, in this case, the currents through S and through the parallel system L, G, C, are the same, the equation for the alternating regime in the (fictitious) parallel circuit C, G, L, S will be

$$C\frac{dv}{dt} + Gv + \frac{1}{L}\int vdt - \sigma v + \frac{\delta v^3}{3} = 0 \qquad . \quad (25)$$

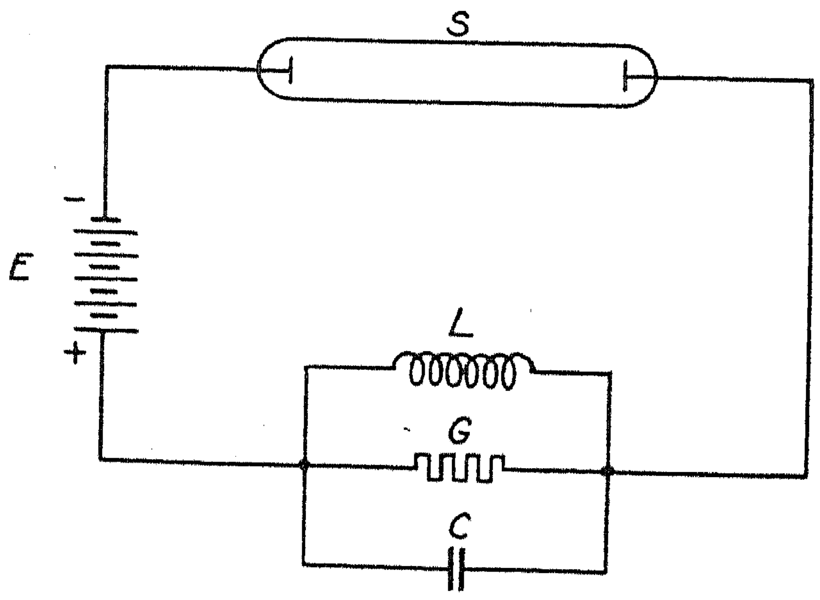

Fig. 19

corresponding, term for term, to equation (10*a*). This regime, however, will only take place when σ is greater than G; in other words, when the continuous regime (I_0, V_0) is unstable.

Now it is clear from the correspondence principle that, as functions of time, i_L, i_G (or v), and i_C will have the same values as v_C, v_R (or i), and v_L, in a series oscillator. Therefore, by a proper choice of units, in the periodic regime they will oscillate like the functions y, z, and u, of x, respectively.

The period of a parallel oscillation can be derived at once from the period of a series oscillator, again by application of the correspondence principle. It is given, in the sinusoidal case, by $2\pi\sqrt{(LC)}$, and in the relaxation regime by $1 \cdot 6L(\sigma - G)$. Exactly as in a series generator, the oscillations being sustained cannot be rigorously sinusoidal, and as there is always some capacitance present (or some stiffness in a mechanical system) they cannot be of the limiting relaxation type either.

The comparison between equations (11) and (20) for y and z leads to a more general conception of an oscillation generator of the second order.* One class, at least, of such generators will correspond to differential equations of the type

$$\ddot{y} = f(y, \dot{y}) \qquad . \quad . \quad . \quad . \quad . \quad (26)$$

Mechanically, this represents the rectilinear motion of a small body of mass 1, on which a force is acting which is any function of its position and velocity. This force will sometimes add to the energy of the body, and sometimes subtract from it; in electrical terms, the resistance will be negative or positive. If we put, as above, $z = \dot{y}$, equation (26) becomes of the first order only and can be written

$$\frac{dy}{z} = \frac{dz}{f(y, z)} \qquad . \quad . \quad . \quad . \quad (27)$$

This equation represents a family of curves in the (y, z) plane. If it happens that these curves are asymptotic to one or more closed curves (and this question can be answered in a general way—see Appendix 2), then to each closed curve will correspond a sustained periodic movement of the small body. In spite of positive resistances being present, the oscillations of the body will not die out, because of the presence of negative resistances which are at times greater than the former, and this is what is meant physically by saying that the periodic movement is *sustained.*

In such a periodic movement the input of energy exactly balances the output at the end of one period, but it would be wrong to conclude from this that no source of energy is needed: nature does not know the credit system, and therefore, outside of the network represented by equation (26), there must exist a source of energy. The question at once arises as to how well this source of energy is utilized, an interesting point to which we shall return later.

Coming back to our two oscillators, series and parallel, they are, it seems, the simplest examples of the general type (26). Given the dissipative network L, R, C, or C, G, L, to be maintained in oscillation, we need a source of energy (I_0 or E_0) and a non-linear, dissipative dipole N or S. The characteristic of this dipole must have a falling branch, so that the continuous regime should be unstable. The simpler characteristics will show one falling branch only; and since the characteristic must run entirely in the positive angle Oyz (the dipole containing no source of energy) this falling branch will lie between two ascending ones. This can be obtained in two ways, and two only, which are precisely the N and S characteristics of Figs. 17 and 18. (The distinction between these two types was pointed out by Rukop;* an interesting paper on the subject is that by G. Crisson.†) One of them has more or less the shape of a capital N, the other of a capital S, and I have found it convenient to name them accordingly.

The important point is that they are not equivalent. According to the linear theory, it is necessary, in order to sustain oscillations in a dissipative resonant circuit, to have at one's disposal a negative resistance. We see, furthermore, that a negative resistance of the N type is unfit to sustain oscillations in a parallel system (C, G, L), and one of the S type in a series system (L, R, C). This

* See Reference (22).

* See Reference (12). † *Ibid.*, (18).

result is again to be credited to non-linear theory; linear theory, by its very nature, is unable to take into account in which direction a falling characteristic may turn.

RETROACTION OSCILLATORS

We may feel that we understand fairly well the working of the two oscillators which we have hitherto considered, but the objection could be raised that they are rather unusual types. Does the theory which we have worked out apply to the more usual oscillators? We shall see that it does go still one step further, and quite an important one.

Let us consider a triode, comprising in its plate circuit a parallel resonant system C, G, L, and let the plate and grid circuits be coupled by a mutual inductance M (Fig. 20). The classical theory takes for the characteristic surface of the triode a plane

$$\rho j = e_a + Ke_g \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and gives for the differential equation satisfied by the alternating potential difference e_a across C, G, L,

$$C\frac{de_a}{dt} + Ge_a + \frac{1}{L}\int e_a dt + \left(\frac{KM}{L} + 1\right)\frac{e_a}{\rho} = 0 \quad . \quad (28)$$

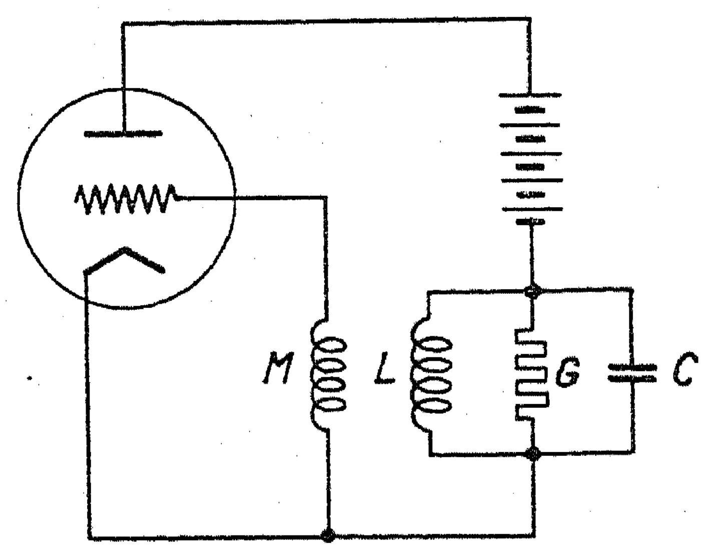

Fig. 20

which corresponds to equation (3) and is subject to the objections which were presented at the beginning.

According to equation (2), the plate current of the triode depends upon the "lumped voltage" $v = e_a + Ke_g$, and it depends upon it linearly. Now the first assumption holds true over a wider field than the second, so that a large part of the characteristic surface of the triode can be assimilated to the cylinder

$$j = \frac{1}{\rho}(e_a + Ke_g) - \delta\frac{(e_a + Ke_g)^3}{3} \quad . \quad . \quad (29)$$

If we use this equation instead of (2), we obtain instead of (28) the following equation:—

$$C\frac{dv}{dt} + Gv + \frac{1}{L}\int vdt + \left(\frac{KM}{L} + 1\right)\left(\frac{v}{\rho} - \delta\frac{v^3}{3}\right) = 0 \quad (30)$$

This is of exactly the same form as equation (25), and therefore, by one differentiation and proper choice of units of time and voltage, reduces to equation (20). (This calculation was made by van der Pol in 1920,* taking also into account a dissymetry in the lumped characteristic of the triode.)

The reason why we find the same equation as for a dynatron is as follows: in the dynatron the grid tension was fixed, and the (i_a, v_a) characteristic of the tube had a falling branch. In the present case the triode has a whole family of static characteristics, one for each value of e_g. When the triode is oscillating the representative point (i_a, e_g, e_a) describes a line on the charateristic surface; this line, projected on the (i_a, v_a) plane, also has a falling branch, the slope of which is the negative resistance we need. It is not always possible to introduce such a "dynamic characteristic" for a given oscillator (because the representative point will in general describe a closed curve); that we can do so here simplifies the result to a great extent. Moreover, the dynamic characteristic of the triode is of the S type, as we see from the curve of Fig. 21 experimentally obtained by E. V. Appleton and B. van der Pol;† so that with a coupled triode we can sustain oscillations in a parallel circuit but not in a series one.

The electrostatic action of the grid of a triode corresponds to the electromagnetic action of an exterior

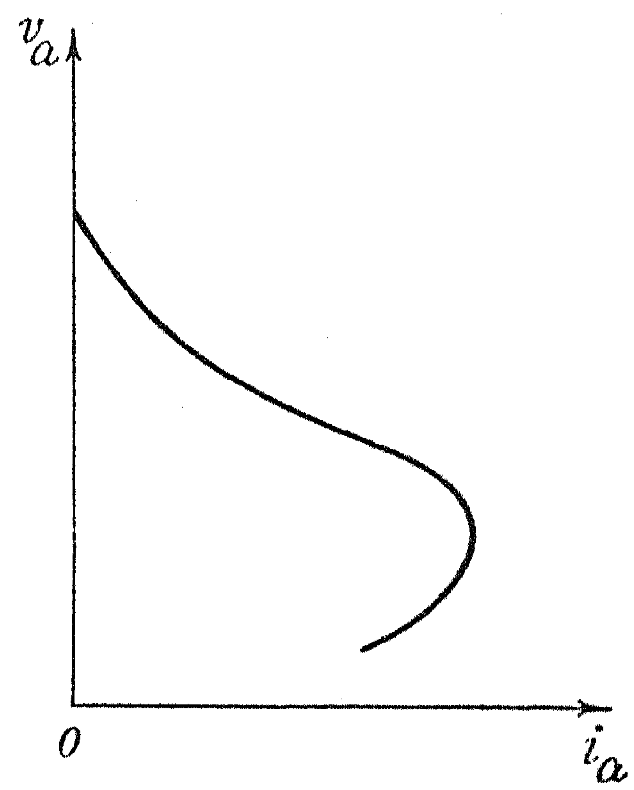

Fig. 21

winding, so that to an ordinary triode corresponds a tube of the magnetron type; and a magnetron, in fact, sustains oscillations in a series L, R, C, circuit. The development is interesting but presents some difficulties, and we shall not dwell upon it here.

OSCILLATORS OF HIGHER ORDER

We see, then, that the reduced equations (11) and (20) hold not only for oscillators using a dipole with falling characteristic, but also for certain circuits using a back-coupled triode. Unfortunately, they do not hold for all triode generators.

The reason for this is quite simple. Let us take a triode oscillator, with a resonant circuit in the grid, coupled to the plate circuit. Using the utmost simplifications, that is, neglecting the resistance of the resonant circuit and the grid current, and assuming linear characteristics, we still find a linear differential equation of the third order.

Linear theory, in this case, considers the algebraic equation, obtained by replacing d/dt by p; this equation, of the third degree, is found to have one real negative root and two conjugate complex ones; if we write that the two complex ones are pure imaginaries, $\pm\, \omega\sqrt{(-\,1)}$,

* See Reference (8).

† See Reference (9).

we find two relations, which correspond to the relations

$$R = \rho \quad \text{and} \quad LC\omega^2 = 1$$

which linear theory gave us as conditions for sustained oscillations in the case of equation (3). (For this calculation see Appendix 4.)

The same objections as in the case of the second order apply to this theory; only, in this case, we are unable to offer a non-linear solution. It is true that, just as equation (11) could be reduced to the first order (equation 13), the present third-order equation could be reduced to the second order; but we cannot go any further, and, while the behaviour of the integral curves of equation (13) is known qualitatively, from the work of H. Poincaré, we have no such knowledge of non-linear equations of the second order.

Our previous non-linear work will, however, help us quite a little in the present, more difficult, case. We know from experiment that, just as in the case of the second order, a third-order oscillator will go on oscillating quietly, after we have passed the critical condition given by linear theory. This, surely, must also happen here on account of the curvature of the triode characteristics. We can then improve the linear theory quite materially by saying that the condition for sustained oscillations is that the conjugate roots of the algebraic equation should have a *positive* real part (instead of *no* real part). This will give us an inequality instead of an equality (just as formerly we wrote $R < \rho$ instead of $R = \rho$). (See Appendix 4.)

Again, if this real part is very small, we may apply Lord Rayleigh's method and develop a quasi-sinusoidal solution in a Fourier series. This was done by A. Blondel in 1919* for a specific equation of the third order. The method is obviously general, and will give us the quasi-sinusoidal solution of an equation of any order, provided such a solution actually exists.

The inadequacy of our mathematical knowledge is all the more regrettable, in that equations of the third order present themselves quite frequently. Not only is it quite usual to place the resonant circuit of a triode oscillator in the grid; but it has been shown, both by R. C. Wegel† and by myself,‡ that a reed instrument (such as the larynx) and a turbine with a Watt governor are mechanical systems belonging to this same type, while I could not give you an instance of a mechanical oscillator of the second order.

Furthermore, triode oscillators of the fourth or even higher order are actually quite common, since the order grows very rapidly when one takes the complexity of actual networks into account. Whatever the order of the oscillator, we may assume that the necessary and sufficient condition for oscillation is that there should be at least one pair of conjugate roots of the characteristic equation on the right side of the imaginary axis. The physical variables would then grow indefinitely with time according to the linear equations, but the curvature of the characteristics would have a limiting effect on their amplitudes, as Blondel also stated in 1919. Two general criteria expressing the above condition have been found, one by A. Hurwitz§ and another more

* See Reference (6).
† *Ibid.*, (14).
‡ *Ibid.*, (16).
§ *Ibid.*, (1).

recently by H. Nyquist.* They are both interesting, having different fields of application.

It is a remarkable fact that any oscillator used in practice must of necessity present asymptotic periodic regimes, independent of initial conditions within finite limits, whereas asymptotic periodic solutions are less and less likely for a differential equation as its order grows. This must then be a special property of the equations which correspond to physical oscillators,† a typical instance in which physical intuition is far ahead of mathematical knowledge.

POWER EXCHANGES IN "N" AND "S" OSCILLATORS

Up to now we have studied the variations of various voltages and currents, in an N or S oscillator, as functions of time. Let us now turn our attention to the exchanges of *power* which take place in these oscillators.‡

We may reason on the N oscillator of the simplest

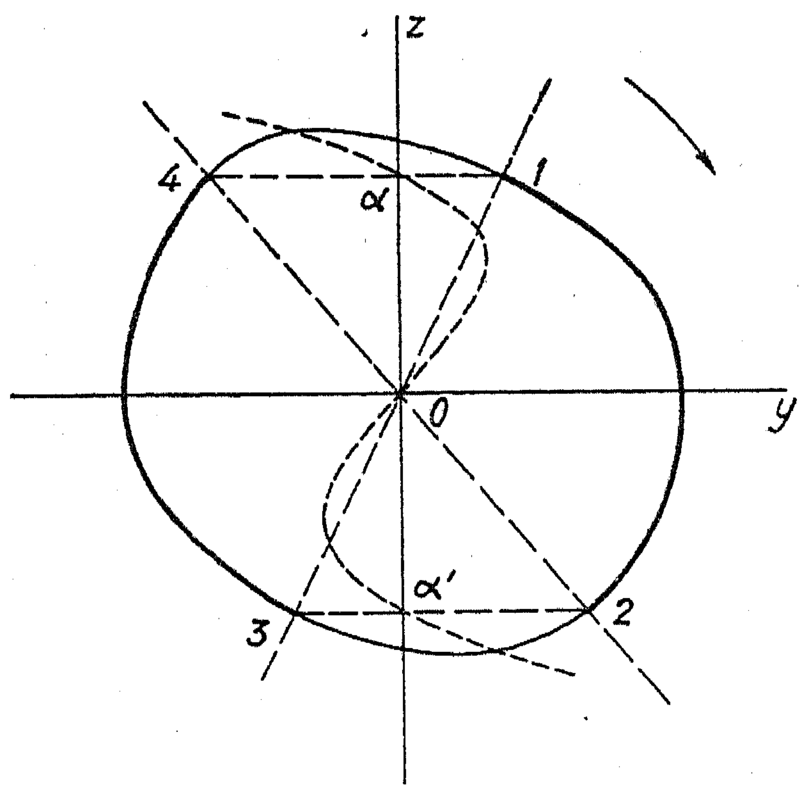

Fig. 22

type (Fig. 5). It consists of a source of power (the constant-current generator), the non-linear resistance N, the pure reactances L and C, and the load R.

Going back to our previous analysis, we see that the non-dimensional quantities which we called y and z are proportional to $q/\sqrt{C}$ and $\sqrt{L}(i)$ respectively, so that, if M is any point in the plane (Fig. 6), $\overline{OM}^2$ is proportional to

$$\tfrac{1}{2}\frac{q^2}{C} + \tfrac{1}{2}Li^2 = U + T$$

the sum of the energies stored in the condenser and in the coil.

Now it is well known that in the course of a sinusoidal movement, such as that of a resistanceless pendulum, a definite quantity of energy goes back and forth indefinitely between the kinetic and the potential type. This corresponds to the fact that in the sinusoidal case the closed integral curve is a circle, U + T = constant.

The circumstances are essentially different in a *sustained* oscillation, where energy is continually dissipated in the load. This energy comes, of course, from the source. We have seen that an essential element in the transfer was the dipole N, and this element also continually dissipates energy. As to L and C, they

* See Reference (20).
† *Ibid.*, (21).
‡ *Ibid.*, (22).

never dissipate energy but at certain times they receive energy from the source and at other times give it out to the dissipative components N and R, according to whether the radius vector $\overline{OM}$ increases or decreases.

If we mark on the closed curve in Fig. 22 points 4 and 1 on the horizontal of α, and points 2 and 3 on the horizontal of α', we can prove that $\overline{OM}$ increases from 1 to 2 (or 3 to 4) and decreases from 2 to 3 (or 4 to 1).

Each half-period of a complete oscillation is therefore divided into two parts. During the first part (1–2 or 3–4), the source feeds energy into all the other components, L, C, N, and the load R. During the second part (2–3, or 4–1), the source and L and C feed energy into N and the load R.

The coil and the condenser together act then as a generalized flywheel, which at times stores up energy coming from the source and at other times delivers it into the load.

EFFICIENCY OF "N" AND "S" OSCILLATORS

We are now ready to take up the question of the efficiency of an oscillator,* which we had postponed. We shall assume that the purpose of the oscillator is to furnish alternating power to the load, and therefore call *useful* the energy W_R spent in the load R during one period, and *lost* the energy W_N furnished to the dipole N. As to L and C, they give out during one period just as much energy as they have received, so that we can ignore them in reckoning the efficiency of the system. This efficiency is then

$$\eta = \frac{W_R}{W_R + W_N} \quad . \quad . \quad . \quad . \quad (31)$$

Now, thanks to the preceding non-linear theory, we know (from a graphical diagram) the value attained at any time by any of the variables in the system, and therefore we are able to calculate η. This calculation is not difficult (see Appendix 5) and gives

$$\eta = \frac{R\mathfrak{I}^2}{I_0 V_0} \cdot \frac{A}{\tau} \quad . \quad . \quad . \quad . \quad (32)$$

where R is the load, and $\mathfrak{I}$, I_0, V_0, suitably defined currents and voltage, A the area of the closed (y, z) curve, and τ the reduced period. These two last quantities can be obtained by mechanical integration, the closed curve being graphically known. We can therefore obtain the efficiency of the oscillator, for any given ϵ, to a certain degree of approximation.

But we can go further, and calculate the value of η exactly, in the two limiting cases of the sinusoidal and relaxation oscillations. Indeed, as we have seen, we know in these two cases the exact equations defining the behaviour of the system. We find, moreover, that the efficiency thus calculated is a maximum when certain relations between the constants of the oscillator are satisfied. These maxima are:—

For sinusoidal oscillations $\eta = 0{\cdot}375$
For relaxation oscillations $\eta = 0{\cdot}525$

For a parallel oscillator the efficiency would obviously be the same as for the series oscillator of the same ϵ.

* See Reference (23).

This result in itself is not very important, as the oscillators belong to two seldom used types and the symmetrical cubic characteristics only qualitatively imitate the actual ones. But a generalization will now be attempted and may be found of interest.

REMARKS ON THE EFFICIENCY OF PRIME MOVERS

Let us observe that a tool of any kind, to be of use, must be given a circular or an alternating movement; in other words a periodic movement, which will almost always be of constant frequency f. A machine, on the other hand, is a system which transmits power from a source of energy to a tool, or generally into a load. We shall divide machines into two main classes:

(A) Machines in which the input frequency and the output frequency are either the same, or are, by construction, in a simple and invariable ratio. These machines are *generalized transformers*.

(B) Machines in which the ratio of the input and output frequencies is continuously variable at will (within certain limits of course). These machines we shall call *motors*.*

There seem to be three types of transformer:—

(*a*) *Ordinary electrical transformers and their mechanical analogues*. These are passive, synchronous systems, i.e. they contain no source of energy and the input and output frequencies are the same.

(*b*) *Synchronous transformers containing a polarization source*, which in theory does no work. Examples of these are: a telephone receiver, an alternator, a synchronous motor, a microphone condenser.

(*c*) *Passive frequency-transformers such as a two-wheel gear*, or an alternator and a synchronous motor mounted on the same shaft and having different numbers of poles. If p and q are the number of teeth on the wheels, and $2p$ and $2q$ the number of poles, then the ratio of the input and output frequencies is $p : q$; it is a simple fraction and is invariable for a given machine.

A machine of any of these types—a transformer—has an efficiency η which is of course smaller than unity; but this efficiency can be brought, by putting enough care and money into the construction, as near unity as we like, at a given frequency.

Motors are of two types, according to whether the input frequency is different from zero or not:—

(*a*) *Machines such as induction motors*. In these machines the ratio $\omega' : \omega$ of the output to the input frequency is essentially variable. As we know, the efficiency of the transformation of power within the rotor is

$$\eta = \frac{\omega'}{\omega} = 1 - g \quad . \quad . \quad . \quad . \quad (33)$$

where g is the slip; that is, ω and ω' being given, the efficiency of the actual motor is smaller than η, but can be brought as near η as we like, in the same sense as above. We notice, then, that there is here a definite, unavoidable loss of power which appears in connection with the change of frequency.

* We are here leaving aside systems of still another class, which do not transmit power from a source to a tool, viz. the repeaters or relays. In these machines a small input power, of frequency f, controls a source of energy placed, so to speak, on the side, and the system delivers a great amount of power at frequency f. These "three-cornered" systems must be studied separately.

(*b*) *Motors in which the input frequency is zero.* This is an all-important class, for it appears as though all sources of energy were, in the last analysis, of zero frequency. Such is the power of solar heat, of coal and wood combustion, of the wind, of a river or a waterfall, of an explosive gas mixture, of steam, of the chemical reaction in a primary cell or battery, etc. We might say, then, that this is the class of *prime movers*.

The problem is then to transform power of zero frequency into power of frequency f. A very general way of doing this (not the only one, but by far the most frequent at the present time) is to put something in the way of the continuous flow of energy issuing from the source so that this flow will become unstable, and hence pulsating. This can be done in two extreme and opposite ways: either very smoothly, as when reeds are swaying in a river, or electrons in a triode with very loose coupling; then the oscillations are sinusoidal; or very brusquely, as when the slide-valve of a double-action steam-engine reverses the flow of steam at the end of one stroke, or as in an Abraham–Bloch multivibrator; then we have relaxation oscillations. We have seen, moreover, that when the system satisfies equations (11) or (20), the passage from one type to the other can be made continuous. (It looks as if this would also be true in the case of oscillators of higher order; but the theory fails us here, as has already been mentioned.)

But the study of the movements of individual parts of a machine is the realm of kinematics. From a dynamical point of view we have to ask ourselves: What are the power relations in our machine? The flow of energy, instead of being constant, has been made to pulsate; in connecting, therefore, the oscillating component to the load, we shall be able to make use of alternating power. Will a certain fraction of the energy pass by the oscillating component and be lost, or is there a possibility of transforming *all* of the continuous power into the alternating form?

I cannot answer this question in a rigorous way, but I shall present two examples for your consideration.

The first one is, of course, the series or parallel oscillator which we have been considering this evening. The qualitative answer is here obvious. It is the non-linear dipole, N or S, that makes the system pulsate; but as its (variable) resistance is always positive some continuous power is of necessity lost in it. We have applied analysis to this problem, and have found the exact limits of the maximum efficiency obtainable—from 0·375 to 0·525, according to the value of ϵ; quite low values, in fact.

The second example is the steam engine. If T_1 is the absolute temperature of the steam in the boiler, T_2 that of the water in the condenser, then we know that the maximum efficiency would be obtained if the cycle of the steam could be made reversible, in which case it would be

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad . \quad . \quad . \quad (34)$$

also, by the way, a small quantity.

Now I ask you to notice that this unavoidable loss of power, which baffled us in our student days, is linked with the fact that the steam describes a cycle, that is, that we have insisted upon changing from zero frequency, which was that of the power in the boiler, to the piston and flywheel frequency. You will find this remark, in a more or less equivalent form, in the better textbooks on thermodynamics.*

If now we recall formula (33) for the efficiency of an induction motor, so strikingly similar to formula (34)—and then our results on N and S oscillators—it appears very unlikely that such similar relations can hold for such different systems, without these relations being particular cases of some more general law. I should like very tentatively to suggest for such a law the following wording:

If a machine transforms power from the frequency f_1 to the frequency f_2, and if this machine is such that the ratio $f_2 : f_1$ can be varied continuously, then the efficiency of the transformation, for given values of f_1 and f_2, is inferior to a fixed number smaller than unity, which can only reach unity when f_1 and f_2 are equal.

Of the many remarks which the subject would appear still to call for, I should like to mention at least one. One way of transforming power of zero frequency into power of frequency f is, as I have said, to render the continuous energy-flow unstable, hence pulsating. There are, however, other ways. For instance, the stator of a hydraulic turbine transforms a flow of water into a rotating vortex, and I am not certain that the maximum efficiency of this transformation might not be unity. Only, the rotor of the turbine behaves in this vortex just as the rotor of an induction motor behaves in the rotating field, so that in the end the example of the hydraulic turbine supports the suggested proposition.

I shall now bring these considerations to an end. I shall be glad if I have succeeded in showing you, on this one problem of the generation of oscillations, what a vast and fascinating field is that of non-linear phenomena.

CONCLUSION

In conclusion I should like to express my thanks to the Engineer-in-Chief of the Post Office and to Messrs. A. C. Cossor, Ltd., for their kind assistance and co-operation in the demonstrations which were given during the lecture.

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* For instance, Prof. Porter [see Reference (19)], after stating the second law of thermodynamics in Sir William Thomson's words, says: "Two comments appear to be necessary. Firstly, 'continued mechanical effect' is intended, otherwise the law is easily violated; e.g. by driving a fan by means of a jet issuing from a cylinder of compressed air which becomes cooled by the process. Secondly, etc." "Continued mechanical effect" is precisely what a non-linear device (such as a slide-valve) provides, ensuring the endless repetition of an aperiodic phenomenon (such as the one quoted), or, in other words, creating a cycle.

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APPENDIX 1

The Series Dynamo Oscillator

In the days of d.c. power transmission it was found* that if a d.c. motor, with separate excitation, were fed by a series dynamo, the motor would reverse periodically, every few seconds or so. The explanation, given by van der Pol,† is that between the brushes of the dynamo, D (Fig. 23), we have an e.m.f. v depending upon the current i according to equation (8) or Fig. 3. In series with this e.m.f. are the impedance of the field coil, (R_0, L_0), and the impedance of the motor M. This last is (R, L) when the motor is at rest, but when M rotates it can be shown to contain a capacitance C, as follows.

Let us call Φ the field of M, $2\pi\nu$ the number of induced conductors on the rotor, $\mathbf{I}$ the moment of inertia of the rotor, $\mathbf{T}$ the torque on the shaft. We have then the two electro-mechanical equations

$$\begin{cases} -\left(R_0 + L_0\dfrac{di}{dt} + v\right) = R + L\dfrac{di}{dt} - \nu\Phi\omega \\ \mathbf{T} = \nu\Phi i + \mathbf{I}\dfrac{d\omega}{dt} \end{cases}$$

If the motor is non-loaded, which is the simplest case,

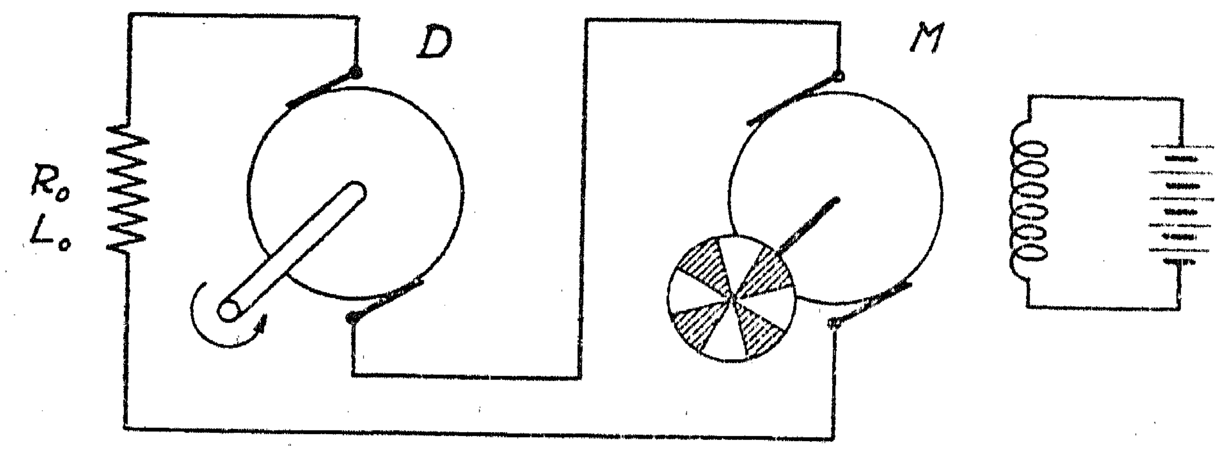


Fig. 23

we must put $\mathbf{T} = 0$, and eliminating ω the first equation becomes

$$-v = (L + L_0)\frac{di}{dt} + (R + R_0)i + \frac{(\nu\Phi)^2}{\mathbf{I}}\int idt$$

We see then that we have in the series electrical circuit, in addition to the obvious L's and R's, a "capacitance" C of electro-mechanical origin, equal to $\mathbf{I}/(\nu\Phi)^2$. Since the p.d. v is connected with the current i by the relation

$$v = -\rho i + \gamma\frac{i^3}{3}$$

the current i satisfies the non-linear differential equation

$$(L_0 + L)\frac{di}{dt} + (R_0 + R)i + \frac{(\nu\Phi)^2}{\mathbf{I}}\int idt = \rho i - \gamma\frac{i^3}{3}$$

which is equation (10) of the text. The capacitance C is very great, of the order of several millifarads for a small motor; the parameter

$$\epsilon = (\rho - R_0 - R)\sqrt{\frac{C}{L}}$$

is usually greater than 1, so that reversals are definitely "jerky"; and the period is of the order of 1 sec. By

* See Reference (2).

† *Ibid.*, (11).

varying the field and the resistances one can change the character of the oscillation (its ϵ), its amplitude, and its period. (This experiment was shown during the lecture.)

APPENDIX 2

Abstract of H. Poincaré's and A. Liénard's Work

H. Poincaré* has investigated the behaviour, in the whole of the (y, z) plane, of the curves (C) which satisfy at every point the differential equation of the first order

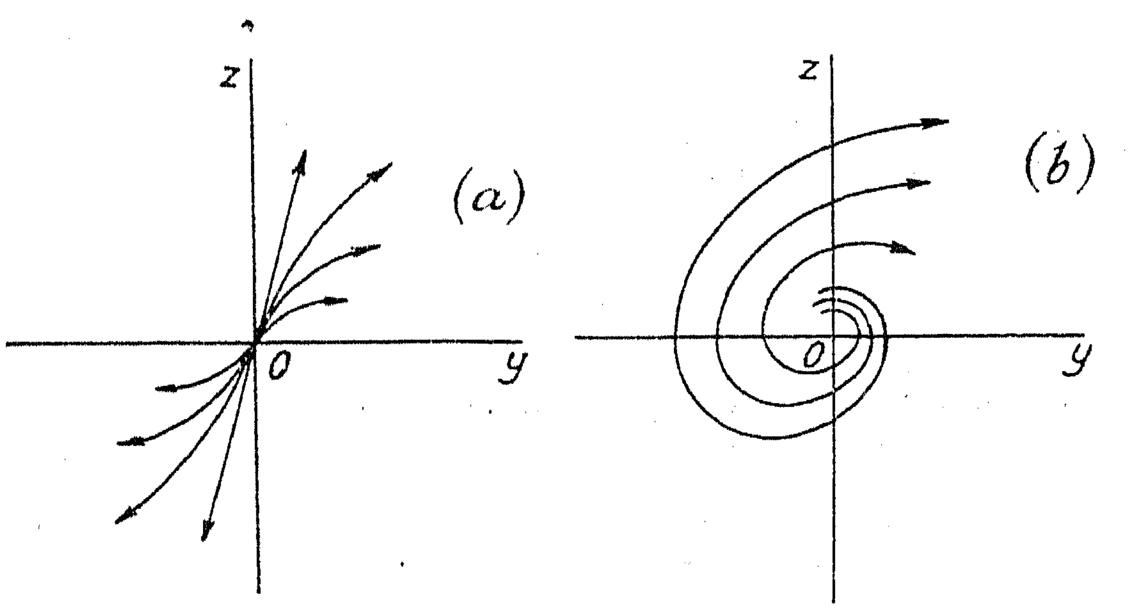
$$\frac{dy}{P(y, z)} = \frac{dz}{Q(y, z)} \quad . \quad . \quad . \quad . \quad (35)$$

where P and Q are both polynomials in y and z. Equation (13) in the text, being

$$\frac{dy}{z} = \frac{dz}{\epsilon\left(z - \frac{z^3}{3}\right) - y} \quad . \quad . \quad . \quad . \quad (13)$$

obviously belongs to type (35).

Through any point (y_0, z_0) of the plane passes one C curve and one only, except for the finite number of



$\epsilon > 2$. $\epsilon < 2$.

Fig. 24

exceptional points where $P = Q = 0$. In our problem the origin is the only exceptional point. Two cases should be distinguished (Fig. 24). If $\epsilon > 2$, an infinite number of curves start from the origin, all of them having the same tangent; if $\epsilon < 2$, an infinite number of curves have O for an asymptotic point. If, as above, $x = \int \frac{dy}{z}$ is taken as the time, then all curves are described in the clockwise direction as time increases. (The distinction between the cases of $\epsilon > 2$ and $\epsilon < 2$ is only the usual one between aperiodic and oscillating damping, except that in this case time starts from $-\infty$ instead of tending towards $+\infty$.)

In the general case it is not difficult to recognize the behaviour of the C curves in the immediate neighbourhood of the several exceptional points. Poincaré then has shown that a C curve leaving an exceptional point either goes to another or curls asymptotically around a certain closed curve $\mathfrak{C}$ (which he calls *cycle limité*). Such a $\mathfrak{C}$ curve is of necessity an integral curve of equation (13), a C curve. There may be several of them, or none at all.

Poincaré has given a process by which it is possible to ascertain for any given equation (35) the number of these closed curves, and even to know, so to speak, "where they are"; that is, the process leaves us finally with a number of oval rings of finite width, and knowing that one $\mathfrak{C}$ curve, and one only, runs between the inner and outer boundaries of any one ring. But nothing more has since been found on the subject. This will account for the curious fact that although we can draw $\mathfrak{C}$ curves with tolerable graphical accuracy, we cannot give even the roughest analytical approximation of the corresponding oscillations (unless ϵ is small).

As it happens, equation (13) falls into one of the exceptional cases which Poincaré did not treat explicitly. Liénard filled the gap as follows:—*

Liénard considers a (Γ) curve of the shape of a cubic, but he does not suppose that it admits of any algebraic or analytical equation; therefore his reasoning applies to the empirical characteristic of a tube, provided certain conditions (for which I would refer the reader to the original paper) are fulfilled. In particular the origin is supposed to be a centre of symmetry for (Γ). Let us consider only the upper half of the plane (Fig. 25). Taking a point such as α on (Γ), there is one arch belonging to a (C) curve, such as $\mathrm{A}\alpha\mathrm{A}'$, passing through α. Liénard shows that (1) if α lies between O and α_0 the difference

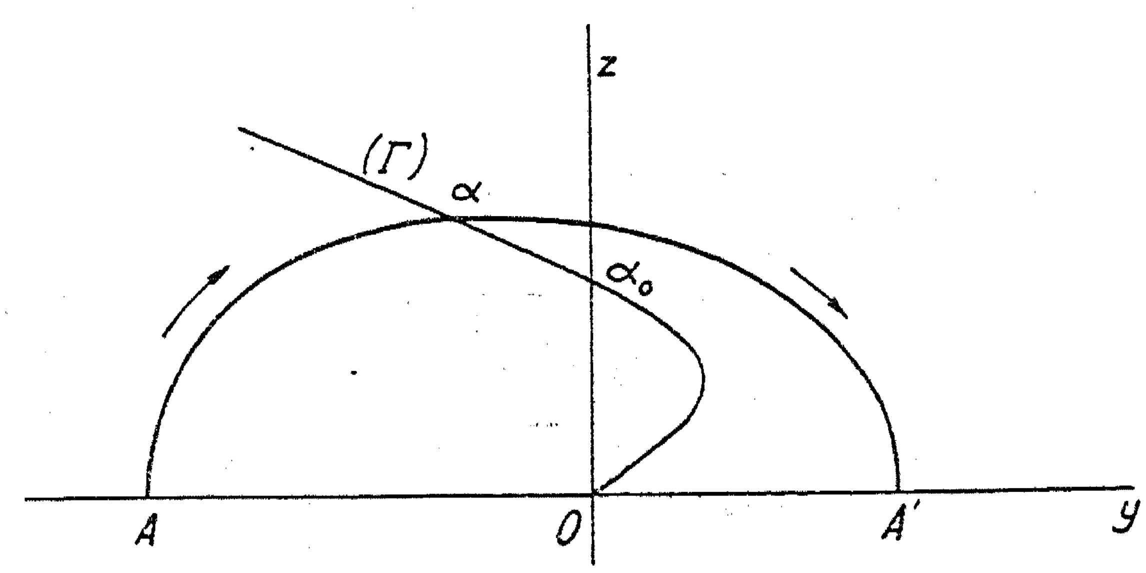


Fig. 25

$\overline{\mathrm{OA}}'^2 - \overline{\mathrm{OA}}^2$ is positive; (2) if α lies very far on (Γ) it is negative; (3) if α goes from α_0 to infinity it is always decreasing. There is, therefore, one point α on (Γ), and one only, for which this difference is zero. (Γ) being symmetrical, the lower half of the plane will provide us with an arch, forming with the upper one a closed (C) curve, which we have already called $(\mathfrak{C})$.

If we start from any point inside of $(\mathfrak{C})$ and follow the (C) curve passing through it in clockwise direction (time increasing) we see that we shall cross the Oy line an infinite number of times at points A, A′, A″, . . . the distance of which to O is always increasing, and in this way we shall always get nearer to the closed curve $(\mathfrak{C})$—without ever reaching it, of course, because two (C) curves can never meet (origin excepted). The reverse holds for the region outside of $(\mathfrak{C})$.

APPENDIX 3

Relaxation Oscillations

Relaxation oscillations are the limiting case of the periodic solution of equation

$$\ddot{y} - \epsilon\left(\dot{y} - \frac{\dot{y}^3}{3}\right) + y = 0 \quad . \quad . \quad . \quad (11)$$

when ϵ is infinite.

* See Reference (3).

* See Reference (13).

If we put

$$x_1 = \frac{x}{\epsilon}, \quad y_1 = \frac{y}{\epsilon}, \quad z_1 = \frac{dy_1}{dz_1}$$

the relation between y_1 and z_1 is

$$y_1 = \left(z_1 - \frac{z_1^3}{3}\right) - \frac{1}{\epsilon^2 z_1}\frac{dz_1}{dy_1} \quad . \quad . \quad . \quad (16)$$

For ϵ infinite, the closed $(y_1 z_1)$ curve is the curve D_1BB_1D of Fig 9.

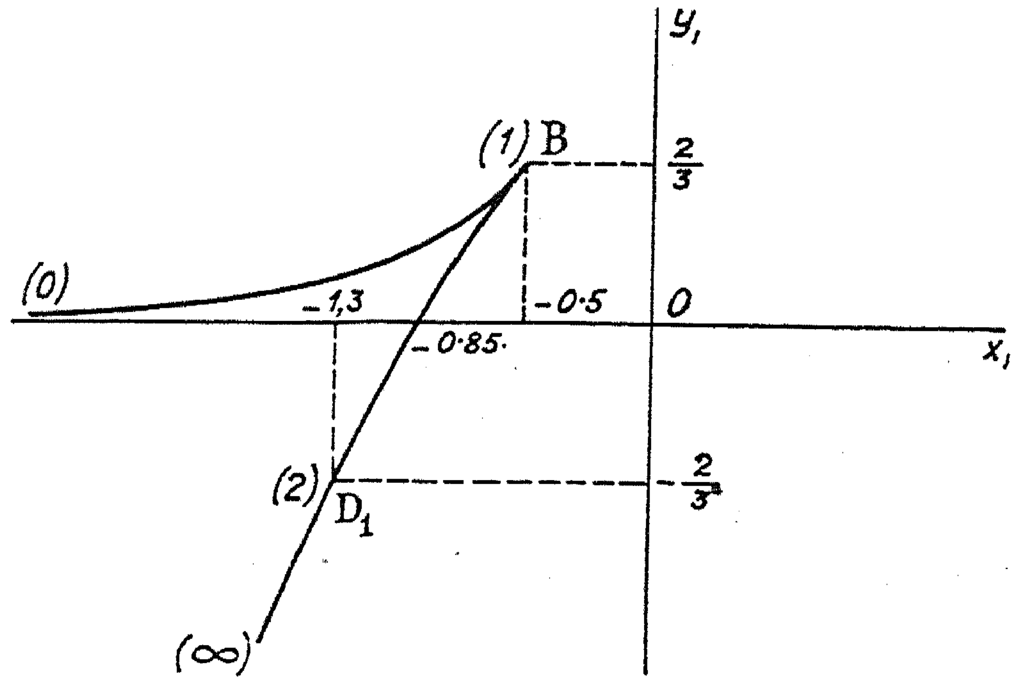


Fig. 26

On arcs D_1B and B_1D we have

$$y_1 = z_1 - \frac{z_1^3}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

and
$$dx_1 = \left(\frac{1}{z_1} - z_1\right)dz_1$$

therefore, on arc D_1B

$$x_1 - x_0 = \log_e z_1 - \frac{z_1^2}{2}, \quad 2 > z_1 > 1 \quad . \quad (37)$$

If we vary the parameter z_1 from zero to infinity, equations (36) and (37) define, for, say, $x_0 = 0$, the curve of Fig. 26. (The values of z_1 are given between brackets.)

Since we deal with arc D_1B of Fig. 10, we may keep only arc D_1B of Fig. 26.

We derive from this the variations of y_1 as a function of x_1 when time (or x_1) grows indefinitely. They are given by Fig. 11(A). Arc *ab* is the same as arc B_1D of Fig. 26; the angular point at *b* corresponds to the sudden drop of z_1 from $+1$ to -2 in Fig. 10; arc *bc* is the same as arc *ab*, but reversed; the angular point at *c* corresponds to the sudden jump of z_1 from -1 to 2 in Fig. 10; and so on, indefinitely.

We can easily calculate the values of the period and of the amplitude. When z_1 goes from 2 to 1, y_1 varies (equation 36) from $-\frac{2}{3}$ to $+\frac{2}{3}$, and x_1 (equation 37) from $(\log_e 2 - 2)$ to $-\frac{1}{2}$. The full period of the oscillation is then

$$3 - \log_e 4 = 1{\cdot}6137\ldots$$

and its amplitude is $\frac{2}{3}$, in the non-dimensional variables x_1 and y_1.

Coming back to an N-oscillator, if we introduce

$$\mathbf{I} = \sqrt{\left(\frac{\rho - R}{\gamma}\right)} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

(a quantity which van der Pol calls the γ-current, and which is typical of the non-linearity of the system), as well as

$$\omega = \frac{1}{\sqrt{(LC)}} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

and connect the variables x and y to time t and electric charge q by

$$x = \omega t, \quad y = \frac{\omega}{\mathbf{I}}q \quad . \quad . \quad . \quad . \quad . \quad (40)$$

then, in the limiting case of ϵ infinite $(L = 0)$ we find for the period and amplitude

$$T = 1{\cdot}6\frac{\epsilon}{\omega} = 1{\cdot}6C(\rho - R) \quad . \quad . \quad . \quad (41)$$

$$Q = \tfrac{2}{3}\epsilon\frac{\mathbf{I}}{\omega} = \tfrac{2}{3}C(\rho - R)\sqrt{\left(\frac{\rho - R}{\gamma}\right)}$$

The amplitude of the current in branch L, R, C is simply $2\mathbf{I}$.

Similarly, in an S-oscillator, where C would be nil, the period would be

$$T = 1{\cdot}6L(\sigma - G) \quad . \quad . \quad . \quad . \quad . \quad (42)$$

and the amplitude of the tension at the terminals of C, G, L would be $2V$, putting

$$V = \sqrt{\left(\frac{\sigma - G}{\delta}\right)}$$

a quantity which we may call the δ-tension.

APPENDIX 4

Oscillating Triode of Order Three

Consider an oscillating triode with resonant circuit in the grid (Fig. 27). The transformer equations are

$$\begin{cases} -v = \left(L_1\dfrac{dj}{dt} + R_1 j\right) + M\dfrac{di}{dt} \\ -u = M\dfrac{dj}{dt} + \left(L_2\dfrac{di}{dt} + R_2 i\right) \end{cases} \quad . \quad . \quad . \quad (43)$$

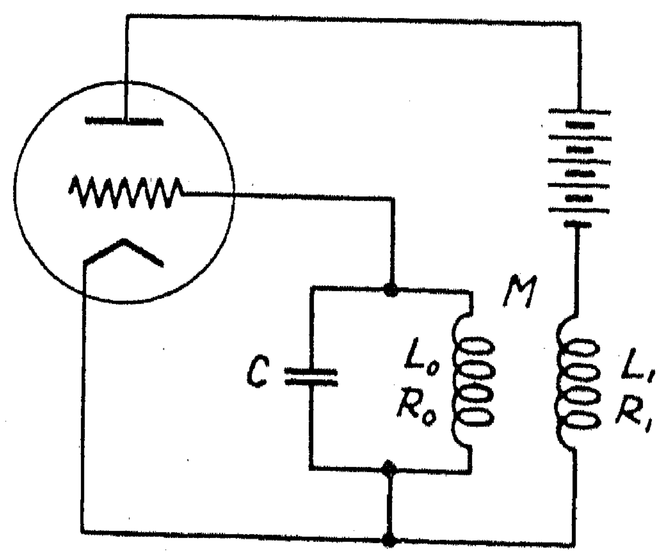


Fig. 27

Moreover,

$$q_1 = \frac{dj}{dt}, \quad q_2 = \frac{di}{dt} = C_2 u, \quad \rho j = v + Ku$$

This gives between q_1 and q_2 the two equations

$$\begin{cases} (\ddot{q}_1 + 2\delta_1\dot{q}_1) + \alpha_1\ddot{q}_2 - \gamma q_2 = 0 \\ \alpha_2\ddot{q}_1 + (\ddot{q}_2 + 2\delta_2\dot{q}_2 + \omega^2 q_2) = 0 \end{cases} \quad . \quad . \quad (44)$$

putting

$$\alpha_1 = \frac{M}{L_1}, \quad \alpha_2 = \frac{M}{L_2}, \quad \delta_1 = \frac{R_1 + \rho}{2L_1}, \quad \delta_2 = \frac{R_2}{2L_2},$$

$$\gamma = \frac{K}{L_1C_2}, \quad \omega = \frac{1}{\sqrt{L_2C_2}}$$

The characteristic (or determinantal) equation of system (44) is

$$(1 - \alpha_1\alpha_2)p^3 + 2(\delta_1\delta_2)p^2 + (\omega^2 + 4\delta_1\delta_2 + \alpha_2\gamma)p + 2\delta_1\omega^2 = 0 \quad . \quad (45)$$

If we want the triode to oscillate, equation (32) must have two complex roots with positive real part. Variables q_1 and q_2 will then be functions of time of the form

$$Ac^{-Kt} + Be^{+\delta t}\cos(\omega t + \phi)$$

A, B, and ϕ, being integration constants. The amplitude of the oscillating term grows indefinitely with time; we assume, without being able to prove it, that if the curvature of the triode characteristics were taken into account, it would, on the contrary, tend toward a limit.

A limiting case will be when equation (45) has two purely imaginary roots. Writing equation (45) as

$$ap^3 + bp^2 + cp + d = 0$$

the condition for this is easily found to be

$$\Delta = bc - ad = 0$$

[The corresponding frequency is then $\omega = \sqrt{\left(\frac{c}{a}\right)}$]

A somewhat delicate discussion shows that if $\Delta > 0$, equation (45) has all its three roots in the half-plane at the left; if $\Delta < 0$, it has one negative real root and two complex conjugate ones with positive real part. The condition for the triode to oscillate is therefore $bc - ad < 0$, or

$$\left(\alpha_1\alpha_2 + \frac{\delta_2}{\delta_1}\right)\omega^2 + \frac{\delta_2}{\delta_1}(4\delta_1\delta_2 + \alpha_2\gamma) < 0 \quad . \quad (46)$$

Since all the factors except $\alpha_2\gamma$ are positive, a necessary condition for oscillations is $\alpha_2\gamma$ negative, or M negative, as in the simpler case of the coupled triode of order two.

APPENDIX 5

Efficiency of an "N" or "S" Motor

This efficiency is

$$\eta = \frac{W_R}{W_R + W_N} \quad . \quad . \quad . \quad . \quad (31)$$

W_R and W_N being the energies dissipated in the load R and in the dipole N during one period.

Evaluation of W_R.—From its definition,

$$W_R = R\int i^2 dt$$

the integral being taken during one period, or, what is the same thing, around the closed curve ($\mathfrak{C}$). But from equations (39) and (40)

$$t = x\sqrt{(LC)}$$

$$i = \frac{dq}{dt} = \mathbf{I}\frac{dy}{dx} = \mathbf{I}z$$

hence

$$W_R = R\mathbf{I}^2\sqrt{(LC)}\int_{\mathfrak{C}} zdy$$

or, calling A the area of the closed curve $\mathfrak{C}$, a non-dimensional number

$$W_R = R\mathbf{I}^2\sqrt{(LC)}A \quad . \quad . \quad . \quad . \quad (47)$$

Evaluation of $W_R + W_N$.—Calling as before I_0 the constant current delivered by the source, v_N the p.d. at the terminals of N (or of the branch L, R, C), the energy delivered by the source during one period T is

$$W_R + W_N = \int_0^T I_0 v_N dt$$

(The integral includes also $W_L + W_C$, which is zero for one period.)

We have supposed the oscillation to be symmetrical, or the N-characteristic symmetrical in regard to its point of inflexion (I_0, V_0), therefore

$$\int_0^T v_N dt = V_0 T$$

Then, as above,

$$T = \int_{\mathfrak{C}} dt = \sqrt{(LC)}\int_{\mathfrak{C}} dx = \sqrt{(LC)}\int_{\mathfrak{C}} \frac{dy}{z}$$

or, calling τ a non-dimensional number, the integral on the right, which can be evaluated from our knowledge of curve (C),

$$T = \sqrt{(LC)}\tau \quad . \quad . \quad . \quad . \quad (48)$$

and hence

$$W_R + W_N = I_0V_0\sqrt{(LC)}\tau \quad . \quad . \quad (49)$$

Finally we have for the efficiency of the N-motor the general formula

$$\eta = \frac{R\mathbf{I}^2}{I_0V_0}\cdot\frac{A}{\tau} \quad . \quad . \quad . \quad . \quad (32)$$

It would appear as though this expression were only valid for a cubic characteristic, because **I**, the γ-current (38), has been so defined. But we could take for **I** any intensity of current in order to define the non-dimensional variable y, and the product I^2A would still have the same value. Therefore expression (32) holds good for any symmetrical characteristic of N shape, empirically given, and for any L, R, and C.

For an S-motor a similar formula evidently holds.

Cubic characteristic ; sinusoidal and relaxation cases.—We can go one step further if the characteristic is a cubic and in the limiting cases $\epsilon = 0$ or ∞. In the sinusoidal case the closed curve (C) is a circle of radius 2, hence $A = 4\pi$, $\tau = 2\pi$, and

$$\eta = \frac{2R(\rho - R)}{I_0V_0\gamma} \quad . \quad . \quad . \quad . \quad (50)$$

In the relaxation case a simple calculation shows that $A = 1{\cdot}5\epsilon$, and (Appendix 3) $\tau = 1{\cdot}6\epsilon$, hence

$$\eta = 2{\cdot}8\frac{R(\rho - R)}{I_0V_0\gamma} \quad . \quad . \quad . \quad . \quad (51)$$

Maximum efficiency.—The N-characteristic must lie entirely in the region of positive i_N, v_N; using the cubic equation this is easily seen to imply

$$I_0 V_0 \gamma \geqslant \tfrac{4}{3}\rho^2$$

On the other hand, algebraically,

$$R(\rho - R) \leqslant \frac{\rho^2}{4}$$

equality being attained when $R = \dfrac{\rho}{2}$.

Therefore, using relation (32), we find

$$\eta_{max.} \leqslant \frac{3}{16} \cdot \frac{A}{\tau} \quad . \quad . \quad . \quad . \quad (52)$$

where A/τ is a non-dimensional number, a function of the ϵ of the cubic characteristic.

This number, we have seen, is 2 for $\epsilon = 0$, and $2 \cdot 8$ for $\epsilon = \infty$. Therefore—

For sinusoidal oscillations $\eta \leqslant 0 \cdot 375$; and for relaxation oscillations $\eta \leqslant 0 \cdot 525$.

It seems likely that A/τ would increase steadily with ϵ, alhough we can offer no proof of this.

DISCUSSION ON "THE THEORY, PERFORMANCE, AND CALCULATIONS OF A POLYPHASE CAPACITOR-TYPE MOTOR."*

Mr. C. W. H. Minchin (*communicated*): The theory developed by the authors has been based on a number of assumptions, which, although of negligible inaccuracy on the motors chosen for test, may prove serious sources of error on machines with high magnetizing current. It is the low-power-factor machine which especially calls for compensation, and advantage may be gained by calculation from a more accurate equivalent circuit.

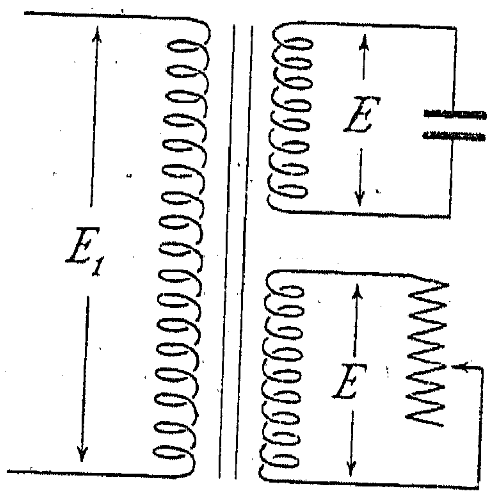


Fig. A

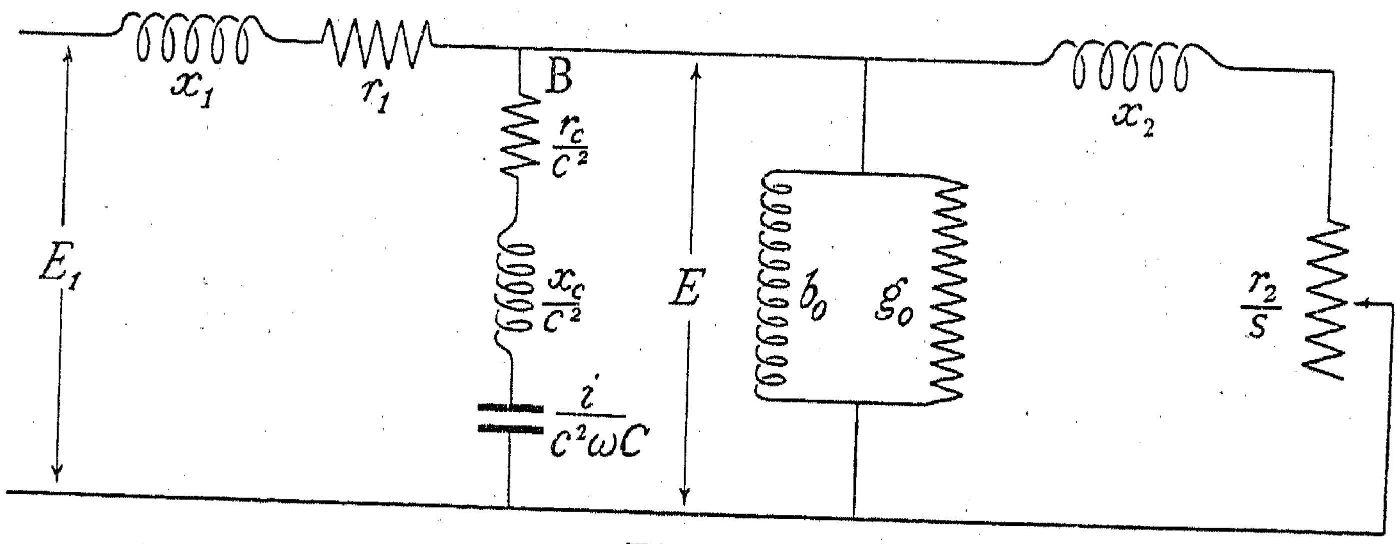


Fig. B

Keeping to the same nomenclature,

r_c/c^2, x_c/c^2, $1/(c^2\omega C)$, = the compensating circuit resistance, reactance, and capacitive reactance respectively, referred to the primary winding.

Z_c, the impedance of the compensating circuit referred to the primary,

$$= \frac{r_c + j\left(x_c - \dfrac{1}{\omega C}\right)}{c^2}$$

Case 1. The motor connections per phase and the corresponding equivalent circuit are shown in Figs. A and B.

Case 2. The equivalent circuit of Case 2 presents more difficulty owing to the compensating circuit being excited both by the internal voltage E and the external voltage E_1. Fig. C shows the motor connections per phase.

If c is infinitely large, the compensating circuit will derive its excitation from voltage E, and point B (Fig. B) must be to the right of the primary impedance.

If c is infinitely small, the compensating circuit will derive its excitation from voltage E_1, and point B must be to the left of the primary impedance.

Sufficient accuracy for all calculations should be obtained if the primary impedance be split at point B with $(r_1 + jx_1)\dfrac{c}{c+1}$ to the left, and $(r_1 + jx_1)\dfrac{1}{c+1}$ to the right.

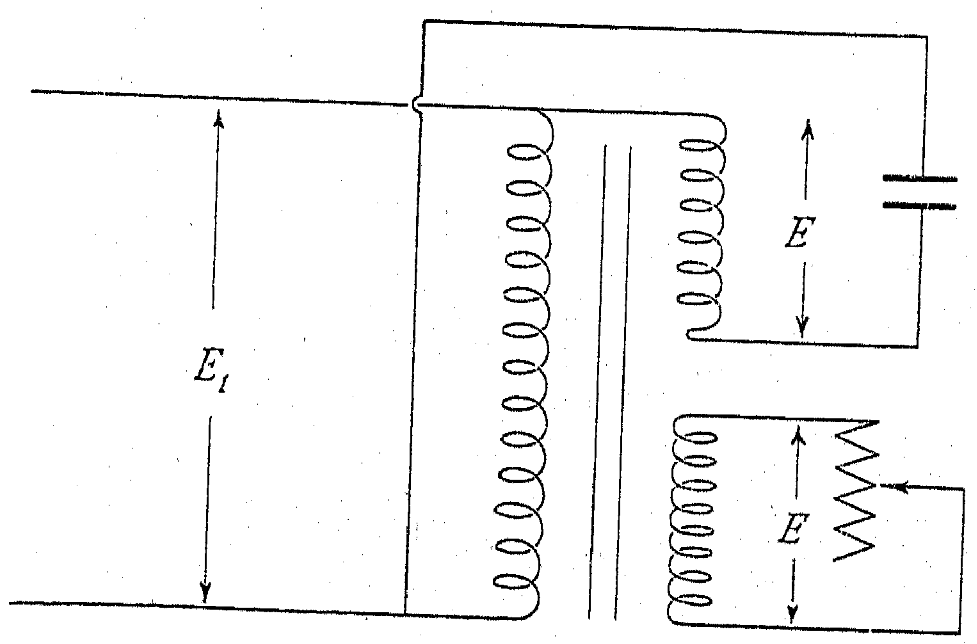


Fig. C

* Paper by Dr. J. J. Rudra and Mr. D. J. Badkas (see vol. 77, p. 420).

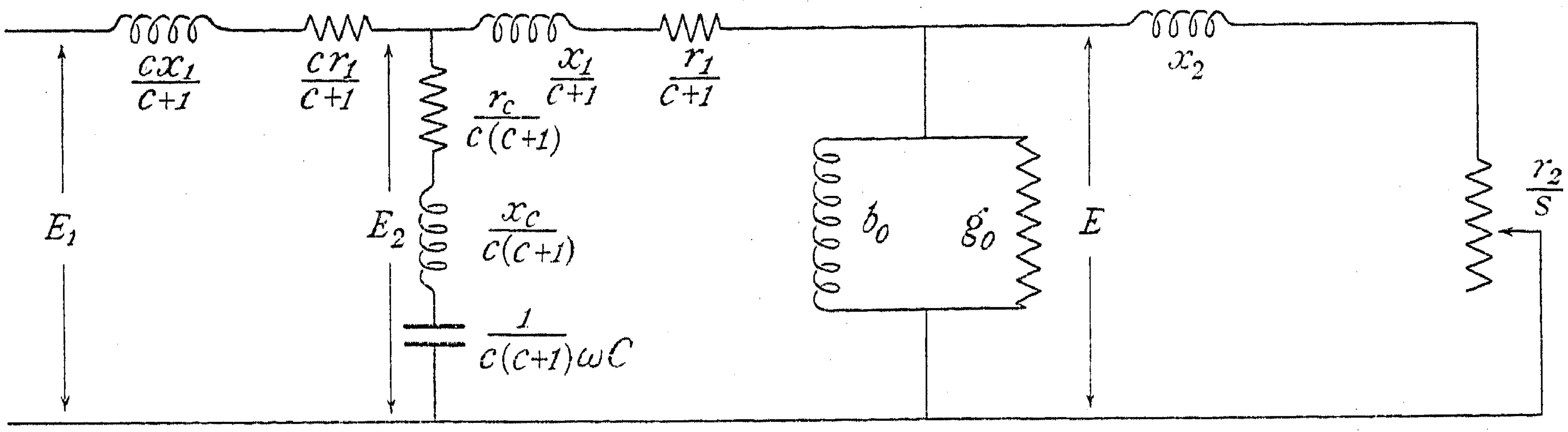


Fig. D

When E_2 = voltage induced in the compensating winding referred to the primary,

E_1/c = actual voltage of the line acting in the compensating circuit and referred to the primary,

$(E_1/c) + E_2$ = total referred voltage impressed on the compensating winding,

x = voltage-drop in $(r_1 + jx_1)[c/(c+1)]$ of the primary winding,

Then $E_2 = E_1 - x$

$I'_{c.ref.}$, the referred current in the compensating circuit,

$$= \frac{E_2 c[c + 1 + (x/E_2)]}{r_c + j[x_c - 1/(\omega C)]}$$

In all cases x/E_2 will be very small compared with $(c + 1)$ and may be neglected, and the equivalent circuit will become as shown in Fig. D.

It is interesting to note that a given power-factor correction calls for an almost constant condenser kVA whichever connection is employed, and that the actual connection and value of c must be chosen chiefly to give an economical condenser voltage.

Dr. J. J. Rudra and **Mr. D. J. Badkas** (*in reply*): We agree with Mr. Minchin that the approximate theory of the motor given in our paper may not suit motors with high magnetizing currents. Although we had given some consideration to the exact theory of the motor, we felt that for a new motor it was much better to put forward first an approximate and simple working theory than a complicated theory such as the exact one is. However, the exact theory has to be tackled sometime or other and we welcome Mr. Minchin's communication in this connection.

The exact equivalent circuit for Case 1 proposed by Mr. Minchin in Fig. B can be confirmed mathematically by the method given below, and the equation of the locus of current in it can be easily deduced from equation (m).

The reasoning employed by Mr. Minchin for deducing the circuit in Fig. D for Case 2 is very interesting and, though the circuit does not appear to be correct and is very difficult to solve, it should not give results very different from those from the circuit which we obtain mathematically below.

Neglecting the interaction of the leakage fluxes of the primary and the compensating windings, as is evidently done by Mr. Minchin in the cases considered by him and which seems quite justified, the equations of the motor for Case 2 may be written as:—

$$\dot{E} = Z_0\dot{I}_0 \quad . \quad . \quad . \quad . \quad (a)$$

$$\dot{E}_1 = Z_1\dot{I}_1 + \dot{E} \quad . \quad . \quad . \quad . \quad (b)$$

$$\dot{E}_1 + c\dot{E} = Z_c\dot{I}_c \quad . \quad . \quad . \quad . \quad (c)$$

[From (b) and (c)] $$\dot{I}_c = \frac{(1+c)\dot{E}}{Z_c} + \frac{Z_1}{Z_c}\dot{I}_1 \quad . \quad . \quad . \quad . \quad (d)$$

$$\dot{E} = Z_2\dot{I}_2 \quad . \quad . \quad . \quad . \quad (e)$$

$$\dot{I}_1 = \dot{I}_0 + c\dot{I}_c + \dot{I}_2 \quad . \quad . \quad . \quad . \quad (f)$$

Substituting in (f) from (a), (d), and (e), and transposing, we get

$$\left(1 - \frac{cZ_1}{Z_c}\right)\dot{I}_1 = \left\{\frac{1}{Z_0} + \frac{c(1+c)}{Z_c} + \frac{1}{Z_2}\right\}\dot{E} \quad . \quad (g)$$

Substituting for $\dot{E}$ from (g) in (b) gives

$$\frac{\dot{E}_1}{\dot{I}_1} = Z_0 + \frac{1}{\dfrac{1}{Z_0\left(1-\dfrac{cZ_1}{Z_c}\right)} + \dfrac{c(1+c)}{Z_c\left(1-\dfrac{cZ_1}{Z_c}\right)} + \dfrac{1}{Z_2\left(1-\dfrac{cZ_1}{Z_c}\right)}} \quad . \quad (h)$$

The right-hand side of equation (h) shows that the exact equivalent circuit of the motor consists of an impedance Z_1 in series with three parallel impedances, viz.

$$Z_0\left(1 - \frac{cZ_1}{Z_c}\right), \quad \frac{Z_c[1 - (cZ_1/Z_c)]}{c(1+c)}, \quad \text{and} \quad Z_2\left(1 - \frac{cZ_1}{Z_c}\right)$$

The circuit given by Mr. Minchin in Fig. D cannot be obtained by any permissible modification of the terms in the right-hand side of equation (h).

The term cZ_1/Z_c should be expected to be small in practical motors, and, as its presence greatly complicates the solution of the circuit of the motor, it may be neglected in the first instance. The equivalent circuit of the motor then reduces to that shown in Fig. E. This circuit is probably as exact as one would like for practical purposes.

To enable the exact theory to be tested, and compared with the approximate theory outlined in the main paper, we obtain below the equation of the locus of the primary-current motor according to the exact circuit. For convenience in calculations, it is advisable to combine the constant impedances Z_0 and $Z_c/[c(1+c)]$ into a single constant impedance $Z'_0 = r'_0 + jx'_0$. The relation between

the current and the voltage of the circuit can now be written as

$$\dot{E}_1 = \frac{Z_1Z_0' + Z_1Z_2 + Z_0'Z_2}{Z_0' + Z_2}\dot{I}_1 \quad . \quad . \quad . \quad (j)$$

Take O, the starting point of the primary current vectors in Fig. F, as the origin of co-ordinates, and the direction of OE, the primary impressed e.m.f., as the axis of Y. Let (x, y) be the co-ordinates of the extremity of I_1 at any load. Then, when I_1 is leading,

$$\dot{I}_1 = I_1 \cos\phi_1 + jI_1 \sin\phi_1 = (y - jx)$$

x in this case being negative.

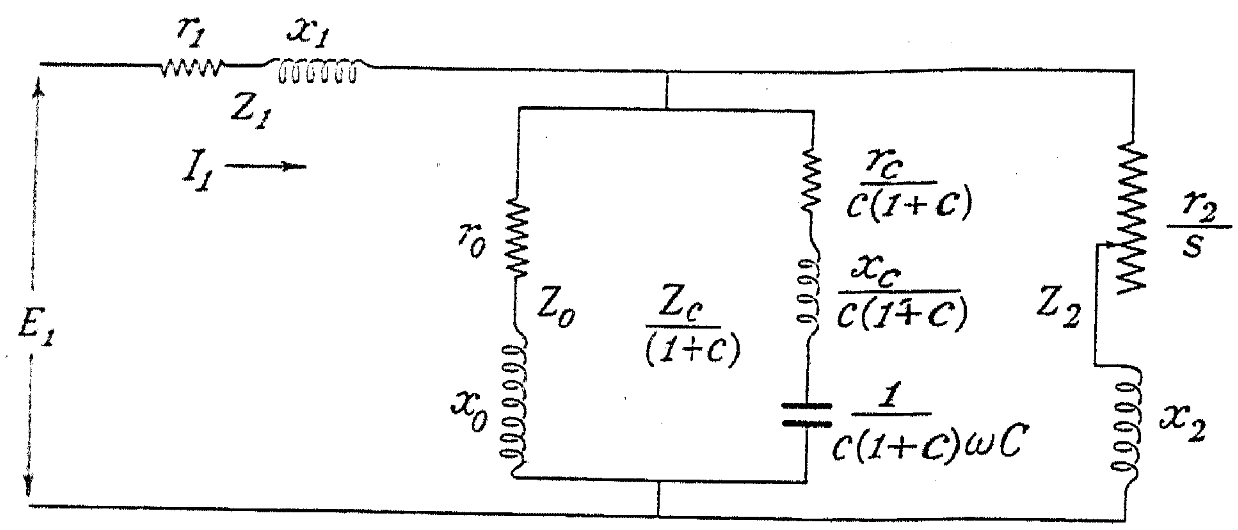


Fig. E.—Exact equivalent circuit for Case 2.

When I_1 is lagging,

$$\dot{I}_1 = I_1 \cos\phi_1 - jI_1 \sin\phi_1 = (y - jx)$$

x in this case being positive. Hence, whether I_1 is leading or lagging, $\dot{I}_1 = (y - jx)$. Therefore, multiplying both sides of equation (j) by $(Z_0' + Z_2)$, putting $(y - jx)$ for $\dot{I}_1$, and simplifying,

$$\left(r_0' + \frac{r_2}{s}\right)E_1 + (x_0' + x_2)E_1 = \left\{\alpha + \frac{(r_0' + r_1)r_2}{s}\right\}y + \left\{\beta + \frac{(x_0' + x_1)r_2}{s}\right\}x + j\left\{\beta + \frac{(x_0' + x_2)r_2}{s}\right\}y - j\left\{\alpha + \frac{(r_0' + r_1)r_2}{s}\right\}x \quad . \quad . \quad . \quad . \quad . \quad (k)$$

where

$$\alpha = r_0'r_1 - x_0'x_1 - x_0'x_2 - x_1x_2$$
$$\beta = r_0'x_1 + x_0'r_1 + r_0'x_2 + r_1x_2$$

Equating the real and imaginary terms on the two sides of equation (k)

$$\left(r_0' + \frac{r_2}{s}\right)E_1 = \left\{\alpha + \frac{(r_0' + r_1)r_2}{s}\right\}y + \left\{\beta + \frac{(x_0' + x_1)r_2}{s}\right\}x \quad (l)$$

$$(x_0' + x_2)E_1 = \left\{\beta + \frac{(x_0' + x_2)r_2}{s}\right\}y - \left\{\alpha + \frac{(r_0' + r_1)r_2}{s}\right\}x \quad (m)$$

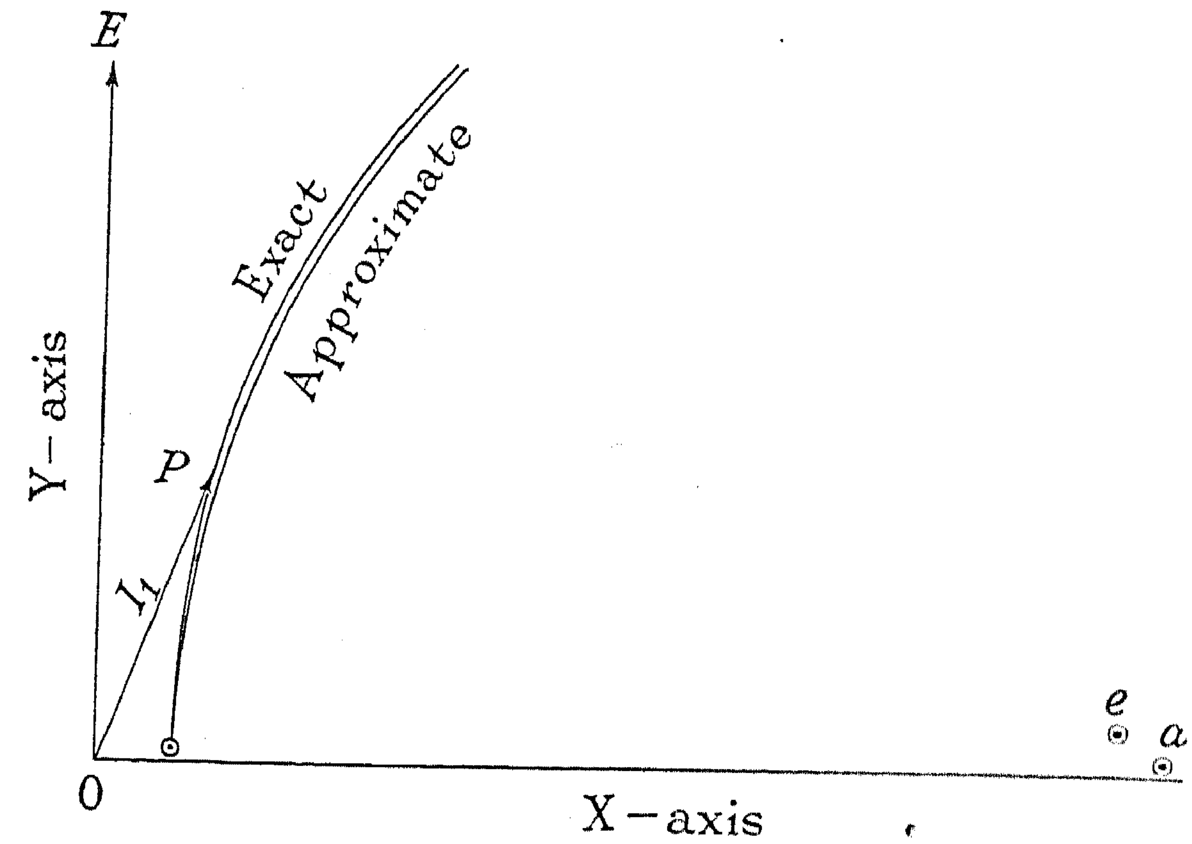


Fig. F.—Exact and approximate primary-current loci for Case 2.

Equating the values of r_2/s given by equations (l) and (m), cross-multiplying, and simplifying,

$$(r_0'^2 + x_0'^2)x_1 + (r_1^2 + x_1^2)x_0' + \left\{(r_0' + r_1)^2 + (x_0' + x_1)^2\right\}(x^2 + y^2) - (r_0'^2 + x_0'^2 + 2x_0'x_1 + 2x_0'x_2 + 2x_1x_2)E_1x - (2x_0'r_1 + 2r_0'x_2 + 2r_1x_2)E_1y + (x_0' + x_2)E_1^2 = 0 \quad (n)$$

(n) is the equation of locus of the primary current, and is a circle whose radius and co-ordinates of the centre are easily obtained from the equation. Fig. F shows the exact and approximate primary circular current loci with their respective centres e and a for the 5-h.p. motor referred to in the paper. The point P on the exact locus corresponds to approximately full load. It will thus be seen that, as far as this motor is concerned, the difference between the exact and approximate loci is small in the working range of the motor.

We regret that it has not been possible to test the exact theory on a motor of large magnetizing current, and we hope that this will be done by those who are in a position to do so.

DISCUSSION ON "REMOTE CONTROL OF POWER NETWORKS"*

NORTH-WESTERN CENTRE, AT MANCHESTER, 7TH APRIL, 1936

Mr. W. Kidd: The authors speak of "remote control" and also of "supervisory control"; what is the line of demarcation?

They stress the point that continuity of supply is one of the main concerns of supply engineers; that is quite true, but I must remind them that supply authorities have a remarkably good record in that respect, and in this area interruptions are extremely rare. I certainly would not advocate the use of supervisory control as a means for reducing the number of interruptions; it has many other features which are assets to the supply industry, and which justify its adoption.

For example, it will help to reduce the period of an interruption when trouble does occur. The first and most important thing in such circumstances is to get to know what has happened; only when he knows the origin of the trouble can the engineer take correct action. Supervisory control gear will bring that information instantaneously to a central control room, where it also provides facilities for instantaneously performing any switching operation. It is quite unlikely that, without supervisory control, a fault in an unattended substation could be dealt with in less than half an hour.

When distribution systems were fed either direct from generating stations or from manually-controlled substations, there was very little use for supervisory gear, and I doubt whether such gear could have been justified even in unattended substations, on account of the then high price of automatic plant. The development of supply-system design in the direction of the superimposition of higher-voltage transmission systems (66 kV and 33 kV), with transformer substations feeding lower-voltage systems, has necessitated a large number of distribution substations; the change-over from direct current to alternating current has further increased the number of unattended substations. This great increase in the ratio of the number of substations to the number of attendants on the system is a feature which has contributed more than anything else to the desirability of the employment of supervisory gear.

In our recent paper,† Mr. Carr and I showed that supervisory gear is a sound commercial proposition, and, judged on financial grounds only, will pay for itself within 3 years. That statement has been substantiated by information received later from other supply systems. Supervisory gear has been used in this area for 5 or 6 years; no record has been kept of the number of operations, but I am informed by my colleagues that there has not been a single case of incorrect operation. A few failures to operate have occurred; they have mainly been due to "dry" joints. These faults do not show themselves during preliminary testing; they are not serious, because they do not cause wrong operation but simply failure to operate. Records from another supply undertaking show that in 10 000 operations they had three failures to operate, but no incorrect operation. It can therefore be asserted that the coded impulse system is thoroughly reliable. There is a tendency to use insulated wire of poor quality with this gear; I strongly advocate a high standard of insulation for the small wiring, especially in the substations, when there may be dampness at certain seasons.

Any system of transmission and distribution which is growing at a reasonable rate can neither have its extensions developed and planned in a thoroughly satisfactory manner, nor can it be controlled effectively, unless there is accurate information available regarding the incidence of loading on plant and cables. The collection of such information on a large system which has unattended substations is not only inconvenient but also costly, and consequently is frequently not properly done. The installation of supervisory gear is obviously the correct procedure; it gives the means for collecting all the required particulars periodically, easily, and at a low cost. In my opinion, it would be foolish not to avail oneself of such useful apparatus.

The maintenance of this type of gear should be done by people accustomed to telephone work rather than by those accustomed to dealing with heavy plant. I often hear supervisory gear characterized as being complicated; yet there are thousands of automatic telephone equipments in use and they give rise to very little trouble. We have found the repairs bill for a large supervisory control installation to be almost negligible. One man can test and adjust a substation with a large number of equipments easily in a day. The total cost is about 14s. per circuit breaker per annum, and I am of the opinion that the cost will be substantially reduced in the future.

The indicating boards which have been shown in the authors' slides are extremely interesting, but none has yet appeared to be anything like ideal. The most suitable type of central control board is, I suggest, still undeveloped, but I think it will soon emerge. The colour of the boards is an important item for those people who have to sit in front of them all day. The white background with black lines is rather glaring and tiring for the eyes, and a black ground and white lines does not look well. I would suggest a green tint. What experience have the authors had on this point? I should like to know their views.

Rapidity of operation and checking is important, and there is little doubt that in recent years the makers have

* Paper by Messrs. G. A. Burns and T. R. Rayner (see page 95).
† *Journal I.E.E.*, 1934, vol. 74, p. 285.

been able to meet the requirements of power supply engineers much better than formerly. The rotary stepper switch is a reliable piece of apparatus.

Mr. T. W. Ross: Supervisory control is now established in practically all countries, and is accepted by operating engineers as a reliable and useful means of controlling apparatus situated at a distance from the control point.

This form of control was originally developed as an addition to the completely automatic generating station or substation, and as the direct result of experience in the operation of such stations. It was found that the automatic starting and stopping of machinery in accordance with some predetermined condition was not, in some cases, entirely satisfactory, and that it was sometimes necessary to anticipate a change in conditions in order to obtain the best results. It is practically impossible to anticipate changing conditions automatically, and therefore the human element has to be introduced. A single operator can, however, control many generating stations or substations if means are available for doing so. Before the advent of supervisory control the only means available was remote control by numerous pilot wires between the control room and the stations. If the distance between these points is short, this method may still prove economically sound, but for longer distances the cost of cable would be prohibitive. Supervisory control, as distinct from remote control, provides a solution to this problem, since by means of selective switching a single pilot or telephone circuit can be made to do the work of any number of pilot wires.

The problem of transferring the pilot or telephone wires to and from the various control circuits is similar to that of automatic telephony, and the obvious solution was to make use of the highly developed automatic telephone apparatus already in successful use in all parts of the world. The authors have contributed largely to the development of supervisory control gear in this country, and the many schemes which have been evolved as the result of their work have considerably simplified and improved the art. For instance, the original supervisory gear required four pilot wires (two for selection purposes and two for control purposes), whereas the modern gear requires only a single channel of communication. This channel of communication can take any form which is suitable for telephony.

It is a pity that the paper does not deal with the use of carrier current over the power lines. Although there are many difficulties in the way, I feel that this method will be much used in the future for telephoning and supervisory purposes. The authors mention that when a G.P.O. channel of communication is used the supervisory gear must be completely insulated from the G.P.O. lines, but they do not mention that in some cases it is also necessary to insulate it completely from the power apparatus with which it is associated. Interposing relays are largely used for effecting this insulation, and although they add to the cost of the apparatus I consider them desirable.

One development for which the authors are responsible is to my mind of great importance. I refer to the phototelemeter. This is the only practical form of remote metering which gives an instantaneous reproduction of instrument indications at the remote point. It is particularly applicable to supervisory control schemes and can be operated through any channel of communication.

I notice on page 98 a statement that the voltage induced into the pilot wires is proportional to the voltage on the power line. I cannot understand this, since the maximum end-to-end voltage is due to magnetic induction brought about by any unbalance in the 3-phase currents flowing over the power line.

I consider that the use of supervisory control gear and channels of communication is not a suitable method for the selective protection of power networks. The reasons which induce me to make this statement are: (*a*) unwanted time-delay introduced by the supervisory gear; (*b*) the channel of communication must always be available for protective-gear purposes; (*c*) the introduction of additional apparatus into the protective scheme.

Mr. J. Venters: A feature of remote control equipment that impresses an outsider is the ease with which it can be tested and altered on site. This feature is valuable on account of the time it saves, because occasions arise when the remote control equipment is only part of the scheme, and the gear cannot be tested as a whole until it is erected in the customer's premises. Factory tests will eliminate errors in construction and in adjustment of the individual parts, but the final adaptation of the equipment to suit service conditions must be postponed until it is erected on site. Any alterations that may then be found to be necessary, either to the adjustment of the relays or to the scheme as a whole, can be made with great rapidity before the equipment is handed over to the customer.

When remote control equipment is employed to transmit impulses to operate protective devices, a compromise has to be made between two conflicting requirements: (*a*) rapidity of transmission of the impulses, and (*b*) complete security against wrong operation of the protective gear. The rapidity of clearance of faults is becoming an increasingly important matter, and the advantage is with those systems and methods of remote control that give the desired results in the minimum time. Against this, however, security against wrong operation of the protective gear means complication, and in general every additional relay in the series adds to the time of operation. The problem arises in its most acute form with pilot-wire types of protective gear, because the pilots are an expensive part of the equipment and have to be reduced to two wires wherever possible. Instances arise where the pilots are the connecting link for two independent sets of protective gear, as well as for all the supervisory and communication equipment. The equipment has to be designed to discriminate between the signals sent over the pilot wires without delaying the transmission of those that operate the protective gear, and to ensure that the equipment will work correctly over a wide range of impedances in the pilots themselves or shunted across them, whether the pilots are correctly connected or accidentally reversed. Provision to meet reversal of the pilots appears to be essential where overhead pilots are hired from the national telephone administration.

Overhead pilot wires may be considered satisfactory for remote control equipments engaged on routine work, but when conditions of emergency arise, or when the pilots are used to operate protective equipment, the vulnerability of overhead pilots is a disadvantage. As an example, with a pilot system of protection guarding an overhead line transmitting a large amount of power, the risk of failure of the power line is greatest when the weather is bad, during thunderstorms, gales, or heavy falls of snow, but under such conditions the risk of failure of the overhead pilot wires is greater still. The wires may become broken or twisted together, and so unable to perform their function just when urgent messages have to be transmitted, or the protective gear operates to trip off the power line. For such conditions underground pilots appear to be the only adequate solution, although carrier-current pilots using the power line itself may develop into an alternative.

Most remote control equipments are designed to operate on direct current at 50 volts, and in general the 50-volt supply is obtained from a battery. As, however, it is not an easy matter even with regular maintenance to keep the battery voltage at 50, it is desirable that the remote control equipment should be designed to operate satisfactorily over a wide range of battery voltage, say 40 to 60 volts. This requirement is of special importance when the remote-control equipment is used to transmit impulses operating protective gear, because the protective gear must operate satisfactorily whenever called upon to do so by a failure on the power system it protects.

Variation in the battery voltage is also of importance when the remote-control equipment transmits the readings of indicating instruments, such as ammeters and wattmeters. The instruments may be some distance from the remote-control equipment, and the voltage-drop in the connecting leads may introduce a complication.

Mr. H. Pearce: Referring to the finish of supervisory control diagrams, having seen a good many diagrams carried out in various colours, including, for example, brown and light blue, I have come to the conclusion that what is really needed is a comparatively dark tone which will not be trying to the eyes of the operators. My own particular taste is for a blue-green which makers describe as "Luton blue." It is a restful colour, and not too bright.

Mr. W. Fennell: In col. 2, page 95, the authors say, "It is also necessary that the system shall be run with the minimum staff possible." Are they prepared to prove that statement? I should like to point out that redundant staff is one of the troubles the electrical industry will have to face in the near future. We shall soon find ourselves paying men to be at home instead of on the job. I know of one case where the situation has arisen of men having to be placed in certain positions about the area in order that the consumers shall have someone to whom they can go when they have trouble with the supply. Unattended substations are being developed to such an extent that there is no staff for miles with whom the consumer can get into touch. Under the old-fashioned system there were attended substations in each small town, and when the supply failed consumers could telephone or go round to find out how long the supply was likely to be off. There could be an indicator in each locality, operated from the control centre, to tell the public locally how long it will be before the supply is available.

On the question of supervisory control and remote control, until to-day I thought I knew the difference between the two. I thought that remote control meant that action at one place produced an operation elsewhere, and that supervisory control provided also information of a distant happening, so that the operator could know what action to take.

With regard to the transmitting of readings of distant instruments, I have in use one piece of apparatus of great importance in receiving a bulk supply. The bulk receiving station is about 2 miles from the centre of control and is up a lane about $\frac{1}{2}$ mile long which is often 6 in. deep in mud. We want to know from time to time what the load is on the system: it is important at peak-load times to operate batteries so as to limit the maximum demand. We have a hired Post Office line which transmits the reading of the necessary instrument to our principal substation. For some time past I have been investigating the possibility of dispensing with the three shifts of attendants at the receiving station as they are so isolated, and have normally no useful duties. I hope that before long we shall be able to use some supervisory control system, not for the purpose of discharging these men, but to transfer them to some worthwhile occupation.

Mr. A. S. Johnstone: I should like to know whether anything has been done in connection with short-distance beam wireless links to replace circuits hired from the Post Office. It is, of course, essential that any circuits hired from the Post Office should be absolutely reliable under all circumstances, and it is questionable whether the Post Office authorities realize this. Experience has shown that the Post Office are apt to reverse lines without first notifying the renter of these lines, and, although apparatus may be made to deal with reversed lines, complications are bound to arise when special circuits used for telemetering are interfered with.

Automatic apparatus is not so reliable as people state. Any maintenance man in a public exchange will explain that apparatus does not function as it should do, and that on many occasions this apparatus has to be locked out of circuit whilst the necessary adjustments are made. In the Post Office, apparatus used for public exchanges is duplicated many times over so as to facilitate repairs. This is not always a financial proposition when apparatus of a similar nature is used for telemetering purposes.

Mr. P. F. Gunning: The use of the visual telegraph has not, in my opinion, been stressed as much in the paper as its usefulness appears to warrant. In these days when generating stations operate to prearranged programmes in order to obtain the highest overall efficiency for a large area, conditions outside the control of the operation department, such as storm, fog, and line or generator failure, may necessitate sudden departure from the programme on the part of one or more stations, depending upon the nature of the abnormal occurrence.

The C.E.B. supervisory equipment in North-West

England provides for the transmission of a common visual signal to all of 26 stations, which takes less than 5 secs. Thus the control engineer can, by means of the telegraph, rapidly cope with any emergency condition that may arise. Particularly valuable is the use of the telegraph in the control of system voltage and transfer of reactive load, by the transmission of signals for the raising or lowering of transformer taps. In the same area it has been found that the following signals are most useful: (*a*) "Hold load"; (*b*) "Increase load"; (*c*) "Decrease load"; (*d*) "Programme"; (*e*) "Raise taps"; (*f*) "Lower taps." The "hold load" signal requires the station to hold its existing load, departing from programme. The "increase load" and "decrease load" signals require the station to raise or lower the load by a prearranged amount, decided by consideration of the efficient generating capacity of the station.

If the fullest use is made of the telegraph in the foregoing manner, for load-despatching and reactive-load control, the control engineer's telephone is left free for the more important messages dealing with such matters as permits, switching, and emergency information regarding plant.

A complaint has been raised during the discussion that the reversal of pilot wires causes dislocation of service. This is a contingency which should be provided for, especially in cases where the lines are hired from an outside telephone administration. The line circuits should be so arranged that reversals do not matter. This can be done by the use of voice-frequency signalling currents; or by "pause" marking when signalling with direct-current impulses; or by a scheme whereby the first two pulses are used to find out the conditions of the lines, whether reversed or normal, and to reverse the line-relay circuits if necessary for the succeeding pulses. Such a circuit principle is in use for the inter-tripping of the line oil circuit-breakers at Crewe, Percival Lane, and Warrington, over G.P.O. channels. A time of 0·8 to 1·0 sec. is obtained for the transmission of the inter-trip code, which is composed of pulses of varying polarity. Such a code ensures correct discrimination between a genuine inter-trip signal and external interference.

The reliability of the types of supervisory equipment described in the paper has been very good. In the early stages of the grid, the routine maintenance had to be neglected owing to the large amount of new equipment being installed and put into commission. The frequency of faults on the gear, even under such adverse conditions, was surprisingly small.

Messrs. G. A. Burns and T. R. Rayner (*in reply*): Mr. Kidd asks for a clear definition of the terms "remote control" and "supervisory control." It is agreed that these two terms are used very loosely in the paper and also in general conversation. We think, however, that the term "remote control" is a general one, embracing all installations where the switches are situated at a point some distance from the controlling devices. Supervisory control is a more restricted term and is usually used to describe a system wherein the operation of a number of switches is controlled over the same pilots. It is, we feel, a term which has come into use owing to the fact that the equipment used for supervisory control is similar to that which was in the first place used for supervisory indications.

The question of the best design for an indicating board is certainly very controversial, but we agree with Mr. Kidd that from the various types now in use a vastly improved board will soon be evolved.

As regards the colour of the mimic diagram, it is agreed that white and black are very trying colours to live with. Black, however, has one great advantage in that it makes it a comparatively easy matter to obtain a good match between old and new panels, whereas white and, in fact, all the lighter colours, change somewhat with time, making it practically impossible to make a change to the board which cannot be detected due to variations in the colour of the ground. From the point of view of producing a board with a pleasing appearance, it seems to us that it is impossible to improve upon the use of a fairly light bluish-green, but a darker colour as favoured by Mr. Pearce has the advantage that it is less disfigured by dirt, etc. It must be borne in mind that the depth of colour employed will be governed to a very large extent by the general level of illumination in the control room.

In reply to Mr. Ross, the suggestion on page 98 is that the maximum voltage which can be induced into the telephone line is equal to that between the faulty phase and earth, whilst the fault current is actually flowing. That is to say, we have assumed a line which is charged from one end with the far end open, and a fault develops on one phase of the power line. Under these circumstances a heavy current will flow in a single phase only, and in fact all the current in the system will be an out-of-balance current. Naturally any current which does flow in the other conductors with opposite phase will tend to reduce this induced voltage.

Mr. Ross also criticizes the use of supervisory apparatus for protecting purposes. With regard to the time-delay introduced by the supervisory equipment, it is often found that the overall time taken to effect a clearance is reduced and not increased by the incorporation of supervisory apparatus. The assumption of a pilot between two stations enables the protective relays to be much faster in operating, and the time saved here is often considerably more than that taken to transmit the impulse to the far end of the line. It is, of course, agreed that the channel of communication must be available for use by the protective circuits.

With modern equipments it is usual either to provide a continuous test to prove the continuity of the pilots or, alternatively, to provide means at some central point for completely checking the operation of the protective gear from end to end. In our opinion this second method of test is much preferable to a continuous line test, since no extra complications are introduced into the protective equipment proper.

The introduction of additional apparatus into the protective scheme is only a disadvantage if it reduces the reliability of the system as a whole, and it has been found that with the judicious use of communication channels and supervisory equipment, more rapid and accurate discrimination is obtained and, therefore, the overall reliability of the system is increased.

Mr. Johnstone is, we think, under a misapprehension

concerning the normal procedure in a P.O. telephone exchange. The administration do not normally provide a large quantity of duplicate apparatus to facilitate repairs. Admittedly, there will usually be one or two spare sets of equipment, but these will act as spares for several thousands of switches. Mr. Johnstone should also remember that the traffic handled by switches in a telephone exchange is very much greater than that which obtains on supervisory equipment. For instance, in a director area some of the switches are in practically continuous use during the busy periods of the day, and in fact the whole exchange is so designed that it is just capable of carrying the maximum traffic.

Various statements concerning the reliability of the telephone apparatus have been made by a number of speakers, and these require no further elaboration.

INSTITUTION NOTES

LIST OF MEMBERS

Copies of the List of Members corrected to the 1st September, 1935, are still available. Any member wishing to receive a copy should apply to the Secretary.

SCHOLARSHIPS

The following Scholarships have been awarded by the Council for 1936:—

Ferranti Scholarship (Annual Value £250; *tenable for* 2 *years*).

W. E. Harper (Birmingham University).

Duddell Scholarship (Annual Value £150; *tenable for* 3 *years*).

P. Hargreaves (Lower School of John Lyon, Harrow).

Silvanus Thompson Scholarship (Annual Value £100, *plus tuition fees; tenable for* 2 *years*).

L. S. Anand (North-Western Railway, India).

Swan Memorial Scholarship (Annual Value £120; *tenable for* 1 *year*).

D. H. Thomas (Metropolitan-Vickers Electrical Co.).

David Hughes Scholarship (Value £100; *tenable for* 1 *year*).

W. H. Penley (Liverpool University).

Salomons Scholarship (Value £100; *tenable for* 1 *year*).

E. F. O. Masters (City and Guilds College).

Thorrowgood Scholarship (Annual Value £25; *tenable for* 2 *years*).

L. G. Leaton (Southern Railway Co.).

OVERSEAS MEMBERS AND THE INSTITUTION

During the period 1st June to 31st August the following members from overseas called at the Institution and signed the "Attendance Register of Overseas Members":—

Anthony, P. A. W., B.E. (*Cairns, Queensland*).
Atchison, A. F. T., B.Sc. (*Cairo*).
Barnard, H. L., B.E. (*Sydney*).
Belfield, R. (*Barbados*).
Blaber, R. W. S. (*Khodaung, Burma*).
Brodie, J. E., M.Sc., B.E. (*Christchurch, N.Z.*).
Bulley, H. S. (*Calcutta*).
Camozzi, P. J. (*Malta*).
Couse, F. A. (*Khaur, India*).
Davies, D. P. (*Rangoon*).
Deshpande, S. B., B.A. (*Calcutta*).
Donkin, H. J., M.B.E. (*Delhi*).
Grainger, O. W. (*Johannesburg*).
Hoey, G. McC., B.A., B.E. (*Lucknow*).
Hooker, A. (*Cairo*).
Horn, D. J. (*Calcutta*).
Jeffs, E. A. (*Wellington, N.Z.*).
Johnstone, A. W., B.Sc. (Eng.) (*Khodaung, Burma*).
Mackenzie, F. J. R. (*Georgetown, British Guiana*).
Metz, G. L. E. (*Vesteras*).
Oliver, C. J., B.Sc. (*Rio de Janeiro*).
Parrott, R. G. (*Buenos Aires*).
Patel, S. P., B.E. (*Nadiad, India*).
Preston, C. E., M.Eng. (*Deccan*).
Priestley, H. T., B.E. (*Brisbane*).
Renaut, J. O. (*Christchurch, N.Z.*).
Sharpley, Prof. F. W., F.R.S.E. (*Dhanbaid, India*).
Smit, N. P. (*Vesteras*).
Woodworth, L. B. (*Johannesburg*).

ACCESSIONS TO THE REFERENCE LIBRARY

[NOTE.—The books cannot be purchased at the Institution; the names of the publishers and the prices are given only for the convenience of members; (*) denotes that the book is also in the Lending Library.]

FERMI, E. Molecole e cristalli. 8vo. 303 pp. (Bologna: Nicola Zanichelli, 1934.) *L.*50.

GAY, C. M., and FAWCETT, C. DE VAN. Mechanical and electrical equipment for buildings. 8vo. viii + 429 pp. (New York: John Wiley and Sons, Inc.; Chapman and Hall, Ltd., 1935.) 20*s*. (*)

GLASSTONE, S., *D.Sc., Ph.D.*, and HICKLING, A., *M.Sc., Ph.D.* Electrolytic oxidation and reduction: in-

organic and organic. Volume nine of a series of monographs on applied chemistry. Under the editorship of E. H. Tripp. 8vo. ix + 420 pp. (London: Chapman and Hall, Ltd., 1935.) 25*s.* (*)

GOLDING, E. W., *M.Sc.Tech.* Electrical measurements and measuring instruments: a textbook covering the syllabuses of the B.Sc. Engineering, City and Guilds (Final), and A.M.I.E.E. examinations in this subject. 2nd ed. 8vo. xii + 812 pp. (London: Sir Isaac Pitman and Sons, Ltd., 1935.) 20*s.* (*)

GRIMSEHL, E. A textbook of physics. vol. 5, Physics of the atom. Edited by R. Tomaschek. Tranlation by L. A. Woodward. 8vo. xiii + 474 pp. (London: Blackie and Son, Ltd., 1935.) 17*s.* 6*d.* (*)

GUILLEMIN, E. A., *Ph.D.* Communication networks. vol. 2, The classical theory of long lines, filters and related networks. 8vo. vii + 587 pp. (New York: John Wiley and Son, Inc.; London: Chapman and Hall, Ltd., 1935.) 37*s.* 6*d.* (*)

GULLIKSEN, F. H., and VEDDER, E. H. Industrial electronics 8vo. xiv + 245 pp. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1935.) 17*s.* 6*d.* (*)

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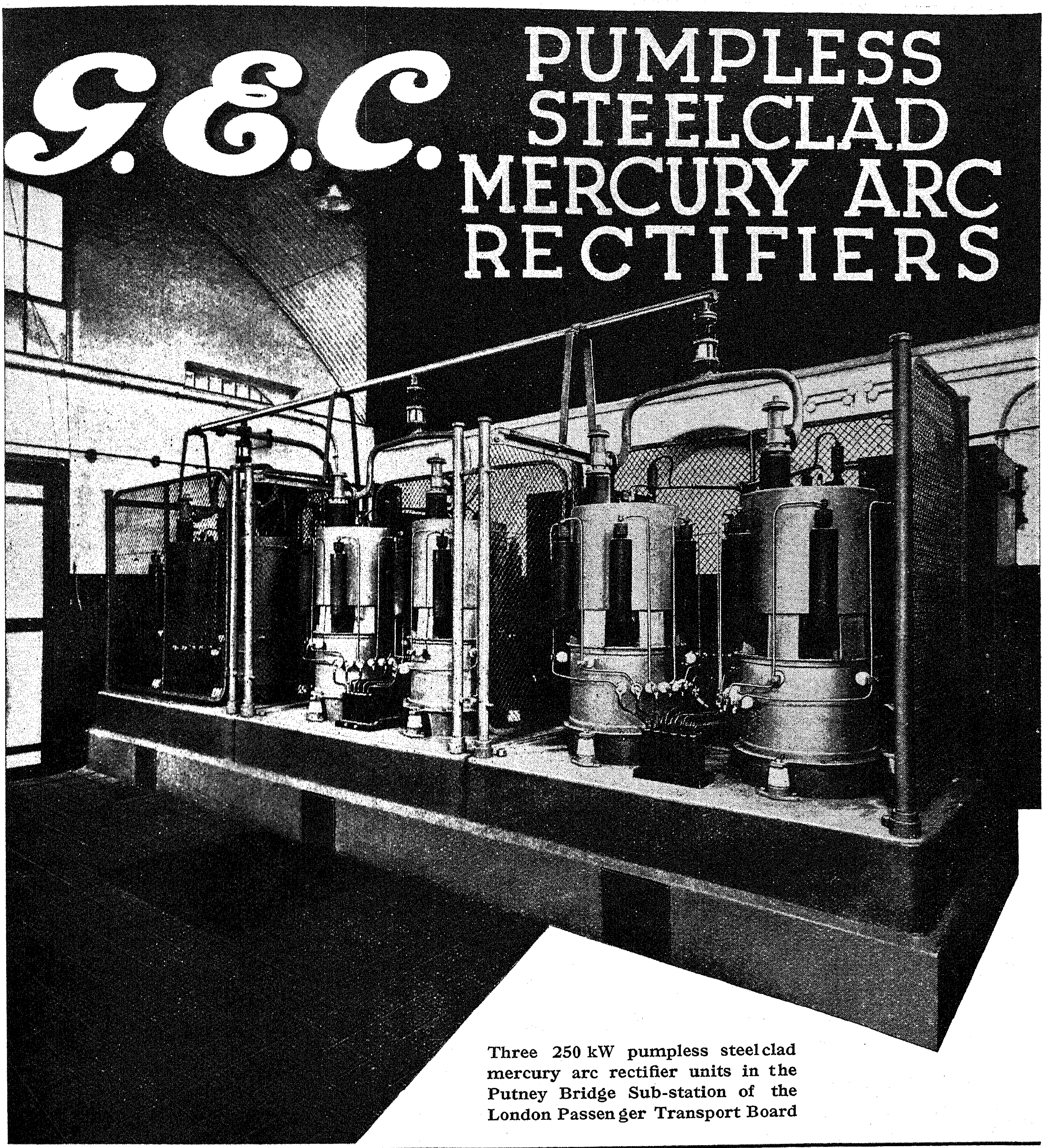

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tricity, the Electricity Commissioners approve of the construction and pattern of the meter (hereinafter described) for the measurement of electrical energy when supplied on the constant pressure, two-wire, single phase alternating current system, and known as the Ferranti, Type F.L. Single Phase Watthour Meter, rated sizes 2.5, 5, 10, 20 and 50 amperes, 230 volts at a frequency of 50 cycles per second.
Electricity Commission,
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10th October, 1934.
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Provided that the meter be
Electricity Commission,
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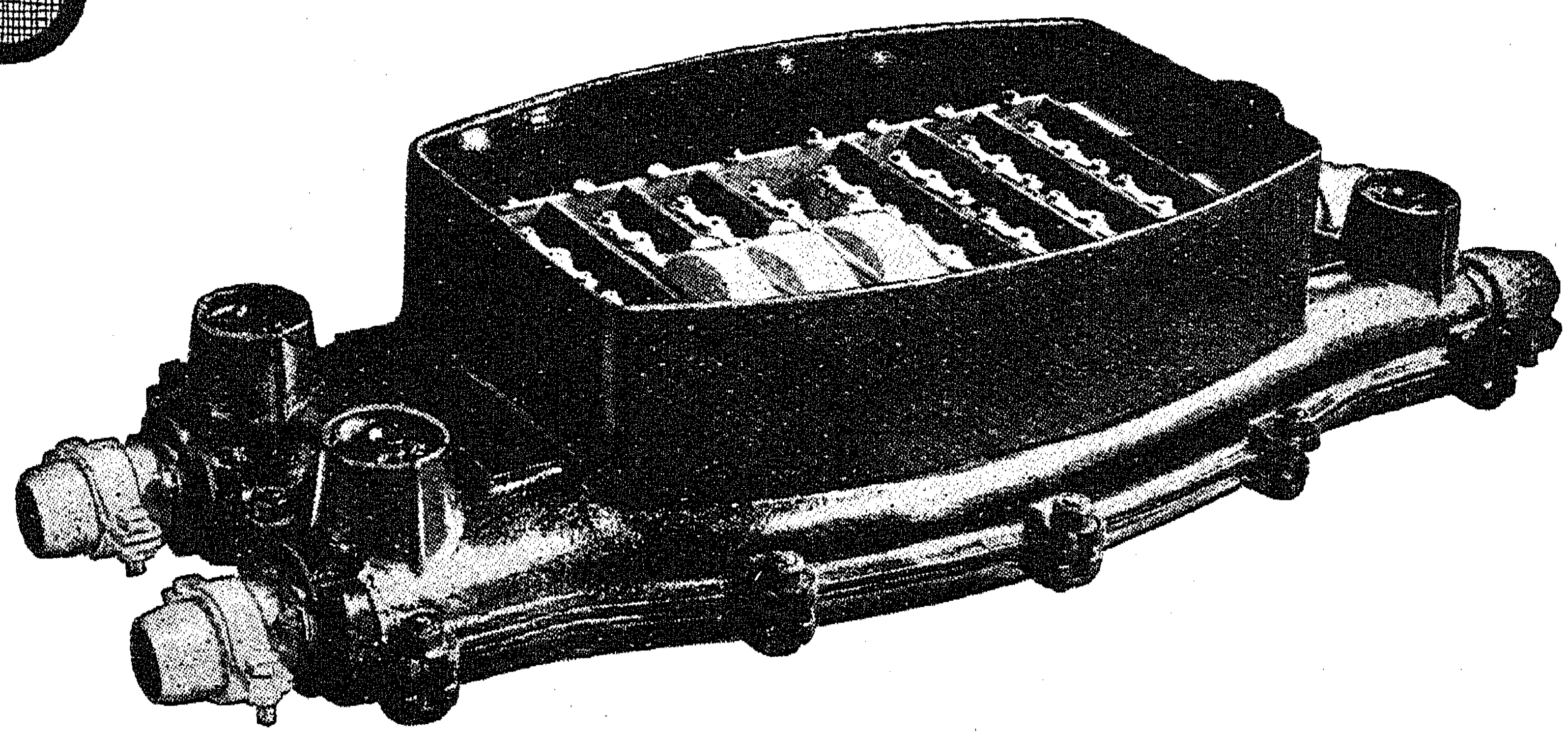

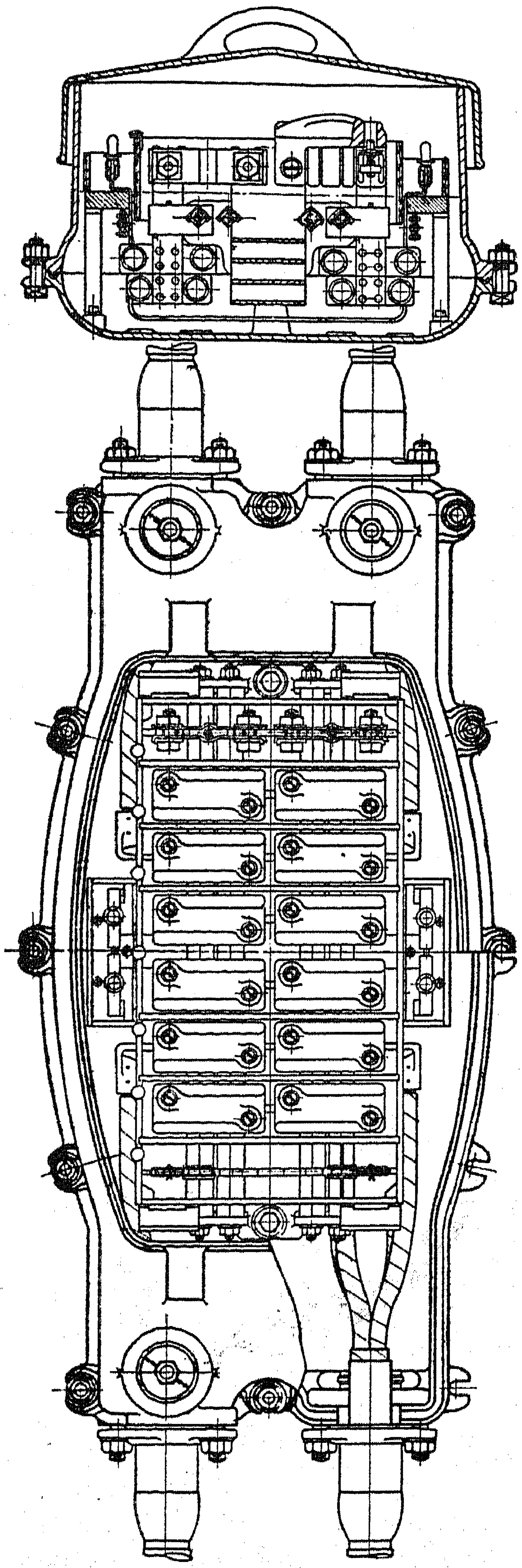

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